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ATTITUDE AND POSITION CONTROL OF TILTING ROTOR MULTI-COPTER

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Abstract. *In this paper, the dynamic model of a tilting rotor multi-copter, with n motors containing vectoring thrust mechanisms on lateral and longitudinal directions, is presented. Once standard configurations of multi-copters (rotor fixed) are classified as underactuated systems, their number of control inputs are insufficient to allow position and attitude control independently. Hence, the addition of tilt mechanisms in the rotors arm increases the number of control inputs enabling the system to perform hover flight in different equilibrium conditions. In this work, the aircraft equations of motion are derived using Newton-Euler formulation assuming that each rotor is capable of tilting in two independent directions, laterally and longitudinally w.r.t. its arm. Later, the obtained dynamical model is written as a Linear Time Invariant (LTI) system in the state-space format so that a control law based on modern control techniques is proposed and designed allowing the vehicle to track a desired position for a given attitude condition. Finally, the derived dynamical model and the control design are verified via numerical simulations.*

Keywords: *Multi-copter, Tilt-rotor, Modern Control, Attitude Control*

1. INTRODUCTION

Nowadays, the Unmanned Aerial Vehicles (UAVs), popularly known as drones, are recognized by their versatility and large potential application either on military or civilian missions. Considering the large group of UAV aircraft configurations, the multi-rotor helicopters are included on the Vertical Take-off and Landing (VTOL) aircraft category and have, as remarkable advantages, the ability to perform hovering flights, the capacity of miniaturization and better maneuverability mainly in restricted space environments (Bouabdallah, 2007).

Essentially, on standard multi-copters arrangement, especially quad-rotors, the attitude and position control relies on the propeller angular speed variation of each motor propulsive group. Further, once the number of control inputs is lower than the number of state variables, the system is classified as underactuated (Badr *et al.*, 2016). As a consequence, it is not possible to control position and orientation independently so that the multi-copter is only able to perform hovering flights when its attitude (roll and pitch) angles are zero. In other terms, one can state that the system has only one equilibrium point represented by the aircraft holding an horizontal position w.r.t. the inertial coordinate frame.

On investigating further the controllability conditions of multi-copter UAVs, many authors have designed new actuation mechanisms in order to solve the underactuation issue concomitantly with control actuation optimization, better maneuverability and increase the versatility for specific tasks. One of the adopted actuation strategies consists in the vectorization of the thrust force produced by each motor-propulsive group. This novel actuation feature enables the motor to tilt w.r.t the multi-rotor's arm and can be used to decouple attitude and position control, allowing the aircraft to hover on a non-horizontal position or/and shift its position without changing its orientation.

The autonomous flight control application on tilt rotor multi-copters has been the research object of previous works. For instance, Ryll *et al.* (2012), Hua *et al.* (2014) and Oosedo *et al.* (2016) investigated a quadrotor UAV whose rotor set axes is able to rotate laterally about the longitudinal arm axis using actuators attached on its extremity. Similarly, Nemati and Kumar (2014) and Badr *et al.* (2016) propose a multi-copter design where the motor set is capable of tilting longitudinally w.r.t the multi-copters arm. On regarding the vectoring capability of the tilting mechanisms, either lateral or longitudinal, their application can be extended to bi-copters (Kendoul *et al.*, 2005), tri-copters (Servais *et al.*, 2015; Salazar-Cruz *et al.*, 2009) and fixed wing VTOL configurations as in (Vuruskan *et al.*, 2014) to guarantee stability, allow the vehicle to be controlled, perform autonomous flights and execute transition maneuvers for vertical to horizontal flight.

Considering the potential applications of tilt mechanisms on UAVs, this present work intends to evaluate the dynamical behavior of multi-copters endowed with vectoring thrust mechanisms exploring their capability to perform hover flight for non-zero attitude angles. The formulations will consider a model with *n* rotors with the capability to rotate laterally and longitudinally w.r.t. to its arm. Furthermore, a feedback control law is proposed to allow the system to track a desired attitude equilibrium point. Lastly, the model is appraised via Matlab/Simulink simulations emphasizing the system stability at different equilibrium points.

2. Dynamical Modelling

This section presents the mathematical formulation which represents the multi-copter motion. The dynamical model is later used for control design and system numerical simulation purposes.

2.1 Reference Frames and Kinematic Relations

Before deriving the dynamical model of the multi-copter, the reference frames must be identified. In this work, three different reference frames will be used in order to model the aircraft dynamics coupled with the rotor tilting. The first is an Earth fixed reference frame designated by $ICS : \{O_E; x_E; y_E; z_E\}$ used to represent the absolute position of the aircraft in space. The second, is a body fixed coordinate frame attached to the aircraft, capable of translating and rotating with the body is denoted by $BCS : \{O_B; x_B; y_B; z_B\}$. The origin of the body fixed reference frame coincides with the multi-rotor center of gravity (CG), and its linear and angular velocity vectors about x_B, x_B and z_B axes are denoted by $\vec{v} = \begin{bmatrix} u & v & w \end{bmatrix}^T$ and $\vec{\omega} = \begin{bmatrix} P & Q & R \end{bmatrix}^T$, respectively.

The aircraft attitude or kinematic relations between the *ICS* and can be defined with respect to the *BCS* using the Euler angles notation, which are represented by $\vec{\Theta} = \begin{bmatrix} \phi & \theta & \psi \end{bmatrix}$ corresponding to the roll, pitch and yaw angles, respectively. It must be remarked that $\vec{\omega} \neq \dot{\vec{\Theta}}$ since the $\vec{\omega}$ vector points in the rotation axis direction, while $\dot{\vec{\Theta}}$ only represents the time derivative of the attitude angles. However, these two vectors are correlated by the following kinematic relationship (Roskam, 2001):

$$\vec{\omega} = \begin{bmatrix} 1 & 0 & -\sin\theta \\ 0 & \cos\phi & \cos\phi\cos\theta \\ 0 & -\sin\phi & \cos\theta\cos\phi \end{bmatrix} \dot{\vec{\Theta}} \tag{1}$$

Any vector represented on the body-fixed frame ($\vec{r}_{BCS}$) is interrelated with the inertial coordinate frame ($\vec{r}_{ICS}$) using Euler angles matrix transformation defined by a rotational matrix (R_B^I) (Valavanis, 2007).

$$\vec{r}_{ICS} = R_B^I \cdot \vec{r}_{BCS} \tag{2}$$

with $R_B^I = R_B^I(\psi)R_B^I(\theta)R_B^I(\phi)$, being $R_B^I(\psi)$, $R_B^I(\theta)$ and $R_B^I(\phi)$ rotation matrices around z_E, y_E and x_E axis, respectively.

One must notice that the R_B^I matrix considers a sequence of rotations on the following order: yaw (ψ), pitch (θ) and roll (ϕ). Since R_B^I is orthogonal, its inverse is equivalent to its transpose. Thus, any vector on the *ICS* can be written on the *BCS* by multiplying $\{R_B^I\}^T$ and the body-coordinate vector ($\vec{r}_{BCS}$).

The motor reference frame MCS: $\{O_{Mi}; x_{m_i}, y_{m_i}, z_{m_i}\}$, $i = 1 \ldots n$ illustrated in Fig.1 is associated to each of the i^{th} propulsive set, with x_{m_i} and y_{m_i} representing the lateral and longitudinal tilting actuation axes, respectively. The z_{m_i} axis is normal to the propeller disc spinning plane and is coincident with the produced thrust force direction (Ryll *et al.*, 2012). Each motor propulsive set is installed at the extremities of the UAV‘s arms at point P_i by an angle $\gamma_i = \frac{\pi}{n}(2i-1)$, $i = 1...n$ w.r.t the x_B axis direction of the *BCS*. Also, similarly as the kinematic relations between the BCS and ICS as function of the aircraft attitude (Euler angles), once the *MCS* translate with the aircraft, the orientation of the *i-th* propeller can be represented on the *BCS* using the following rotation matrices:

$$R_M^B(\gamma_i) = \begin{bmatrix} \cos\gamma_i & -\sin\gamma_i & 0 \\ \sin\gamma_i & \cos\gamma_i & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{3}$$

$$R_M^B(\alpha_i) = \begin{bmatrix} \cos\alpha_i & 0 & \sin\alpha_i \\ 0 & 1 & 0 \\ -\sin\alpha_i & 0 & \cos\alpha_i \end{bmatrix} \tag{4}$$

$$R_M^B(\beta_i) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\beta_i & -\sin\beta_i \\ 0 & \sin\beta_i & \cos\beta_i \end{bmatrix} \tag{5}$$

where α_i and β_i represents the longitudinal and the lateral tilt angles about x_{m_i} and y_{m_i}, respectively.

The rotation matrix R_M^B gives the coordinate transformation from MCS to BCS and is composed by the multiplication of Eqs. (3), (4) and (5):

$$\vec{r}_{BCS} = R_M^B(\gamma_i) \cdot R_M^B(\alpha_i) \cdot R_M^B(\beta_i) \cdot \vec{r}_{MCS} \tag{6}$$

2.2 Equations of Motion

Similarly the flight dynamics concepts applied to fixed wing aircraft as in Valavanis (2007), the dynamical mathematical formulation of the multi-copter is obtained using Newton-Euler formulation for a 6 degree of freedom rigid body. The equations of motion are derived in the body coordinate frame $BCS : \{O_B; x_B; y_B; z_B\}$ presented in Fig. 1, therefore, the set of equations that describes the vehicle motion are given by:

$$\begin{bmatrix} \sum F_x \\ \sum F_y \\ \sum F_z \end{bmatrix} = m \begin{bmatrix} \dot{u} - Rv + Qw \\ \dot{v} - Pw + Ru \\ \dot{w} - Qu + Pv \end{bmatrix} \tag{7}$$

$$\begin{bmatrix} \sum M_x \\ \sum M_y \\ \sum M_z \end{bmatrix} = \begin{bmatrix} J_{xx}\dot{P} + QR\,(J_{zz} - J_{yy}) - J_{xz}(\dot{R} + PQ) + J_{yz}(R^2 + Q^2) + J_{xy}(\dot{Q} + PR) \\ J_{yy}\dot{Q} + PR\,(J_{xx} - J_{zz}) + J_{xz}(P^2 + Q^2) + J_{xy}(\dot{P} + RQ) + J_{yz}(\dot{R} + PQ) \\ J_{zz}\dot{R} + PQ\,(J_{yy} - J_{xx}) + J_{xz}(QR + \dot{P}) + J_{yz}(\dot{Q} - PR) + J_{xy}(Q^2 - R^2) \end{bmatrix} \tag{8}$$

where $\vec{F} = \begin{bmatrix} F_x & F_y & F_z \end{bmatrix}$ and $\vec{M} = \begin{bmatrix} M_x & M_y & M_z \end{bmatrix}$ are the external forces and moments applied on the vehicle's the center of mass. J_{xx}, J_{yy}, J_{zz},, J_{xy} J_{yz} and J_{xz} are the body's rotational inertia matrix elements with respect to the BCS. It must be recognized that most of airplanes are symmetrical about $x_b z_b$ plane and, in this case, it follows that: $J_{xy} = Jyz = 0$ (Roskam, 2001). Multi-copters, specially quad-copters, generally have two planes of symmetry, thus $J_{xz} = 0$ is also satisfied.

For a standard multi-copter, the external forces acting on the vehicle while flying are mainly composed of: thrust, drag and gravitational force, while the external moments are due the motor generated thrust unbalance, the resultat gyroscopic effect of the spinning propellers and the propeller drag torque. Hence, the left hand side of Eqs. (6) and (7) is given by:

$$\vec{F} = \vec{F}_T^{BCS} + \vec{F}_D^{BCS} + \vec{F}_{grav}^{BCS} \tag{9}$$

$$\vec{M} = \vec{\tau}_T + \vec{\tau}_{gyro} + \vec{\tau}_{fan} \tag{10}$$

Following Marques (2018) formulation to describe the acting forces, the thrust is considered as proportional to the propeller angular speed and normal to the propeller blade disc acting in z_m direction of the MCS. Thus, the propeller generated thrust can be written in the BCS using the matrix transformation in Eq. (6) which leads to:

$$\vec{F}_T^{BCS} = \begin{bmatrix} \sum_{i=1}^{n}(\sin(\gamma_i)\sin(\beta_i) + \cos(\gamma_i)\sin(\alpha_i)\cos(\beta_i))k\Omega_i^2 \\ \sum_{i=1}^{n}(-\cos(\gamma_i)\sin(\beta_i) + \sin(\gamma_i)\sin(\alpha_i)\cos(\beta_i))k\Omega_i^2 \\ \sum_{i=1}^{n}\cos(\alpha_i)\cos(\beta_i)k\Omega_i^2 \end{bmatrix} \tag{11}$$

where γ_i is the $n - ith$ motor angular position w.r.t the x_B axis and k is a proportional constant relating the generated thrust force and the propeller angular speed (Ω).

The body drag force $\vec{F}_D^{BCS}$ is mainly due the viscous interaction of the system with the surrounding air is assumed as proportional to the linear velocity of the aircraft and always acting in the opposite direction of the body movement. Lastly, the gravitational force $\vec{F}_{grav}^{BCS}$, acts on the body center of gravity always pointing int the negative direction of the *ICS* and can be projected on the *BCS* using the inverse matrix transformation from Eq. (2).

Concerning the external moments acting in the *BCS*, they are mainly generated by the propeller system actuation and how they are distributed on the UAV center of mass. Hence, the torque produced by the spinning propeller can be expressed as the cross product between the rotor vector position ($\vec{r}_{CG/P}$) and the thrust force vector (F_T^{BCS}) for each motor set:

$$\vec{\tau}_T = \vec{d}_i \times \vec{F}_T^{BCS} \tag{12}$$

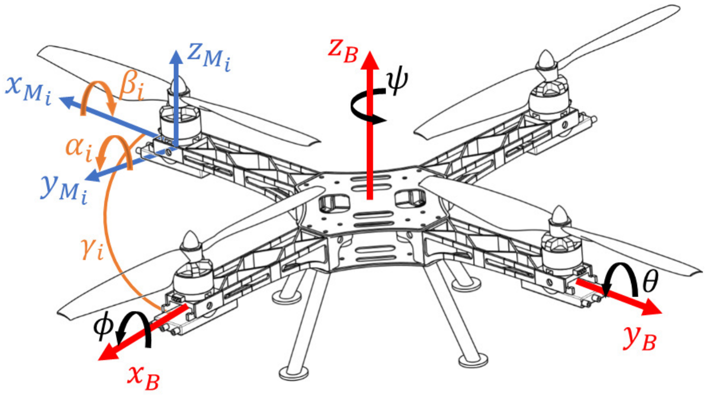


Figure 1: Multi-copter BCS, MCS and tilt angles definition.

The development of the forces and moments formulation is out of scope of this work, for further information it is recommended to consult the reference Marques (2018).

2.3 Linear Model

As stated previously, the multi-copter motion is represented by a set of nine highly coupled nonlinear equations (Eqs. (7), 8 and 1). However, in order to investigate the system performance and apply a attitude tracking control strategy, the derived equations of motion must be linearized in order to form a first order derivative LTI (Linear Time Invariant) system state space model which can be written on the state-space format. Thus, the adopted linearization procedure is based on the small perturbation theory in which the vehicle movement is assumed slightly perturbed from its steady state operating condition (trimmed condition) (Roskam, 2001).

For attitude control, the trimmed condition may vary depending on the desired pitch and roll angles reference signal in order to keep the is set as the equilibrium point, where the linear and angular velocities are zero and, consequently, the approximation is valid only for small perturbations in the surroundings of the trimmed condition. First, the state vector is defined as $\vec{x} = \{ \begin{matrix} u & v & w & P & Q & R & \phi & \theta & \psi & x_E & y_E & z_E \end{matrix} \}$ being $\vec{x}$ perturbations around the stated equilibrium condition. For control purposes, the net thrust force and torques on three BCS directions (F_x, F_y, F_z, τ_x, τ_y and τ_z) are considered as the state space input vector $\vec{u}$ whose perturbed approximation, is given by:

$$\vec{u} = \vec{u_0} + \Delta\vec{u} = \begin{bmatrix} F_{x_0} + \Delta F_x \\ F_{y_0} + \Delta F_y \\ F_{z_0} + \Delta F_z \\ \tau_{x_0} + \Delta\tau_x \\ \tau_{y_0} + \Delta\tau_y \\ \tau_{z_0} + \Delta\tau_z \end{bmatrix} \tag{13}$$

It must be regarded that, assuming the multi-copter hovering equilibrium condition where $\vec{x} = 0$ and $F_{z_0} = mg$, all other equilibrium forces and torques (F_{x_0}, F_{y_0}, τ_{x_0}, τ_{y_0} and τ_{z_0}) are zero.

Once the state variables, forces and moments were linearized considering hoovering condition on Eqs. (1), (7) and 8, the resulting linear first order differential equations can be written in the state space format:

$$\begin{aligned} \dot{\vec{x}} &= A\vec{x} + B\Delta\vec{u} \\ \vec{y} &= C\vec{x} + D\Delta\vec{u} \end{aligned} \tag{14}$$

with $\vec{y}$ being the output measured signals, *A* the dynamic matrix and *B* the input matrix. Since the measured signals are considered to be exactly the state vector, then *C* is an identity matrix and *D* is a zero matrix.

2.4 Control Design

For autonomous flight operation of multi-copters, it is common to develop a control strategy based on two main loops: one responsible for regulating the angular velocities and attitude to maintain the aircraft on stable flight, and the other to track the desired position error (Smeur *et al.*, 2018; Czyba and Szafranski, 2013). Compared to standard multi-copter configurations, the addition of the tilting mechanism in the model increases the complexity of the dynamical model once introduces non-linear and coupling terms.

Hence, the first step for the control design is to establish the state variables, the reference input signals and the control inputs. In this work, the states of the model are represented by a set of 12 state variables which are the linear and angular velocities and position. On the other hand, the reference input signal are chosen as the three space positions on the Inertial Reference Frame *ICS* (x_E, y_E and z_E) and the multi-copter orientation (ϕ and θ).

2.5 Control Law

Furthermore, once defined the model states, inputs and reference signals, a control law is used to compute the required $\vec{u}$ for each time step. The LQR control theory is used to calculate the loop gains such that the system's stability and good tracking capability is guaranteed throughout the operational domain. This technique is widely used on multi-state models since all the loop gains are calculated simultaneously, guaranteeing the system stability and good robustness (Burns, 2001). Also, as proposed in Lavretsky and Wise (2013), the control law structure can be modified in order to establish a tracking controller, for a known reference input signal, assuming the system as a servo-mechanism model. In this work, it is considered that the multi-copter is able to track a desired position and orientation which can be represented as a step (constant) input signal and, therefore, a first order controller is required.

Hence, mainly concerned on tracking a given attitude and position in space, the tracking error is defined as:

$$\vec{e}(t) = \vec{r}(\mathrm{t}) - \vec{x}_T(\mathrm{t}) \tag{15}$$

being $\vec{x}_T(\mathrm{t})$ the tracking states represented by the system position and attitude and $\vec{r}(\mathrm{t}) = \begin{bmatrix} \phi_d & \theta_d & x_E & y_E & z_E \end{bmatrix}$ the tracking command vector containing the reference values for the tracking states which can be represented as a p order time differential equation:

$$\overset{(p)}{r} = \sum_{i=1}^{p} a_i \overset{p-i}{r} \tag{16}$$

Furthermore, the LQR can be extended to the LQT (*Linear Quadratic Tracking*) formulation increasing the system order following the internal model principle, where the error (Eq.(15)) is driven to zero. The problem can be treated as a servomechanism design which contains the reference model equation (Eq.(16)) written in the state space form as (Lavretsky and Wise, 2013):

$$\dot{\vec{z}} = \tilde{A}\vec{z} + \tilde{B}\vec{\mu} \tag{17}$$

where the vector $\vec{\mu}$ represents the plant input vector and $\vec{z}$ is the expanded state vector containing the plant and p error derivatives ($\dot{\vec{z}} = \begin{bmatrix} e & \dot{e} & \dots & \overset{(p-1)}{e} & x \end{bmatrix}^T$). It must be noted that, for step (constant) input signal $p = 1$.

For the control law, it is assumed that the inputs are proportional to the state vector by a feedback gain matrix K_c:

$$\vec{\mu}(t) = -K_c\vec{z} \tag{18}$$

The feedback gain matrix $K_c = \begin{bmatrix} K_p & K_{p-1} & \cdots & K_1 & K_x \end{bmatrix}$ can be calculated in a manner that the closed loop matrix system ($\tilde{A} - \tilde{B}K_c$) is stable. The K_c matrix is obtained solving a cost-function minimization problem given by (Burns, 2001):

$$J = \int_0^\infty \left((\vec{z})^T Q_k (\vec{z}) + \vec{\mu}^T R_k \vec{\mu} \right) \, dt \tag{19}$$

where $Q_k = Q_k^T \geq 0$, $R_k = R_k^T \geq 0$ and $\left(\tilde{A}, Q_k^{1/2}\right)$ are detectable.

Therefore, for a given state space LTI system, there is a gain matrix K_c which can minimize the cost function and bring all states (including the error) to zero simultaneously. The matrix is composed by a regulator matrix K_x responsible to maintain all the states stable, while the matrices $\begin{bmatrix} K_p & K_{p-1} & \cdots & K_1 \end{bmatrix}$ are incumbent to track the reference input signals driving the steady state error to zero.

The cost function minimization problem (Eq. (19)) can be solved using the Riccati equation Burns (2001) where the control behavior (aggressiveness and robustness) can be tuned choosing the values of the weighting matrices Q_k and R_k.

2.6 Desired Tilt Angles

For this problem, the LQR control law is responsible to estimate the forces and moments required to hold the multi-copter in a required attitude and position. Thus, once the multi-copter actuators depends on the propeller angular speed and tilt angles to produce the demanded forces and moments, they must be calculated from the given inputs.

From the kinematic relations of the body, its possible to estimate the desired tilt angles necessary to maintain the thrust force always in the z_E direction using the system orientation angles (ϕ and θ). As stated before, the forces and moments variation w.r.t. the equilibrium condition are set as the controller output and, through them, knowing the tilt direction of each motor-propulsive group, the motor angular speed can be calculated. It must be remarked that, for this work, both longitudinal and lateral tilt mechanism are considered represented by α_i and β_i, respectively.

Thus, firstly, for the tilt angle estimation, one can consider a vector $\vec{r}_p^{ICS} = [0 \quad 0 \quad 1]$ representing the direction of the gravitational force vector. The direction vector $\vec{r}_p^{ICS}$ can be written in *BCS* using the matrix transformation from Eq.(2) and, similarly, using $\{R_M^B\}^T$ matrix, the resulting vector can be represented in *MCS* considering that $\alpha_i = 0$ and $\beta_i = 0$. Hence, the $\vec{r}_p^{ICS}$ in the MCS is given by:

$$\vec{r}_p^{MCS} = \begin{bmatrix} -\cos\gamma_i \sin\theta - \sin\gamma_i \sin\phi \cos\theta \\ -\sin\gamma_i \sin\theta - \cos\gamma_i \sin\phi \cos\theta \\ \cos\phi\cos\theta \end{bmatrix} \tag{20}$$

Moreover, the tilting angles for each rotor can be obtained from Eq.(20) assuming that the desired longitudinal (α_i) and lateral (β_i) angles are represented by the projection of $\vec{r}_p^{MCS}$ on x_M and y_M directions being represented, respectively, by:

$$\alpha_i = \arctan\left(\frac{\sin\gamma_i \sin\phi\cos\theta + \cos\gamma_i \sin\theta}{\cos\phi\cos\theta}\right) \tag{21}$$

$$\beta_i = -\arctan\left(\frac{\cos\gamma_i \sin\phi\cos\theta - \sin\gamma_i \sin\theta}{\cos\phi\cos\theta}\right) \tag{22}$$

From Eqs. (21) and (22) one can conclude that the desired tilting angles for each rotor are function of the motor distribution on the $x_b - y_b$ plane and also the aircraft attitude represented by the γ_i angles and the Euler state angles ϕ and θ, respectively. Hence, they can be estimated from the model state vector $\vec{x}$ for each time step.

2.7 Propeller Angular Speed

Using the control law described in Section 2.5represented by Eq.(19) the incremental forces (F_x, F_y and F_z) and torques (τ_x, τ_y and τ_z) on three *BCS* directions are calculated for each time step. Therefore, the desired propeller angular velocities, which is actually the variable controlled on a real multi-copter, can be calculated from the control law output vector in order to keep the aircraft in balance and track the desired signal. The present work presumes that the propeller angular speed of each rotor can be estimated using the calculated input signal $\vec{u}(t)$ and the desired tilt angles for each motor (α_i and β_i). Thus, assuming stationary equilibrium condition, one can shows the actuation forces and moments can be written as:

$$\begin{bmatrix} \Delta F_x \\ \Delta F_y \\ F_{z_0} + \Delta F_z \\ \Delta\tau_x \\ \Delta\tau_y \\ \Delta\tau_z \end{bmatrix} = \begin{bmatrix} a_{11} & \cdots & a_{1i} \\ a_{21} & \cdots & a_{2i} \\ a_{31} & \cdots & a_{3i} \\ a_{41} & \cdots & a_{4i} \\ a_{51} & \cdots & a_{5i} \\ a_{61} & \cdots & a_{6i} \end{bmatrix} \begin{bmatrix} \Delta\Omega_1^2 \\ \vdots \\ \Delta\Omega_i^2 \end{bmatrix} \tag{23}$$

$$\vec{u}(t) = [A_m]\left[\Delta\Omega^2\right] \tag{24}$$

where the $[A_m]$ coefficients a_{ii} are obtained from the forces and moments equations (Eqs.(11) and (12)):

$$\begin{cases} a_{1i} = (\sin\gamma_i \sin\beta_i + \cos\gamma_i \sin\alpha_i \cos\beta_i)k \\ a_{2i} = (-\cos\gamma_i \sin\beta_i + \sin\gamma_i \sin\alpha_i \cos\beta_i)k \\ a_{3i} = \cos\alpha_i \cos\beta_i k \\ a_{4i} = l \sin\gamma_i \cos\alpha_i \cos\beta_i k + z_{cg} \sin\beta_i k - b \sin\alpha_i \cos\beta_i \\ a_{5i} = -l \cos\gamma_i \cos\alpha_i \cos\beta_i k + z_{cg} \cos\beta_i \sin\alpha_i k + b \sin\beta_i \\ a_{6i} = -l \cos\gamma_i \sin\beta_i k + l \sin\gamma_i \sin\alpha_i \cos\beta_i k - b \cos\alpha_i \cos\beta_i \end{cases} \tag{25}$$

Hence, given the tilt angles α_i and β_i from Eqs. (21) and (22) and the desired control forces from Eq.(19), the angular velocities for the i^{th} motor propulsive group for each time step can be calculated using Eq. (24):

$$\left[\Omega^2\right] = [A_m]^{-1}\, \vec{u}(t) \tag{26}$$

2.8 Control Architecture

Considering the control law and inputs calculations were presented previously, the control architecture used for the system to track a desired reference command is presented in Fig.2.

The figure illustrates the block diagram architecture where two main control blocks can be identified. One is the state regulator, which regulates the state feedback signal from the multi-copter plant ($\vec{x}$) multiplying it by the gain matrix K_x guaranteeing state stability. The second is the integral error control which receives the desired attitude and position reference signals ($\vec{r}$) and the desired tracking states signal (y_e).

The computed forces and moments from the feedback stability are subtracted from the integral error control and, later, the tilting angles and propeller angular velocities model inputs are calculated using Eqs.(21), (22) and (26). The calculated outputs (α_i, β_i and Ω_i) are the inputs to the multi-copter dynamical model represented by Eqs. (7) and (8), which can be numerically integrated to estimate the states.

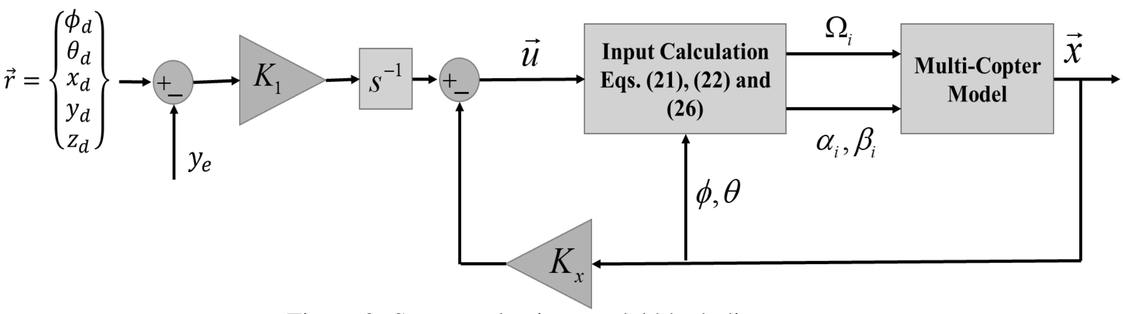


Figure 2: Servomechanism model block diagram.

3. Numerical Simulation

3.1 Simulation set up

In order to validate the dynamical model and the proposed control law presented in the previous sections, a numerical simulation was carried out using Matlab/Simulink following the control block diagram presented in Fig. 2. The main purpose of the simulations is to evaluate if the system is capable to hold a desired position with attitude different from the hovering condition where all the Euler angles and velocities are zero. The vehicle's initial position is set as $\vec{x} = 0$ while the desired states are set to $\vec{r} = [0.5, -0.5, 0.5, 20, 0]$. The position are given in meters while the angles are given in degrees.

Furthermore, for illustration, a quad-rotor MAV is used as case of study for the simulations containing the specific characteristics presented in Table 1. The properties were based on previous works from Marques (2018).

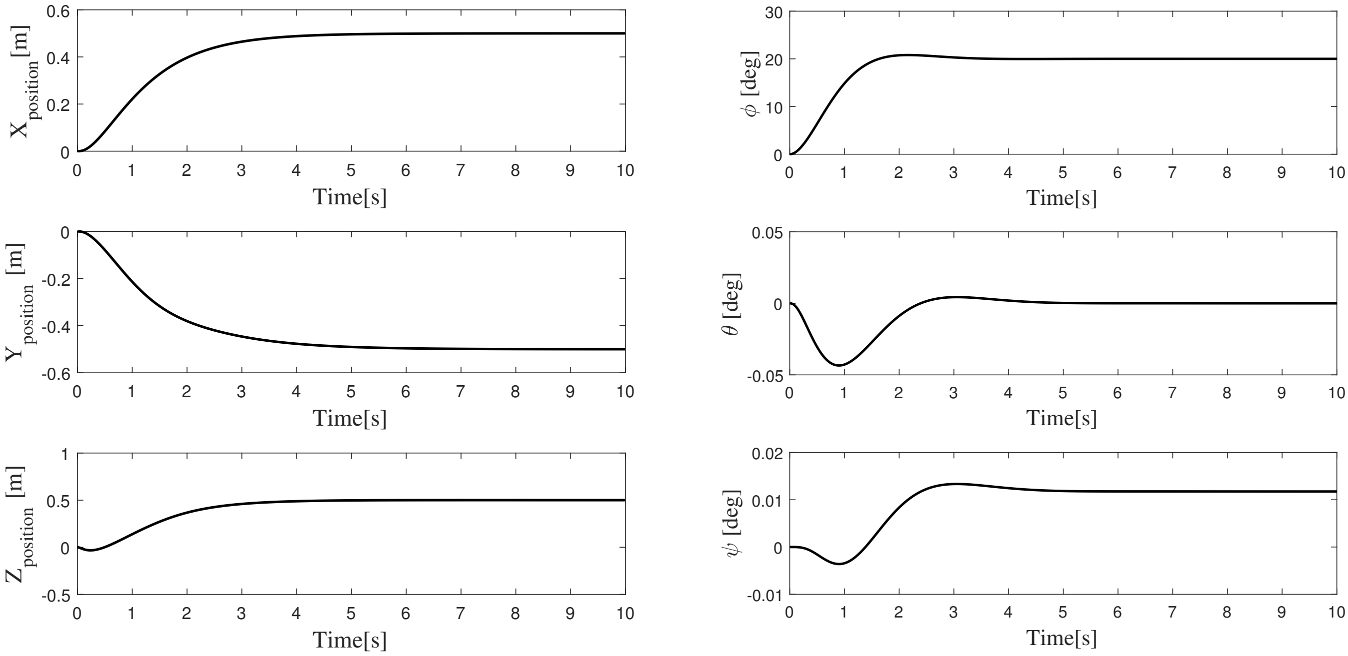
3.2 Simulation results

The procedure to accomplish the proposed simulation is to make the vehicle reach a desired position with a reference attitude establishing a new equilibrium condition.

Figure 3a shows the multi-copter position along the maneuver. It can be seen that the vehicle reaches the reference position, which represents the system steady-state, in approximately $t = 5$ sec. Also, the attitude angles changes are

Table 1: Quad-rotor properties.

Property	Parameter	Value	Unit [SI]
Number of rotors	n	4	-
Mass	m	0.620	kg
Inertia on x_b axis	J_{xx}	0.007	kg.m^2
Inertia on y_b axis	J_{yy}	0.007	kg.m^2
Inertia on z_b axis	J_{zz}	0.013	kg.m^2
Drag coefficient	k_d	4.8e-3	kg/s
Arm Length	l	0.180	m
Thrust coefficient	k	8.901e-6	N.s^2
Propeller drag coefficient	b	1.1e-6	N.m.s^2



(a) (b)

Figure 3: Multi-copter position (a) and attitude (b)

presented in Fig. 3b. It can be perceived that the change in the pitch and yaw angles are close to zero while the roll angle reaches the desired reference value.

Figure 4 illustrates the change in the lateral and longitudinal tilt angles estimated from Eqs. 21 and 22 . From the figure, one can infer that for a roll angle command, only motors 1 and 3 tilts laterally, while motors 2 and 4 tilts longitudinally in order to maintain the thrust vector always pointing in the vertical direction. The tilt angles change basically follows the pitch angle during the maneuver.

Finally, Fig. 5 presents the propeller angular speed variation through the simulation for all 4 rotors estimate from Eq. 26. It can be seen that the motor rotation is increased in order to compensate the change in the weight vector direction and counterbalance the force produced in the y_B direction.

4. Conclusions

This works presented a numerical formulation for tilt rotor UAVs which can be applied for any multi-copter configuration and also expanded to a VTOL aircraft. Further, compared to standard multi-copters, the proposed control methodology enables the system to operate on multiple equilibrium conditions, enlarging the possibility of mission applications.

From the obtained results, it was observed that the system is able to perform hover flight with the orientation different from zero and also track a desired position in space. One might conclude that the new tilt angle and motor angular speed

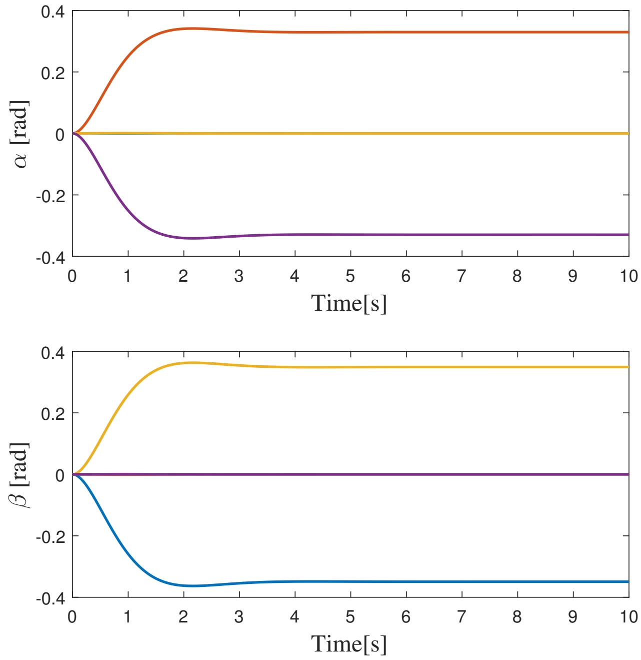


Figure 4: Multi-copter longitudinal (α_i) and lateral (β_i) tilt angles.

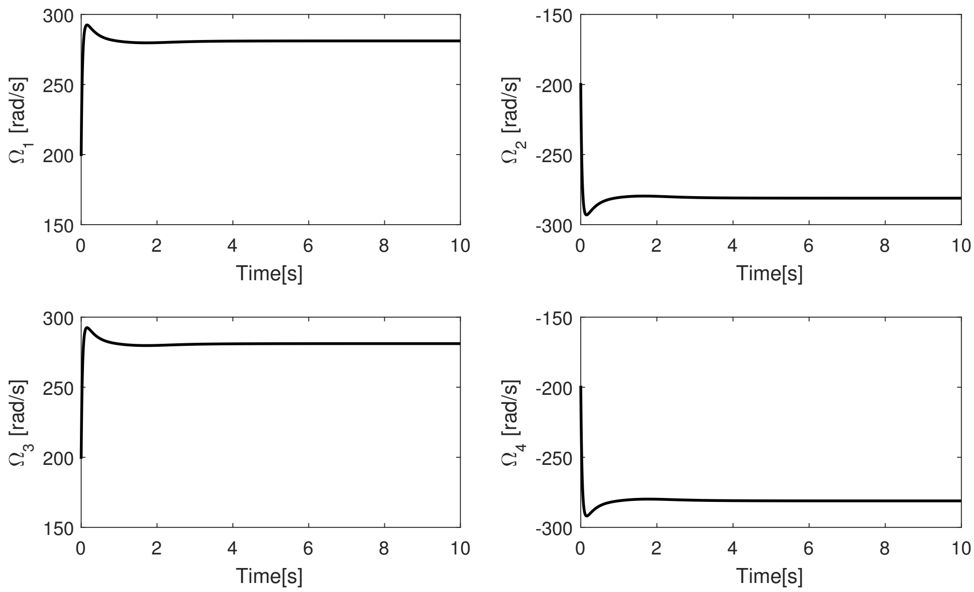


Figure 5: Motor angular speed.

defines a new equilibrium condition different from the hoover flight on which the system equations were linearized. It must be remarked that this new equilibrium condition is not feasible standard multi-copter configurations.

Finally, once the control law is validated numerically, it shall be ready to be implemented in a real micro-controller.

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