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INFLUENCE OF THE LOCATION OF FORCES APPLICATION IN THE NONLINEAR ANALYSIS OF I-GIRDERS SUBMITTED TO BENDING

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Abstract. *This paper discusses the behavior of supported steel beams with I cross-section, which are able to move laterally with free warping and restrained twist in the both ends, defined as the lateral torsional buckling (LTB). It was applied a distributed load subjected the beam to a pure bending, and the model considered the residual stresses, geometric nonlinearity and elastic-plastic behavior of the material. The cases where loads act on the upper and lower lines of the web, as well as the halfway, were evaluated under the light of ABNT NBR 8800:2008. Numerical models with nonlinear analyses were developed in Abaqus software, in order to detect the occurrence of the stabilizing effect and the sensitivity of the load discretization in the results of elastic critical moment and moment resistant to LTB. As expected, the results obtained for load acting on the halfway of the web for elastic critical moment converged to the normative value of ABNT NBR 8800:2008. Moreover, the result in relation to load discretization converged for the analytical result as the number of discrete loads were increased. In other words, it can be said that more discretization led to a good representation of the distributed load in the profile.*

Keywords: *lateral torsional buckling, stabilizing effect, numerical modeling, nonlinear analysis.*

1. INTRODUCTION

According to Fakury et. al (2016), when the behavior of beams subjected to bending is analyzed, the collapse of the structure may be due to lateral torsional buckling (LTB), local buckling of compressed cross-sectional elements or, if none of these limit states occurs, the failure of the structure can be a result of the total cross-section plastification.

This paper therefore discusses the behavior of supported steel beams subjected to simple normal bending due to static actions (distributed loading), specifically I-girders, considering important aspects in the numerical models, such as the residual stresses, geometric nonlinearity and elastic-plastic behavior of the material.

The standard ABNT NBR 8800:2008, used in the design of steel structures in Brazil, considers as a reference the elastic critical moment that causes the elastic buckling of a perfectly straight beam in an elastic regime.

In accordance with Silva (2017), the guidelines of the Brazilian standard do not consider the effect of web distortion and state only that the beam is able to move laterally with free warping and restrained twist in the both ends of the effective length, with loads on the cross-section halfway.

It is known that, if a transverse force is applied to a location other than the torsional center (S) of the cross-section girder, its line of action moves away from it after the LTB has started, the torsion increases and the bending moment resistant to LTB is reduced, being these forces called destabilizers. If the direction of the action line is applied to a location other than the torsional center (S), the torsion reduces after starting buckling, the bending moment resistant to LTB increases, being these forces called stabilizers. Forces applied to the torsional center are neither stabilizers nor destabilizers.

ABNT NBR 8800:2008 standard recommends loads to act on halfway line of the I-profile web. In this paper, however, cases where loads act on the upper, lower and halfway lines of the web were evaluated. Thus, the loading was divided into localized loads, according to the number of divisions of the load line. Numerical models were developed in Abaqus, a commercial software for analysis by finite element method, in order to observe not only the occurrence of the stabilizing effect, but also the numerical sensitivity of the localized loads numbers in the results of elastic critical moment and moment resistant to LTB. The elastic critical moment results obtained in the numerical analysis will be compared to the theoretical values of the ABNT NBR 8800:2008 standard.

2. METHODOLOGY

Considering the lateral torsional buckling of beams with doubly symmetrical I cross-section submitted to bending in relation to the perpendicular axis to the web, ABNT NBR 8800:2008 standard presents the Eq. (1) to obtain the elastic critical moment values:

$$M_{cr}=\frac{C_b.\pi^2.E.I_y}{L_b^{\,2}}\sqrt{\frac{C_w}{I_y}\left(1+0.039\frac{J.L_b^{\,2}}{C_w}\right)} \tag{1}$$

Where E is the Young's modulus, I_y is the moment of inertia in relation to the y-axis, L_b is the effective length of the beam, C_w is the warping constant, J is the torsion constant and C_b is the lateral-torsional buckling modification factor for nonuniform moment diagrams, obtained by Eq. (2):

$$C_b = \frac{12.5|M_{max}|}{2.5|M_{max}| + 3|M_A| + 4|M_B| + 3|M_C|} \leq 3.0 \tag{2}$$

Where M_{max} is the maximum moment absolute value present in the unbraced length, M_A is the moment absolute value at a quarter point of the unbraced length, M_B is the moment absolute value at the centerline of the unbraced length and M_C is the moment absolute value of at a three-quarter point of the unbraced length.

In numerical modeling, the Abaqus software analyzed the profile submitted to bending moment. This computational program used the finite element method, that is, a numerical procedure to determine approximate solutions of value problems on the boundary of differential equations. In other words, it is based on the approximation of balanced body conditions from the Lagrangian point of view to achieve balanced equations from the principle of virtual works.

As the bending loads are applied slowly, and the inertial forces can be neglected, the analysis used in the models can be static. However, once the general static analysis deals with linear problems, such as issues regarding eigenvalues and eigenvectors; it is not suitable for the cases studied here. Thus, it was chosen to implement the technique of Linear Arc Length, proposed by Riks (1972) for the study of geometrically nonlinear structures, taking into account that the elastic critical moment was determined through buckling analysis.

In order to obtain the critical buckling load, the distributed linear load situation (q) along the length of the beam (L_b) was considered as shown in Fig. 1. The I-profile dimensions shown in Fig.2 correspond to the skeleton line measurement.

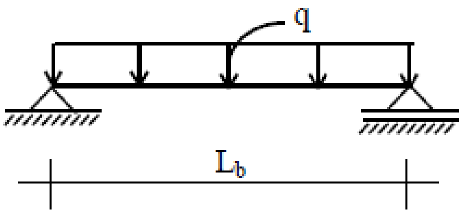

Figure 1. Distributed linear load in simple supported beam.

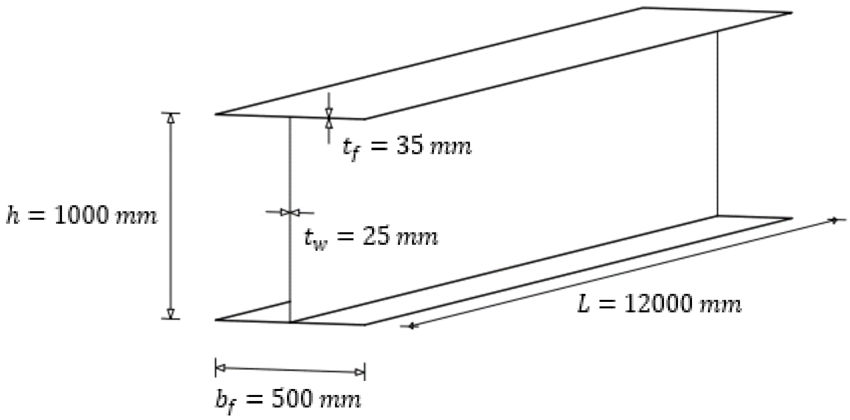

Figure 2. I-profile considered in analysis.

In addition, the elastic-plastic behavior was adopted in the analyses, and in all subsequent analyses the longitudinal elasticity modulus is equal to 200x10^9 Pa and the Poisson coefficient is equal to 0.3. Mechanical properties of steel ASTM A572 used in numerical modeling are presented in Tab. 1.

Table 1. Mechanical properties for the ASTM A572 based on ABNT NBR 8800:2008.

Steel grade	f_y (MPa)	f_u (MPa)
A572	345	450

Considering that more precise results will be found once the model presents a finer mesh, and that the fine mesh enables a better residual stress distribution in nonlinear analysis, the mesh adopted had an approximate global size of 0.05 m, and the element chosen for analysis was the S4, a 4-node doubly curved general-purpose shell.

To represent the load q, its line of action was divided into divisions and equivalent localized loads were applied to each of them. Specifically, q was divided in q_c and q_{ext}, that is, concentrated loads in the middle of the span and at the ends, respectively. Three cases (load acting on the top flange, the halfway and the lower flange) were studied for six different divisions (5, 10, 15, 30, 40 and 60), as exemplified in the models of Fig. 3, which also represents the boundary conditions, according to Tab. 2.

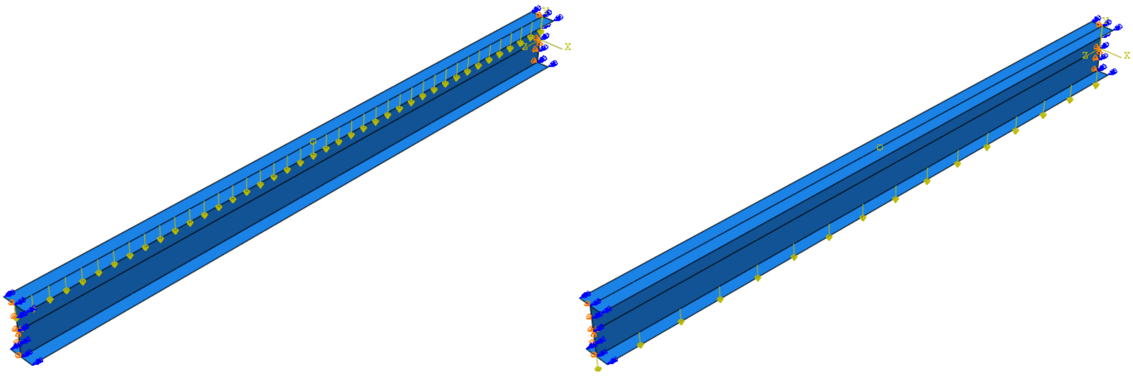

Figure 3. Loads acting on 41 points of the upper flange (to the left) and on 16 points of the lower flange (to the right).

Table 2. Boundary conditions of supported steel beams with I cross-section which are able to move laterally with free warping and restrained twist in the both edges.

Position	Restrictions
End webs	x-displacement (1)
Central point of the webs	y-displacement (2)
Central point of one web	z-displacement (3)
End sections	z-rotation (3)

(1) x-axis corresponds to the width axis.
(2) y-axis corresponds to the depth axis.
(3) z-axis corresponds to the longitudinal axis.

For the welded I-profile, a residual stress distribution was used to evaluate those effects. It was therefore adopted the average distribution proposed by Castro e Silva (2006) for the profile whose residual tensile stresses (σ_{t1} and σ_{t2}) and compression stresses (σ_{c1} and σ_{c2}), depended on steel yield stress (f_y), according to Fig. 4. Fig. 5 shows the residual stress applied to the profile.

In nonlinear analyses, the behavior of the material was assumed to be perfectly elastic-plastic, and the coefficient of imperfection was achieved in terms of the value of the longitudinal length of the beam (a) divided by 1500. Moreover, once the process is iterative, in which the applied load is incrementally done, upon reaching the ultimate moment according to the yield stress (f_y) of the material, this load continues to be exerted, but with a decreasing value.

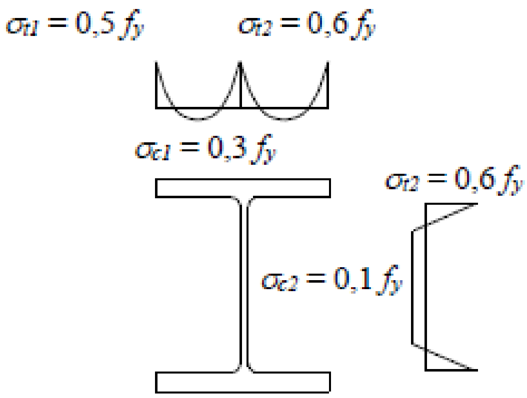

Figure 4. Residual stress distribution on welded I-profile (Castro e Silva, 2006).

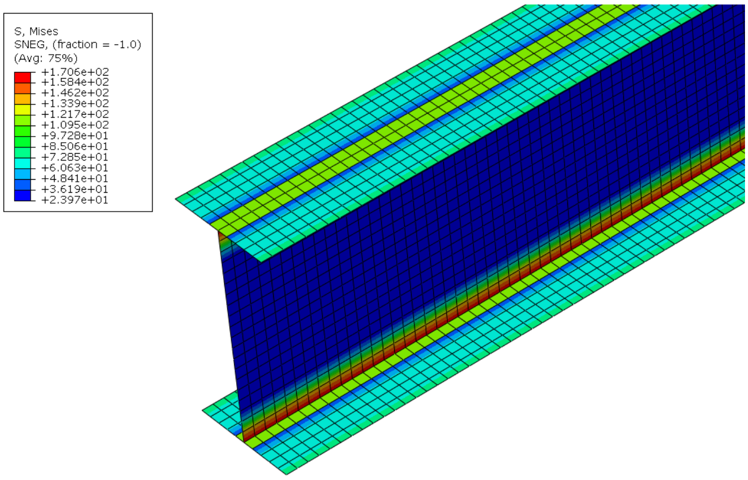

Figure 5. Residual stress distribution applied to the supported beam.

3. RESULTS AND DISCUSSIONS

After performed the buckling analysis considering residual stresses, the elastic critical moment results obtained were compared to the theoretical values of the ABNT NBR 8800:2008 standard, as shown in Tab. 3.

Table 3. Relative difference between numerical elastic critical moments and ABNT NBR 8800:2008 prescriptions.

Divisions	Top line		Centre line		Lower line	
	$M_{cr,Abaqus}$ (N.m)	Relative difference (%)	$M_{cr,Abaqus}$ (N.m)	Relative difference (%)	$M_{cr,Abaqus}$ (N.m)	Relative difference (%)
5	4,773,495	34	6,824,310	5	10,032,630	39
10	4,703,520	35	6,765,525	6	9,751,800	35
15	4,689,855	35	6,757,110	6	9,701,970	34
30	4,682,190	35	6,750,795	6	9,674,460	34
40	4,680,465	35	6,748,605	7	9,669,315	34
60	4,678,620	35	6,746,025	7	9,664,800	34

It was considered the initial imperfection in the nonlinear analysis and the ultimate moments are presented in the chart of Fig. 6.

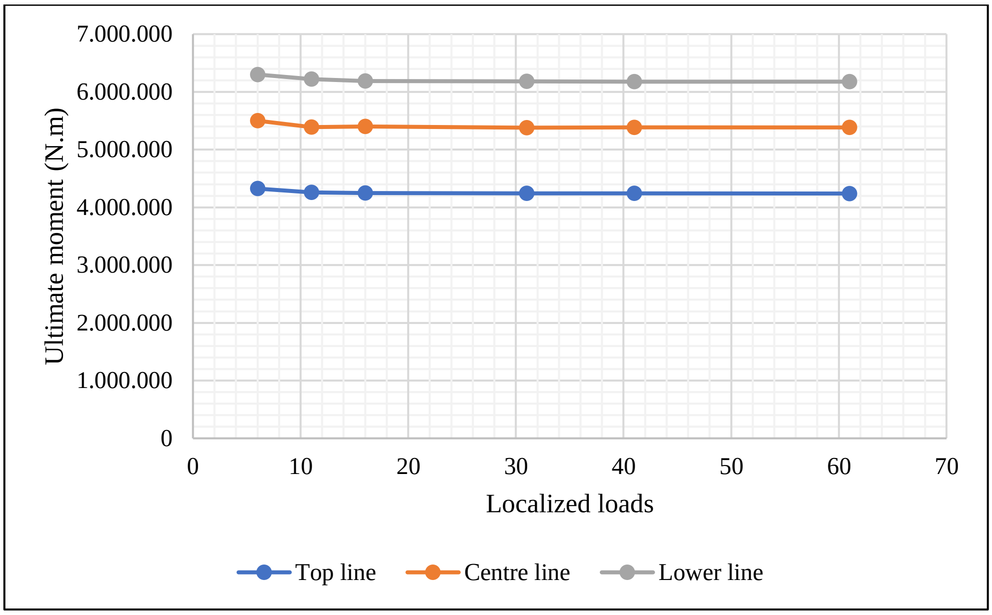

Figure 6. Ultimate moment *versus* number of localized loads.

According to the curves shown in the chart of Fig. 6 and the results of Tab. 3, it is possible to conclude that the convergence was improve when the load discretization increased. Figure 7 shows the deformed shape of the profile for the cases where the load line acts upon the upper flange and the lower flange.

The stabilizing effect is observed when comparing the three curves of Fig. 6 and the numerical elastic critical moment values in Tab. 3, which presents the relative difference of numerical elastic critical moments in relation to the values proposed by ABNT NBR 8800:2008, Eq. (1), equal to 7,218,723.32 N.m. When the load was applied to the upper flange line of the profile, the beam reaches the elastic critical moment early, presenting a higher value compared to when it is applied to the lower flange and the halfway. This phenomenon is evidenced by the deformed configurations of Fig. 7.

The results of Tab. 3 show that the elastic critical moment results obtained numerically were closer to the code values whenever the loads are applied to the profile halfway, which proves to be a good numerical representation of the model.

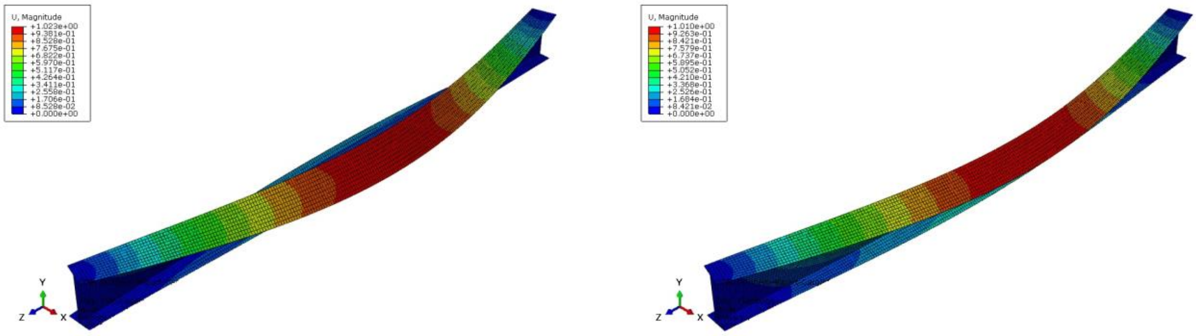

Figure 7. Deformed configuration for loads acting on the upper (to the left) and on the lower (to the right) flange.

4. CONCLUSIONS

This work tackled the study of the simple supported beams which were able to move laterally with free warping and restrained twist and doubly symmetrical I-profile subjected to bending in relation to the major axis (x-axis), modeled in the Abaqus software. Residual stresses and imperfections were added in the analyses in order to obtain the resistant moment for the case of linearly distributed loading in three locations of the section: on the upper, lower and halfway lines of the web.

The phenomenon of lateral torsional buckling (LTB) was observed in the numerical modeling. A comparative analysis of the elastic critical moment results with the normative values of ABNT NBR 8800:2008 indicated a good representation of the numerical model when the loads was applied to the halfway of the web. Regarding the discretization of the loads, it was noted that, for all cases, the result converged as the discretization was refined, that is, the more localized loads there were, the results approached the distributed loading situation.

5. ACKNOWLEDGEMENT

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7. STATEMENT OF RESPONSIBILITY

The authors of this essay are the ones solely responsible for the printed material here included.