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ROTOR PERFORMANCE OPTIMIZATION OF A CENTRIFUGAL COMPRESSOR OPERATING WITH CO_2 IN SUPERCRITICAL CONDITIONS

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Abstract. *Due to its low capital cost, high efficiency with a simple layout and compact turbomachinery, Supercritical CO_2 (S-CO_2) power cycles have been arisen as an advantageous technology. The present work deals with a computer code implemented in MATLAB® by using mean line method to predict the rotor performance of a centrifugal compressor at design point. S-CO_2 is considered as working fluid and the real-gas Span and Wagner equations of state have been used to calculate the thermodynamic properties of the fluid. Since a lot of parameters can affect the rotor performance, a multi-objective optimization procedure is coupled to the computational code by using Non-dominated Sorting Genetic Algorithm genetic (NSGA-II) for maximization of the rotor isentropic efficiency. The input parameters used in the numerical simulations are: mass flow rate, total pressure and total temperature at the inlet of the centrifugal compressor, rotational speed, target pressure ratio and basic geometric parameters of the compressor. The outputs are blade geometry parameter and isentropic efficiency. The results from optimization procedure are compared to available data in the open literature and the main results are discussed.*

Keywords: *Mean line method, Centrifugal compressor, Supercritical carbon dioxide.*

1 INTRODUCTION

The centrifugal compressor is an ancient technology that has been used in many applications throughout history (Krain, 2005). Due to its wide variety of applications, centrifugal compressor performance prediction and design are largely studied topics (Eckardt, 1976) (Yoshinaka, et al., 1989) (Aungier, 1995) (Romei, et al. 2017) (Xu, et al., 2018) Among these applications can be highlighted on the energy production and oil and gas industries. Carbon dioxide emissions are considered as an environmental problem and the discussion on how to reduce them has played attention on the oil and gas industry (Nyquist, et al., 2010). A common solution for this problem is to compress the CO_2 and inject it into oil and gas fields for rejuvenating active producing fields or only to storage in depleted or unused reservoirs (Ansarizadeh, 2015). Using carbon dioxide as working fluid in the energy generation field is a promising branch of research because of its relatively low power for the compression train when it is above the critical point (Khadse, 2016).

Augier (1995), (2000) studied performance prediction and design of centrifugal compressors using the mean line method. Oh (1997) proposed an optimum set of loss models for performance prediction of centrifugal compressors using air as working fluid. Ameli et al. (2018) validated the use of the loss models suggested by Oh (1997) for the design of centrifugal compressors using supercritical CO_2 (S-CO_2) as working fluid. Li et al. (2015) presented a mean line based on optimization method for the NASA HPCC compressor using an adaptative simulated annealing algorithm. Oka et al. (2017) designed a centrifugal compressor using air as working fluid. The authors analyzed the compressor geometry by using an inverse method based on meridional viscous flow. The optimization was performed by Non-dominated Sorting Genetic Algorithm II (NSGA II).

The present work is focused on the application of an optimization procedure which combines a mean line method implemented in MATLAB® to predict the centrifugal compressor geometry and NSGA-II in order to maximize the rotor isentropic efficiency. For the numerical simulations, the compressor is operating at design point. The real-gas Span and Wagner equations of state have been used to determine the thermodynamic properties of the carbon dioxide operating in supercritical conditions.

Firstly, the numerical model is validated by comparing the results to available rotor dimensions and operating conditions of a centrifugal compressor using S-CO_2 as working fluid described in Khadse et al. (2016), named herein as baseline case. In the next step, this baseline rotor is optimized for the same mass flow rate of that used by Khadse et al. (2016) and the most important results are pointed out.

2 MEAN LINE METHOD

The mean line method is a simplified technique based on 1D calculations of the centrifugal compressor kinetic energy and thermodynamic properties. It is well known that this method is still one of the most used preliminary design methods for both compressor geometry parameters and performance predictions. Since the calculations are made on the mean streamline of the flow based on Euler's equation, loss models are needed to include flow phenomena and heat transfer characteristics which are not considered in Euler's equation and that occurs away from the mean streamline. For the determination of the velocity triangles presented on Figs. 1 and 2, it is necessary to calculate some thermodynamic properties of the working fluid. In this sense, Span and Wagner (1996) equations of state are coupled to the mean line method to predict the CO_2 thermodynamic properties in supercritical conditions.

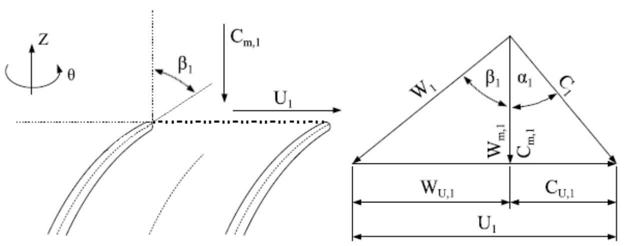

Figure 1 Inlet velocity triangle

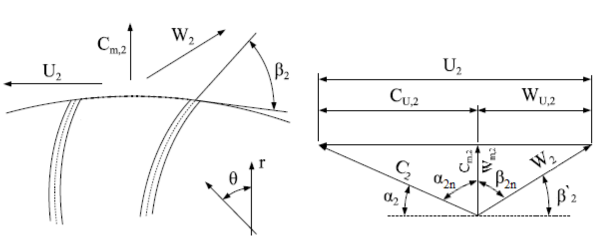

Figure 2 Outlet velocity triangle

3 LOSS MODELS

In order to provide a more realistic flow and heat transfer characteristics present in centrifugal compressors, empirical loss models have been developed by many researchers and some of the most used models are implemented in the numerical routine. Table 1 presents the loss correlations used in the present work.

Table 1 - Implemented loss correlations

	Loss Model	Reference
Slip factor, σ	$\sigma = 1 - \frac{\sin\alpha_2 \sqrt{\sin\beta_2'}}{Z^{0.7}}$	Augier (2000)
Incidence loss, Δh_{inc}	$\Delta h_{inc} = f_{inc}\frac{W_{u1}^2}{2}$	Conrad et al. (1980)
Blade loading loss, Δh_{BL}	$\Delta h_{BL} = 0.05 D_f^2 U_2^2$ $D_f = 1 - \frac{W_2}{W_{1s}} + \frac{0.75\Delta h/U_2^2}{(W_{1s}/W_2)[(Z/\pi)(1 - r_{1s}/r_2) + 2r_{1s}/r_2]}$	Coppage et al. (1956)
Skin friction loss, Δh_{SF}	$\Delta h_{SF} = 2C_f \frac{L_b}{d_H}\overline{W}^2$ $\overline{W} = \frac{C_{1s} + C_2 + W_{1s} + 2W_{1h} + 3W_2}{8}$ $d_H = d_2\left[\frac{\cos\beta_{2n}}{\left(\frac{Z}{\pi} + \frac{d_2\cos\beta_{2n}}{b_2}\right)} + \frac{\frac{1}{2}\left(\frac{d_{1s}}{d_2} + \frac{d_{1h}}{d_2}\right)\left(\frac{\cos\beta_{1s} + \cos\beta_{1h}}{2}\right)}{\frac{Z}{\pi} + \left(\frac{d_{1s} + d_{1h}}{d_{1s} - d_{1h}}\right)\left(\frac{\cos\beta_{1s} + \cos\beta_{1h}}{2}\right)}\right]$ $L_b = \frac{\pi}{8}\left(d_2 - \frac{d_{1s} + d_{1h}}{2} - b_2 + 2L_z\right)\left(\frac{4}{\cos\beta_{1s} + \cos\beta_{1h} + 2\cos\beta_{2n}}\right)$	Jansen (1967)
Clearance loss, Δh_{CL}	$\Delta h_{CL} = 0.6\frac{\varepsilon}{b_2}C_{U2}\sqrt{\frac{4\pi}{b_2 Z}\left[\frac{r_{1s}^2 - r_{1h}^2}{(r_2 - r_{1s})\left(1 + \frac{\rho_2}{\rho_1}\right)}\right]C_{U2}C_{a1}}$	Jansen (1967)
Mixing loss, Δh_{mix}	$\Delta h_{mix} = \frac{1}{1 + \tan^2\alpha_2}\left(\frac{1 - \varepsilon_{wake} - b_3/b_2}{1 - \varepsilon_{wake}}\right)^2\frac{C_2^2}{2}$ $\varepsilon_{wake} = 1 - \frac{C_{m,wake}}{C_{m,mix}}$ $C_{m,wake} = \sqrt{W_{sep}^2 - W_{U2}^2}$ $C_{m,mix} = \frac{C_{m2}A_2}{\pi d_2 b_2}$ $\begin{cases} W_{sep} = W_2; & D_{eq} \leq 2 \\ W_{sep} = W_2\frac{D_{eq}}{2}; & D_{eq} > 2 \end{cases}$ $D_{eq} = \frac{W_1 + W_2 + 2\pi d_2\frac{U_2C_{U2} - U_1C_{U1}}{U_2 Z L_b}}{2}$	Johnston and Dean (1966)

Disk friction loss, Δh_{DF}	$\Delta h_{DF} = f_{DF}\frac{\bar{\rho} r_2^2 U_2^3}{4\dot{m}}$ $\begin{cases} f_{DF} = \frac{2.67}{Re_{DF}^{0.5}}; & Re_{DF} < 3x10^5 \\ f_{DF} = \frac{0.0622}{Re_{DF}^{0.2}}; & Re_{DF} \geq 3x10^5 \end{cases}$	Daily and Nece (1966)
Recirculation loss, Δh_{RC}	$\Delta h_{RC} = 8.0x10^5 \, sinh(3.5\alpha_2^3) D_f^2 U_2^2$	Oh (1997)
Leakage loss, Δh_{LL}	$\Delta h_{LL} = \frac{\dot{m}_{LL} U_{LL} U_2}{2\dot{m}}$ $U_{LL} = 0.816\sqrt{\frac{2\Delta P_{LL}}{\rho_2}}$ $\Delta P_{LL} = \frac{\dot{m}(r_2 C_{U2} - r_1 C_{U1})}{Z\left(\frac{r_1 + r_2}{2}\right)\left(\frac{b_1 + b_2}{2}\right) L_b}$ $\dot{m}_{LL} = \rho_2 Z \Delta_{CL} L_b U_{LL}$	Augier (2000)

4 MULTI-OBJECTIVE OPTIMIZATION ALGORITHM

Non-dominated Sorting Genetic Algorithm genetic (NSGA-II) was coupled to the mean line code for maximization of the isentropic efficiency and minimization of the compressor required power. The population size was 120 individuals and 500 generations. The crossover operator was set 1.0 and the mutation operator was equal to 0.9 to provide a broad exploration and exploitation of the design space.

The NSGA-II is a genetic algorithm type (GA) which consists on identifying solutions of the non-dominated front in a population. Each solution is then compared to every other solution in the population to find if it is dominated. This process is continued to seek all members of the first nondominated level in the population. In order to find the individuals in the next non-dominated front, the solutions of the previous front are discounted temporarily and the same procedure is repeated (Deb, 2002).

The optimization process is applied considering two different approaches: the first case (Case 1) considers a power plant already installed; so, the geometrical, total pressure, total temperature and mass flux parameters are defined. The second case (Case 2) considers the power plant on the conceptual design; hence the geometrical parameters can be optimized, but the total pressure, total temperature and mass flux parameters remains constant.

The design variables and the objective functions are defined for the optimization process. Initially, the initial population is built by using Latin Hypercube Sampling (LHS) and the isentropic efficiency and required power values are calculated by the mean line code. Then, the optimization process is sought out by using NSGA II.

The thermodynamic state of the working fluid was determined by using Coolprop (BELL, *et al.*, 2014) libraries. The constraint imposed during the optimization procedure was that the carbon dioxide was in the supercritical condition (except in supercritical liquid state).

Based on objective functions and the operating ranges of the input parameters, the statement of the optimization process can be addressed as shown in Fig. 3.

Case 1	Case 2
5000 RPM $\leq$ Rotation Speed of Rotor $\leq$ 20000 RPM 0° $\leq$ Inlet Swirl Angle $\leq$ 20° 0.02 $\leq$ Blade thickness over radius $\leq$ 0.04 6 $\leq$ Number of Full Blades $\leq$ 20	0.4 $\leq$ Inlet shroud to tip radius ratio $\leq$ 0.7 0.3 $\leq$ Inlet hub to shroud radius ratio $\leq$ 0.7 0.3 $\leq$ Isentropic Stage loading coefficient $\leq$ 1.1 5000 RPM $\leq$ Rotation Speed of Rotor $\leq$ 20000 RPM 0.02 $\leq$ Exit blade width over radius $\leq$ 0.8 0.02 $\leq$ Blade thickness over radius $\leq$ 0.04 6 $\leq$ Number of Full Blades $\leq$ 20

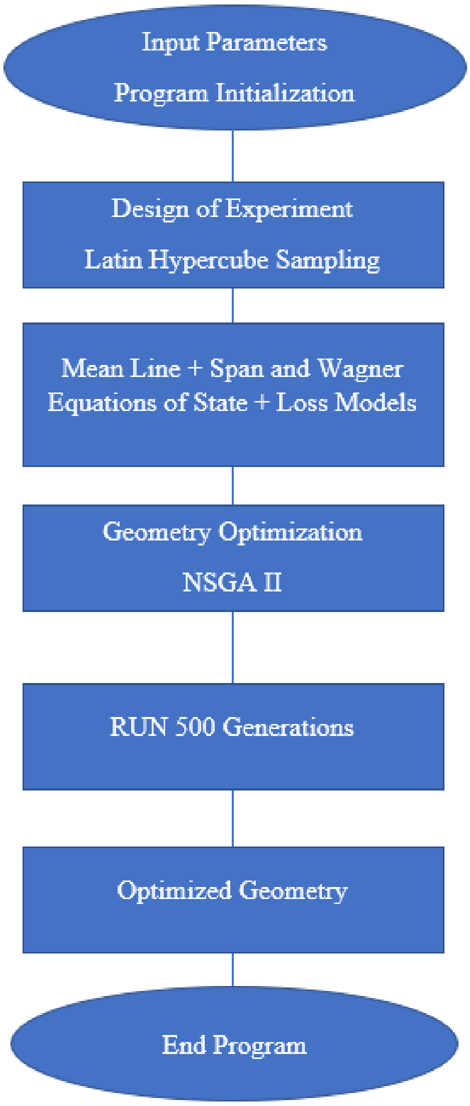

Figure 3 – Optimization Flowchart for Case 1 and Case 2

5 RESULTS AND DISCUSSION

Firstly, validation of the numerical model is needed. The code has been used to predict the performance of the real data of a centrifugal compressor operating with carbon dioxide in supercritical conditions (Khadse, 2016). This compressor is named "baseline" in the present research and its input parameters are presented at Tab. 2. To this end, the results from optimization procedure, called "optimized solution", are compared to the baseline compressor in order to identify the main differences between the results from optimization procedure and a real centrifugal compressor under S-CO_2 conditions.

Table 2 - Real-data for a centrifugal compressor operating with S-CO2.

Impeller Shroud [mm]	384.8
Impeller Hub [mm]	264.4
Impeller Overall Diameter [mm]	527.0
PR	2.5
Total Temperature [K]	320
Total Pressure [MPa]	9.5
Mass Flow Rate [kg/s]	472.19
Rotation Speed of Rotor [RPM]	6560
Inlet Swirl Angle [deg]	0
Blade Thickness [mm]	5.7
Number of Full Blades	15
Clearance Gap [mm]	0.372
Depth of Exit Channel [mm]	23.1
Axial Length [mm]	144.0

Table 3 presents a comparison between the baseline data and the predicted results from the mean line for centrifugal compressor operating with S-CO_2. As it can be seen on Tab. 3, all results have a relative error below 2%, which shows that the implemented code can be considered in a good agreement with the experimental data. Moreover, the results indicated that the set of loss used in the present research for compressor loss predictions can be used for centrifugal compressors operating with S-CO_2.

In the next step, the mean line code is coupled to the optimization algorithm. The optimized isentropic efficiency and the compressor required power are detailed in Tab. 4.

Table 4 also show the values of the isentropic efficiency and compressor power required for the best solution of the Pareto-optimal front as well the baseline compressor. The isentropic efficiency is greatly increased compared to the baseline and the required power is reduced about 2MW, for the same operating conditions.

Table 3 - Validation of the implemented mean line code

	Baseline	**Mean Line Code**	**Relative Error %**
T_{02} **[K]**	378.9	376.93	0.52
P_{02} **[MPa]**	24	23.54	1.92
Total Specific Enthalpy [kJ/kg]	420.5	417.17	0.79
Total Density [kg/m^3]	544.16	546.25	0.38
Total Specific Entropy [kJ/kg*K]	1.6	1.59	0.63
Pressure Ratio	2.5	2.48	0.8

Table 4 - Baseline and optimized results

	Baseline	Optimized solution Case 1	Optimized solution Case 2
Efficiency	78.21	86.04	94.13
Required Power [MW]	17.2	16.2	15

Table 5 shows the design information about baseline and optimized compressors. From Tab. 5, it can be noted that the same mass flow rate can be obtained even if the rotational speed of the impeller is reduced, which contributes with a reduction on the required power.

Regarding the optimized centrifugal compressor from Case 2 compared to the baseline many differences can be noted. In the optimized solution, the inlet dimensions of the compressor have been reduced while the output dimensions are increased. It is also observed that the rotational speed is reduced in order to decrease the compressor required power.

Table 5 - Comparison of results from optimized solutions and the baseline compressor

	Baseline	Optimized solution Case 1	Optimized solution Case 2
Impeller Shroud [mm]	384.8	384.8	223.8
Impeller Hub [mm]	264.4	264.4	81.3
Impeller Overall Diameter [mm]	527.0	527.0	556.3
Inlet Total Temperature [K]	320	320	320
Inlet Total Pressure [MPa]	9.5	9.5	9.5
Mass Flux [kg/s]	472	472	472
Rotational Speed [RPM]	6560	5220	5700
Inlet Swirl Angle [deg]	0	19.95	0
Blade Thickness [mm]	5.7	5.7	5.7
Number of Blades	15	18	14
Clearance Gap [mm]	0.372	0.372	0.372
Depth of Exit Channel [mm]	23.1	23.1	12.3
Axial Length [mm]	144.0	144.0	122.9

For a better understanding of the influence of these changes on each loss mechanism in the centrifugal compressor, a theoretical analysis of every losses was performed. Table 6 presents the loss percentage in the compressor regarding to the total required power, and then the losses encountered for baseline and optimized cases can be quantified and compared. As observed in Tab. 6, all the losses were reduced except for the ones affected by the number of blades, which has increased. For the Case 2, the losses had also been reduced, except the losses related to the leakage and clearance, which increase in percentage due to the reduction of the overall compressor dimensions.

Table 6 - Compressor Losses for the baseline compressor and optimized solutions

	Baseline	Optimized solution Case 1	Optimized solution Case 2
Clearance Loss [%]	0.51	0.81	1.3
Incidence Loss [%]	12.39	7.00	2.78
Disk Friction Loss [%]	0.40	0.18	0.4
Blade Loading Loss [%]	2.63	0.72	0.8
Recirculation Loss [%]	21.22	11.27	1.4
Skin Friction Loss [%]	1.18	3.73	0.8
Leakage Loss [%]	0.57	0.65	0.9
Mixing Loss [%]	0.24	0.0065	0.03
Total Required Power [MW]	17.20	16.2	15

6 CONCLUSION

A Mean line code was implemented in MATLAB® to predict the rotor performance of centrifugal compressors operating with carbon dioxide in supercritical conditions (S-CO_2) as working fluid. It was used the real gas Span and Wagner equations of state for the prediction of the thermodynamic properties of the fluid. The multi-objective optimization procedure consists on the coupling of the implemented mean line code with a robust design of experiments method (Latin Hypercube Sampling (LHS) algorithm) and Non-dominated Sorting Genetic Algorithm genetic (NSGA-II). The objective functions are the maximization of the isentropic efficiency and minimization of the compressor required power. Furthermore, the results from optimization procedure were compared to real-data centrifugal compressor operating with S-CO_2.

Based on the outlined results, the following conclusions can be drawn:

- The combination of Mean Line method and NSGA-II can be considered as a fast and reliable numerical tool for preliminary predictions of the centrifugal compressor performance operating at the design point.
- The results have indicated that the set of loss models used in the present research can be used for centrifugal compressors operating with S-CO_2.
- For a given compressor size, the impeller efficiency can be maximized with few changes on the blade geometry.
- With regard to the Case 1, the optimized solution found a compressor isentropic efficiency 17% higher than baseline compressor; the compressor required power was reduced about 16% on the optimized solution.
- For the Case 2, the isentropic efficiency from optimization procedure increased 20% with an associated decreasing of the compressor required power of 20% when compared to the baseline compressor.
- For a preliminary design, the results from optimization procedure suggested that impeller losses can be drastically reduced.

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