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STUDY OF AEROVISCOELASTIC SYSTEMS IN A PLATE MODELING USING DOUBLET LATTICE METHOD

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Abstract. *The aeronautical industry is always facing the flutter phenomena. The usefulness of flutter as a design metric leads the requirements for the optimization of aeroelastic performance. In order to achieve flutter suppression, the use of viscoelastic materials aims to control vibration amplitudes, in such a way to benefit the structural and aerodynamic performance. A plate wing model is used to investigate the influence of viscoelastic treatment in the aeroelastic response. The structural calculation is performed independently via the finite element method combined with Mindlin's theory, using the Fractional Derivative Model (FDM) for the viscoelastic layer. In addition, industry-standard tools are used to perform the unsteady aerodynamics calculations using the Doublet Lattice Method (DLM). A coupled scheme to assembly the aerodynamic and the structural viscoelastic effects are used in order to solve the aeroelastic model, via usual matching frequency methods. Finally, the influence of the viscoelastic treatment on the proposed structure will be evaluated as well the influence of environmental and operational parameters in the critical flutter velocity.*

Keywords: *flutter suppression, passive flutter control, aeroviscoelastic, doublet lattice method*

1. INTRODUCTION

The general concept of passive flutter suppression is not new. A large number of papers have been published on the subject. Commonly the techniques take advantage of the use of composite materials in order to benefit the aeroelastic performance, in a concept called Aeroelastic. Many authors have investigated the use of passive control using composite fiber orientation, tow steering, and variable fiber spacing in the suppression and reduction of flutter velocity (Stanford et al., 2014; Alyanak and Pendleton, 2016). On the other hand, viscoelastic materials are also commonly used in the industry to mitigate vibrations as a tool of passive control (Cunha-Filho et al, 2016). However, an application of these materials to the flutter phenomena is not numerous, and has been receiving relatively little attention. Many of the applications involving viscoelastic materials in flutter suppression are related of supersonic flutter of panels, generally using methodologies such as the Piston theory, to model the supersonic unsteady aerodynamics (Moon and Kim, 2001; Hasheminejad and Motaaleghi, 2015). Merret (2010) conducts an analytical investigation about the influence of viscoelastic treatment on the flutter velocity, in the time, and frequency domains. Cunha-Filho et al. (2016) explores the use of constrained viscoelastic layers in order to suppress the aeroelastic instability of flat panels by conveniently accounting the frequency and temperature dependent behavior of the viscoelastic material.

The use of viscoelastic materials has some advantages among the passive techniques, in order to be applied to real structures, such as inherent stability, effectively in broad frequencies range, moderate development and maintenance costs (Rao, 2003). Despite these advantages, the sensibility of these materials under distinct operational conditions must be studied to better understand the limitations for possible future applications.

Furthermore, most of aerodynamic modeling in the aeroviscoelastic system is convenient made using linearized and simplified techniques. Among these applications, the Piston theory was the broadly used due the mainly applications in supersonic flows. In the subsonic flows, another commonly used technique is the Strip theory, which gives the increment aerodynamic load necessary to account the flutter solution. Without doubt, the large used method to model the unsteady aerodynamics in an aeroelastic problem is the Doublet Lattice (Albano and Rodden, 1969). The doublet lattice method (DLM) is in used worldwide for flutter and dynamic responses of aircraft at subsonic speeds. As a well know tool in the industry, the DLM take advantages over another aerodynamics modeling, for the capacity to extend over multiple lift surfaces, reliability, and relatively low computational cost. The method allows the calculation of unsteady aerodynamics loads and is applicable to very general aircraft configurations. First introduced by Albano and Rodden (1969) and further refined by Rodden et al. (1999), the DLM is still extensively in used today, even among the high-fidelity methods of Computational Fluid Dynamics (CFD), due to the low computational cost and the advantages taken mainly on the preliminary design of aircraft.

The present work is focused on the use of DLM to give the unsteady aerodynamic load, using industrial tools (MSC NASTRAN), in order to solve the aeroviscoelastic system proposed. Despite the DLM is a common industrial technique to solve aeroelastic problems, few works have proposed applications of the method using viscoelastic materials. In this sense, the work explores the capabilities of the use of viscoelastic in flutter suppression, and the coupling of modified structural dynamics with the output aerodynamics matrices given by DLM.

2. MATHEMATICAL METHODOLOGY

According to Van Zyl (1998), the Doublet-Lattice Method (DLM) is a finite-element method for the solution of the oscillatory subsonic pressure-normalwash integral equation for multiple surfaces. It is one of the most used to study non-stationary aerodynamics that considers the fluid compressible. However, it is useful in preliminary phases of a project in combination with finite element methods, becomes effectively in estimate the flutter speed.

2.1 DLM Modeling

The DLM method can be expressed, as follow:

$$w(x,s)=\left(\frac{1}{8}\pi\right)\sum_{n=1}^{N}\iint_{S} K\left(x,\xi;s,\sigma\right)p\left(\xi,\sigma\right)d\xi d\sigma \tag{1}$$

Where w is the complex amplitude of dimensionless normalwash, p is the complex amplitude of lifting pressure coefficient, (x,s) are the orthogonal coordinates on the *n*th surface S, K is the complex acceleration potential kernel for oscillatory subsonic flow. The Kernel function has been given in the form (Rodden et al, 1972):

$$K=\exp\left(-i\omega x_o/U\right)\left(K_1T_1+K_2T_2\right)/r^2 \tag{2}$$

Where, ω is the frequency, x_o is the distance between the sending and receiving points parallel to the freestream, U is the velocity of the freestream:

$$T_1=\cos(\gamma_r-\gamma_s) \tag{3}$$

$$T_2=\left(1/r^2\right)\left(z_o\cos\gamma_r-y_o\sin\gamma_r\right)\left(z_o\cos\gamma_s-y_o\sin\gamma_s\right) \tag{4}$$

$$r=\left(y_o{}^2+z_o{}^2\right)^{1/2} \tag{5}$$

y_o and z_o are the Cartesian distances between the sending and receiving points perpendicular to the freestream, and γ_r and γ_s, are the dihedral angles at the receiving and sending points, respectively. The coordinate system is illustrated in Fig. 1 and to refer K_1 and K_2 as the planar and nonplanar parts of Kernel. So, as follow:

$$K_1=r\left(\partial I_o/\partial r\right) \tag{6}$$

$$K_2=r^3\left(\partial/\partial r\right)\left[\left(1/r\right)\partial I_o/\partial r\right] \tag{7}$$

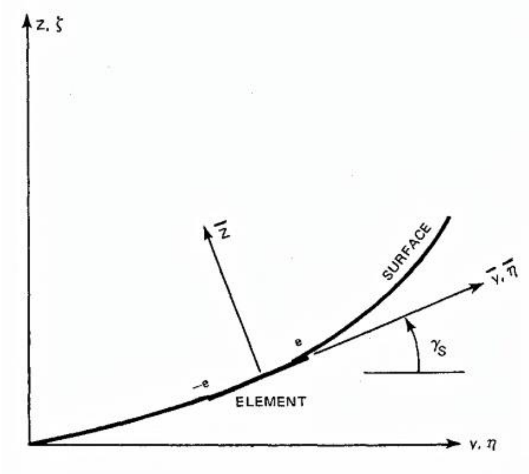


Figure 1. Lifting surface coordinate system (Rodden et al, 1972).

Where the integral I_o is:

$$I_o = \int_{u_1}^{\infty} \frac{\mathbf{exp}\left(-i\omega ru / U\right)}{\left(1+u^2\right)^{1/2}} du \tag{8}$$

Which:

$$u_1 = \left\{ M \left[{x_o}^2 + \left(1-M^2\right) r^2 \right]^{1/2} - x_o \right\} / \left(1-M^2\right) r \tag{9}$$

And M is the Mach number. Expanded forms for K_1 and K_2 are given by Landahl (1967).

The original method was based on the assumption that the lifting pressure could be concentrated along a line. So, the line is located at the ¼ chord line of the element and is formed by division of the boxes. The surface boundary condition is a prescribed normalwash at the control point the each box which is located at the ¾ chord point along the centerline of each box as illustrated in the Fig. 2.

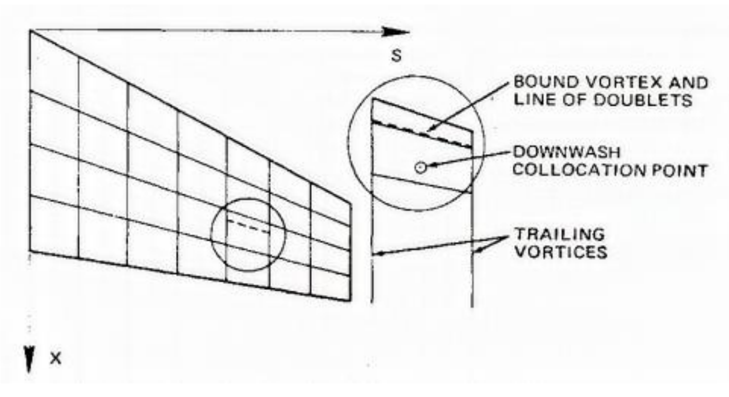


Figure 2. Lifting surface using trapezoidal boxes (Rodden et al, 1972).

2.2 Viscoelastic Modeling

The use of viscoelastic materials has been regarded as a convenient strategy in many types of industrial applications, that these materials can be applied either as discrete device or surface treatments (Nashif et al., 1985). Another interesting strategy consists in incorporating viscoelastic materials as a means of adding damping to vibration neutralizes (Espindola and Bavastri, 1997). As a result, it is currently possible to perform finite element modeling of complex real-world engineering structures such as automobiles, airplanes, communication satellites, buildings and space structures.

According to the linear theory of viscoelastic (Christensen, 1982), the one-dimensional stress-strain relation can be expressed, in Laplace domain, as follows:

$$\boldsymbol{\sigma}(s) = \left(G_r + H(s)\right)\boldsymbol{\varepsilon}(s) \tag{10}$$

In the eq. 10, G_r is the *static modulus*, representing the elastic behavior and $H(s)$ is the *relaxation function*, associated to the dissipation effects. When evaluated the imaginary axis of the s-plane $s = (i\omega)$, the eq. 10 can be expressed under the following form (Lima and Rade, 2005):

$$G(\omega) = G'(\omega) + iG''(\omega) = G'(\omega)\left(1 + i\eta(\omega)\right) \tag{11}$$

Where $G'(\omega)$, $G''(\omega)$ and $\eta(\omega) = G''(\omega)/G'(\omega)$ are the *storage* and *loss moduli* and *loss factor*, respectively. The complex modulus provides a straightforward model the viscoelastic behavior in the frequency domain.

The properties of viscoelastic system depend on of environment conditions. So, the temperature is usually considered to be single most important environmental factor which exerts influence upon the properties of viscoelastic materials. Then, it becomes important to taking into account for temperature variations in the modeling of viscoelastic stiffness. This can be done by making use of the called Frequency Temperature Superposition Principle – FTSP, which establishes a relation between the effects of the excitation frequency and temperature on the properties of the thermorheologically viscoelastic materials (Nashif et al, 1985).

2.3 Aerodynamic Modeling

The aerodynamic model must be capable of interacting effectively with the structure to be able to calculate the aeroviscoelastic behavior of the system. So, it is necessary to generate the normalwash distribution for a single deformation that is given for the structural mesh. In addition, the effect of aerodynamic forces applied in the structural model should be computed to the system. Finally, has been desired the model in the frequency domain to analyzed the aeroviscoelastics interactions.

The first step to the modeling is to make the connection between structural and aerodynamic meshes. In order to understanding it has been considered a surface with only four boxes, such as Fig. 3.

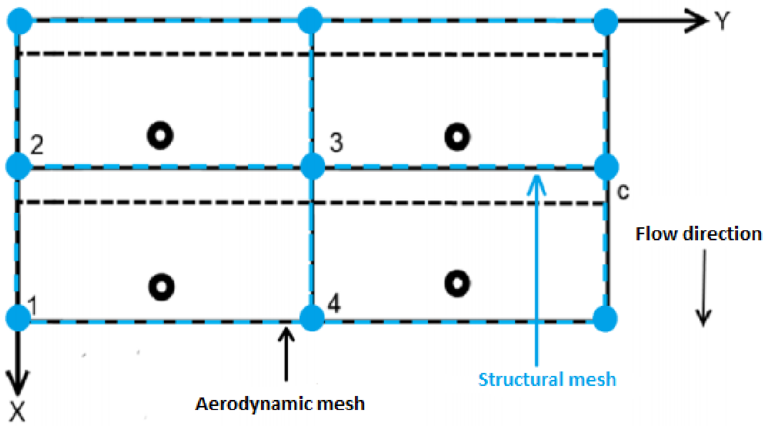


Figure 3. Example of aerodynamic mesh 2x2 with an structural mesh the same size.

$$M_{hh}\ddot{\boldsymbol{\eta}} + C_{hh}\dot{\boldsymbol{\eta}} + K_{hh}\boldsymbol{\eta} = F_m \tag{12}$$

Where M_{hh}, C_{hh} and K_{hh} .represents the mass, damping coefficient and the frequency-and-temperature dependent viscoelastic stiffness, respectively. So, K_{hh} is the sum of elastic and viscoelastic stiffness and applied it in the structural system. The F_m is the unsteady aerodynamic force vector in modal coordinates. The second step has been the built of the spline based on the existing structural mesh that is responsible for the connection of the both meshes which is illustrated by Fig. 4.

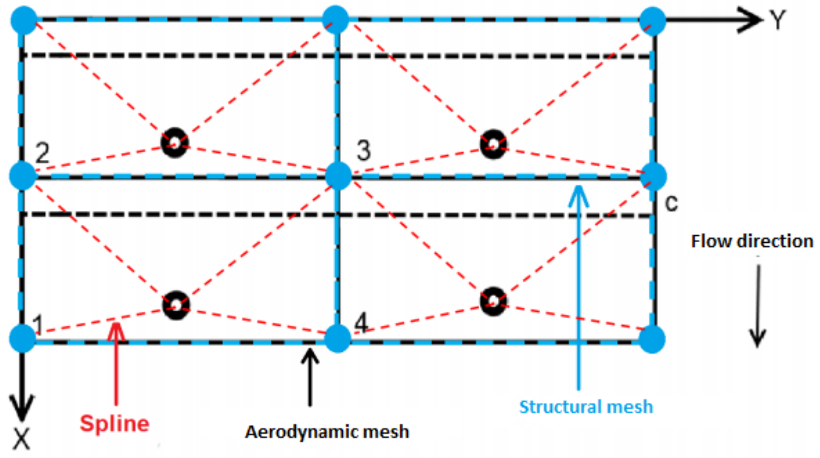


Figure 4.Example of spline construction.

The aerodynamic strength in the modal domain can be written as follow:

$$F_m = \frac{1}{2}\rho_{ar}U_{\infty}^{2}\phi^T T_{as}^{\ T} AIC(k)S(D_r + ikD_I)T_{as}\phi\eta \tag{13}$$

Where S is the matrix of aerodynamic integration that in this case thin plate that is composed of the panel area. For D_r is the matrix the amplitudes of inclinations, D_l is the matrix of the displacements, h is the model of each panel. Finally, the AIC represents the aerodynamic behavior, which has been obtained by the DLM equations.

To simplified, the eq. 13 can be rewritten in function of the generalized aerodynamic matrix Q, as follow:

$$F_m = \frac{1}{2}\rho_{ar}U_{\infty}^{2}\left[Q(k)\right]\{\eta\} \tag{14}$$

Where, $Q(k) = \phi^T T_{as}^{\ T} AIC(k)S(D_r + ikD_I)T_{as}\phi$.

So, ϕ is the eigenvalue of the system and $T_{as} = T_{interpol}T_{spline}$, that each of the red connections of the spline mesh is rigid. The purpose of the spline is to transform the movement of all degrees of freedom of a structural mesh in pure motion of elevation of the nodes connected in the spline. In this way, has been assumed this transformation is represented by the matrix T_{spline} (Kotikalpudi et al, 2015).

Finally, the eq. 12 can be written as:

$$\left(-\omega^2 M_{hh} + i\omega C_{hh} + K_{hh}\right)\{\eta\} = \frac{1}{2}\rho_{ar}U_{\infty}^{2}\left[Q(k)\right]\{\eta\} \tag{15}$$

Therefore, to evaluate the stability of the system it is necessary evaluate the eigenvalues of the system. Then, when the imaginary part of this becomes positive, is an unstable system. However, the Q is a complex matrix, that makes the analysis of stability more complicated, because the reduced frequency of the system depending on the natural frequencies. This means that the problem of the eigenvalues to be solved is nonlinear. Finally, some methods have been developed to solve the problem of nonlinear eigenvalues associated with aeroelastic or aeroviscoelastic stability. (Santos et al, 2005).

2.4 Discretization based on Finite Element

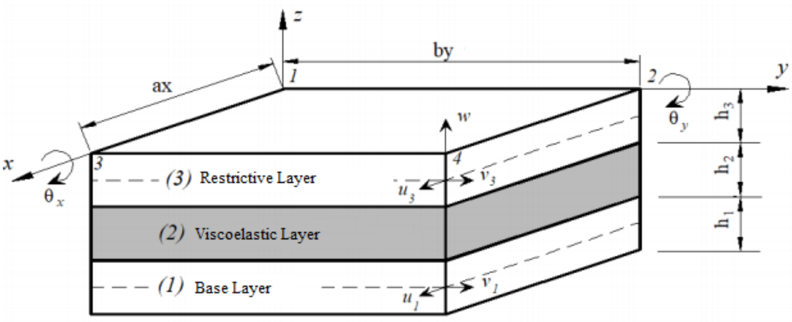


Figure 5. Illustration of a three layer board element (Lima, 2007).

For this paper, has been applied a flat rectangular plate with four nodes and seven degree of freedom for each elementary node, according to developments by Lima et al. (2010). Thus, the degree of freedom are: longitudinal displacement of upper constraining layer u_1 and v_1; displacement of base layer u_3 and v_3; transverse deflection w; rotations $\boldsymbol{\theta}_x = \partial w/\partial x$ and $\boldsymbol{\theta}_y = \partial w/\partial y$. The terms a and b , represent, respectively, the dimensions of the element in the direction x and y . So, the vector of nodal degrees of freedom can be showed:

$$u_{(e)_j}(t) = \left(u_{1_j} \;\; v_{1_j} \;\; u_{3_j} \;\; v_{3_j} \;\; w_j \;\; \boldsymbol{\theta}_{x_j} \;\; \boldsymbol{\theta}_{y_j}\right)^T \quad j = 1,\ldots,4 \tag{16}$$

So, the vector of degrees of freedom at elementary level:

$$u_{(e)}(t) = \left(u_{(e)_1} \;\; u_{(e)_2} \;\; u_{(e)_3} \;\; u_{(e)_4}\right)^T \tag{17}$$

The field of longitudinal and transverse displacement inside of each finite element is interpolated according to:

$$U(x,y) = N(x,y)u_{(e)}(t) \quad 0 \le x \le a \,;\; 0 \le y \le b \tag{18}$$

Where, $U(x,y) = \left(u_1 \;\; v_1 \; u_3 \; v_3 \; w \; \boldsymbol{\theta}_x \;\; \boldsymbol{\theta}_y\right)^T$ and $\mathbf{N}(x,y) = \left(N_{u_1} \;\; N_{v_1} \; N_{u_3} \; N_{v_3} \; N_w \; N_{\boldsymbol{\theta}_x} \;\; N_{\boldsymbol{\theta}_y}\right)^T$ is the interpolation of the functions.

It can be defined the vector of displacements:

$$\boldsymbol{\varepsilon}(x,y,z,t) = B(x,y,z)u_{(e)}(t) \tag{19}$$

Finally, $B(x,y,z)$ is the matrix has been obtained by applying the differential operators of the relations strain-tension in the matrix containing the shape functions $N(x,y)$. The equation of motion has been acquired through the Lagrange equation Thus, it is necessary to formulate the expressions of the kinetic and deformation energies of the sandwich plate formed by three layers. (Lima, 2007)

3. NUMERICAL RESULTS

The results have been obtained from the MATLAB ® programming environment. In order to understand the influence of the viscoelastic damping in the flutter phenomenon has been assessed a three-layer viscoelastic sandwich plate: an aluminum base layer, a viscoelastic layer of the 3M ISD112 and restrictive layer of aluminum. The geometrics proprieties and of each layer are described in tab.1 and tab.2, respectively.

Table 1.Geometrics proprieties on the viscoelastic sandwich plate.

Layer	Length (mm)	Width (mm)	Thickness (mm)
Base	300	500	1,5000
Viscoelastic (1)	300	500	0,0254
Restrictive	300	500	0,5000

(1) Operating temperature has been considered at 27º C.

Table 2. Mechanical proprieties on the viscoelastic sandwich plate.

Layer	Elasticity Module (GPa)	Poisson Coefficient	Density (kg/m^3)
Base	70	0,34	2700
Viscoelastic (1)	-	0,49	950
Restrictive	70	0,34	2700

(1) Operating temperature has been considered at 27º C.

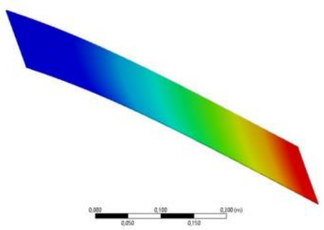
3.1 Verifying of structural modeling without viscoelastic layer

Initially, it is necessary to verify the natural frequencies of the implemented code in MATLAB ® and the similar in the commercial program ANSYS ®. It has been obtained the four initial mode of vibration that illustrated in tab. 3.

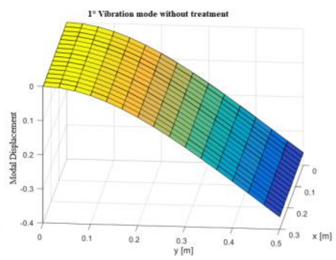
Table 3.The comparison of the natural frequencies has been implemented in MATLAB ® with the commercial program ANSYS.

Mode	ω_n (ANSYS ®)	ω_n (MATLAB ®)
1	5,16	5,25
2	18,58	18,58
3	32,07	32,90
4	62,08	62,50

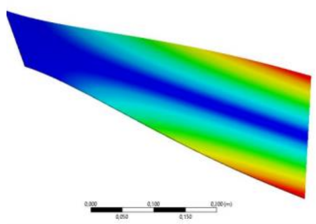
In addition, it is necessary to compare the mode shape to validate the code. So, has been considered only the four plate vibrate modes because the flutter phenomenon is usually associated with the lower frequencies. The comparison of the modal shape is illustrated in fig. 6.



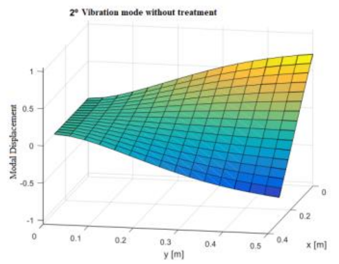
a) 1º vibration mode using the commercial program ANSYS ®.



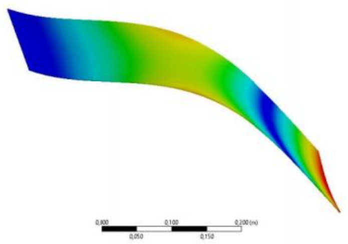
b) 1º vibration mode using the implemented code in MATLAB ®.



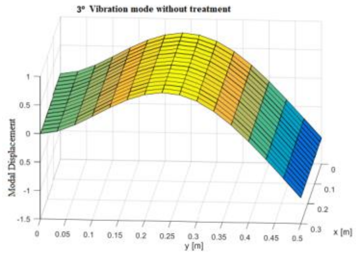
c) 2º vibration mode using the commercial program ANSYS ®.



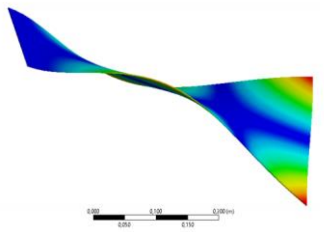
d) 2º vibration mode using the implemented code in MATLAB ®.



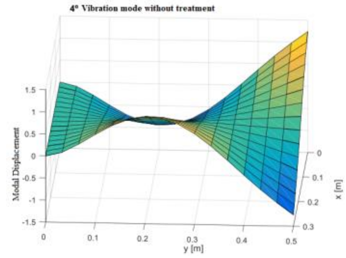
e) 3º vibration mode using the commercial program ANSYS ®.



f) 3º vibration mode using the implemented code in MATLAB ®.



g) 4º vibration mode using the commercial program ANSYS ®.

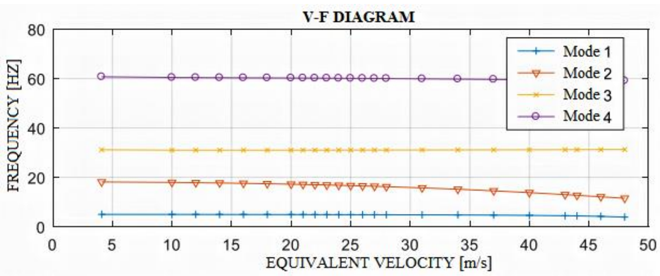


h) 4º vibration mode using the implemented code in MATLAB ®.

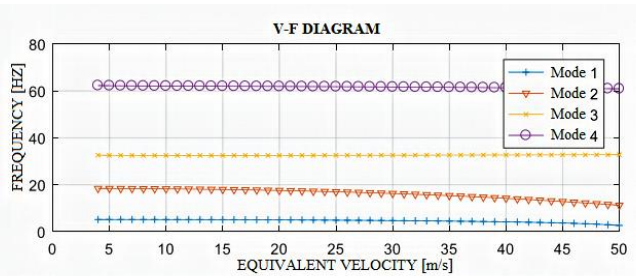
Figure 6. Comparison of the mode shape: a,c,e,g) commercial program ANSYS® and b,d,f,h) implemented code in MATLAB ®.

3.2 Verifying of aeroelastic modeling without viscoelastic layer

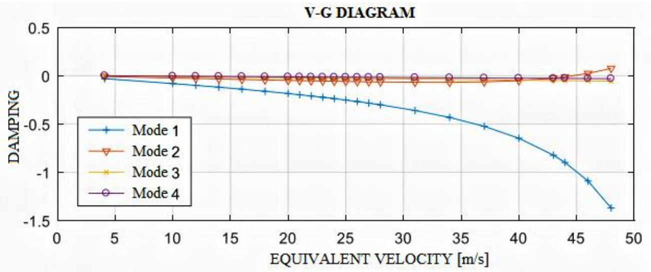
It is necessary two evaluations to predicting the flutter points: a) using the solution 145 of the commercial program NASTRAN ® that taken into account the flutter velocity trough V-G diagram and b) the non-stationary aerodynamic loading has been acquired in NASTRAN ® with the finite element structure implemented in environment of MATLAB ®. The validation can be seen illustrated in the fig. 7.



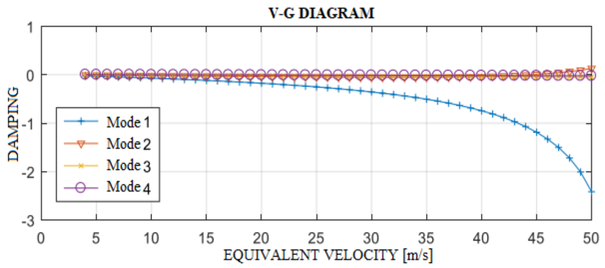
a) V-F diagram has been obtained with solution 145 of the NASTRAN ® using P-K method for solution.



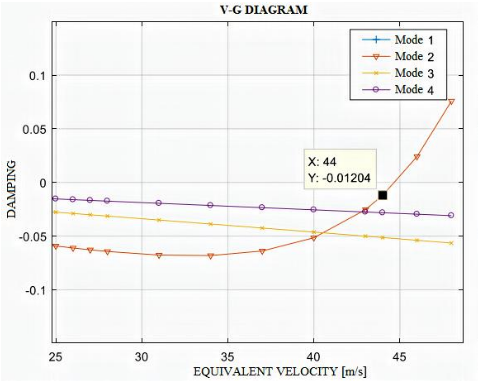
b) V-F diagram has been obtained with the aerodynamic method of NASTRAN ® and implemented code in MATLAB ®.



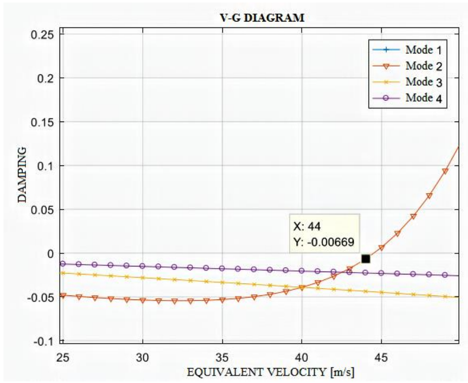
c) V-G diagram has been obtained with solution 145 of the NASTRAN ® using P-K method for solution.



d) V-G diagram has been obtained with the aerodynamic method of NASTRAN ® and implemented code in MATLAB ®.



e) Detail of V-G diagram between the velocities of 25 m/s and 50 m/s, has been obtained with solution 145 of the NASTRAN ® using P-K method for solution.



f) Detail of V-G diagram between the velocities of 25 m/s and 50 m/s, has been obtained with the aerodynamic method of NASTRAN ® and implemented code in MATLAB ®.

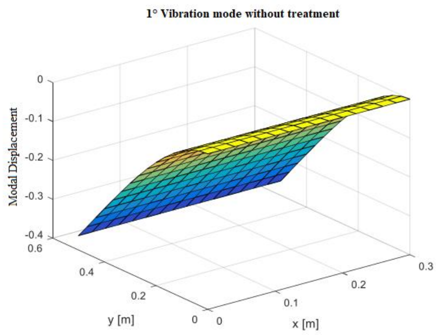
Figure 7. V-G diagram: a,c,e) It has been obtained with solution 145 of the NASTRAN using P-K method for solution and b,d,f) Numerical finite element structure implemented in MATLAB ® joined with commercial program NASTRAN ®.

In the fig. 6, it is observed that the critical flutter velocity is $44\,m/s$ in both cases. However, in fig. 7e and 7f, can be noted the damping is $-0,01204$ and $-0,00669$, respectively. This difference in manly caused by the interpolation function. In Figures 7a and 7b it is also possible to note that the frequency of the second mode has an increase, as long as the frequency of the third mode decrease, this represents the coalescence of these two modes that with the increase of the velocity can cause the flutter phenomenon.

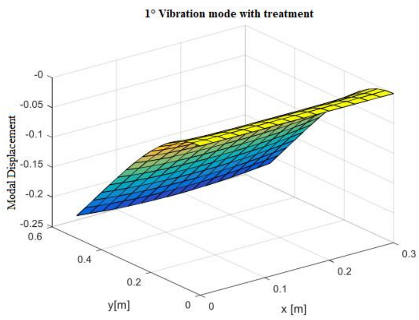
Therefore, it has been showed the efficiency of the strategy used in this approach to evaluate the aeroelastic modeling of viscoelastic systems. In this way, the technique allows to feed the viscoelastic structural model generated in MATLAB ® with the aerodynamic generated on NASTRAN ®.

3.3 Influence of aeroelastic modeling with viscoelastic layer

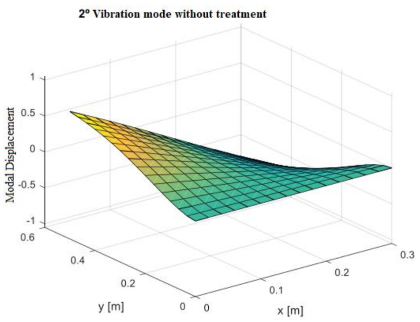
So, it has been necessary to verify the modeling of the sandwich plate containing viscoelastic material The addition of viscoelastic materials increase the damping of the mechanical system and because of the its dissipative properties, these materials has been used in many function of passive vibration control. The fig. 8 below demonstrates their efficiency, since it is possible to notice the decrease of the modal amplitude in all examples in fig. 8 analyzed. It can be visualized the variation of the phase between the degrees of freedom of a system, but without changing of mode shape.



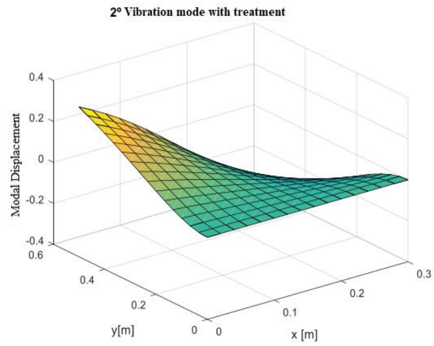
a) 1º vibration mode of the structure without treatment.



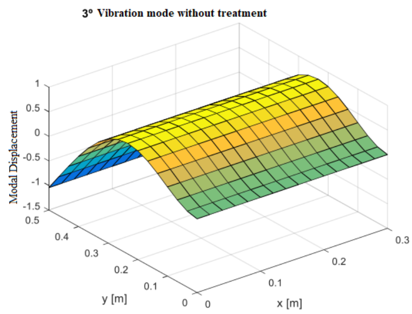
b) 1º vibration mode of the structure with viscoelastic treatment.



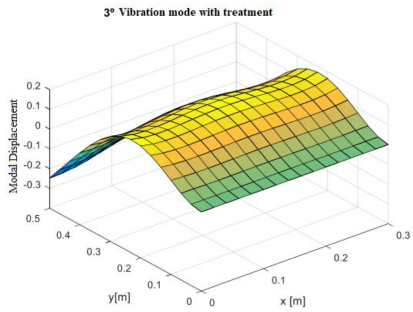
c) 2º vibration mode of the structure without treatment.



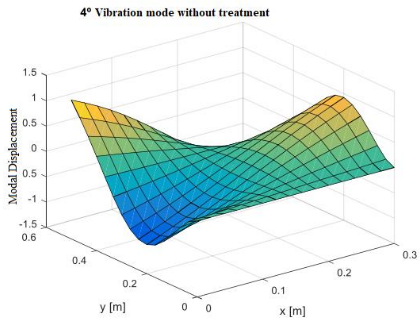
d) 2º vibration mode of the structure with viscoelastic treatment.



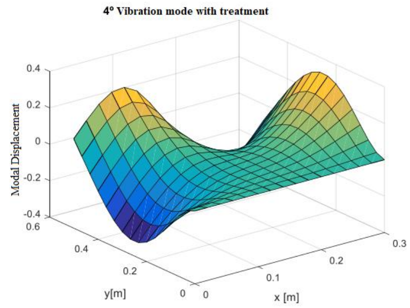
e) 3º vibration mode of the structure without treatment.



f) 3º vibration mode of the structure with viscoelastic treatment.



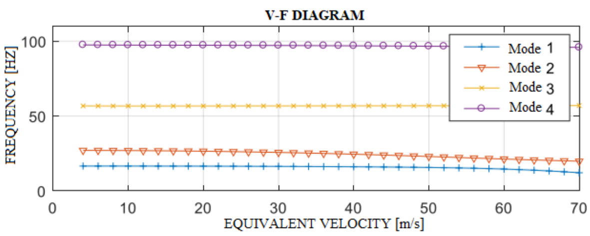
g) 4º vibration mode of the structure without treatment.



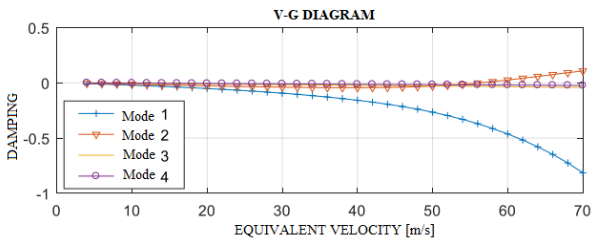
h) 4º vibration mode of the structure with viscoelastic treatment.

Figure 8.Camparison of the structure mode: a,c,e,f) without viscoelastic treatment and b,d,f,h) with viscoelastic treatment.

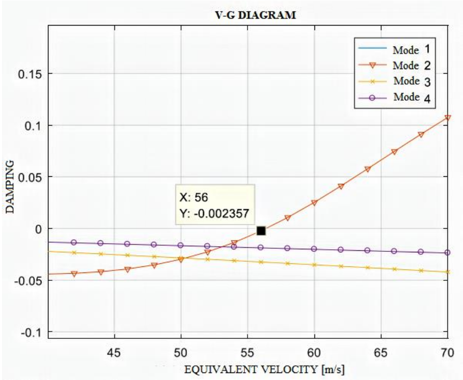
In the fig. 7, can be seen the critical flutter velocity without viscoelastic layer was approximately $44\,m/s$. On the other hand, the fig. 9 can be noticed the efficiency of the viscoelastic treatment in the suppression of the flutter phenomenon. It has been calculated the critical flutter velocity of the $56\,m/s$. It represents a 27% increase in the flutter velocity of the system.



a) V-F diagram to sandwich plate.



b) V-G diagram to sandwich plate.



c) Detail of V-G diagram between the velocities of 40 m/s and 70 m/s to sandwich plate.

Figure 9. V-G diagram to sandwich plate considering the viscoelastic treatment.

4. CONCLUDING REMARKS

In the present work, has been showed a contribution to the recently development of the study about viscoelastic treatment in dynamics systems that have been submitted in the flutter phenomenon, consequently, the passive control of it.

Considering the results of the simulations have been carried out with sandwich plates containing viscoelastic damping, it has been proven that the viscoelastic materials have feasibility for use in aeronautical panels with the objective of delaying the occurrence of the flutter phenomenon. In this case, such materials may present an alternative to the application of existing structures for the suppression of flutter phenomenon, because it is a passive control of easy implementation in the structure and low cost of maintenance.

Another factor that must be taken into account in the design of aeroviscoelastic systems is the operating temperature, since it usually extreme variations of the temperature during the flight and can have a direct impact on the performance of the superficial viscoelastic treatment. In general, increasing the operating temperature of the aeroviscoelastic system leads to a decrease the efficiency of it in terms of the attenuation of the critical flutter velocity

Therefore, in this work the first part refers to the associated conservative system which showed the inclusion of the static stiffness layer and the second part the insertion of the dynamic viscoelastic stiffness increase the flutter velocity of approximately 27%. Then, it is believed that this strategy can be advantageously used for the studies about passive control of flutter phenomenon.

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