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FAULT TOLERANT SUPPRESSION OF GROUND RESONANCE IN HELICOPTERS

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Abstract. *Ground Resonance (GR) in helicopters is a potentially catastrophic instability commonly caused by coalescence of the regressive cyclic blade lag mode with the fuselage motion in certain rotor speed ranges. It can limit the helicopter operational envelope and become the design of this type of vehicle a difficult task. This works introduces an alternative methodology to design an active control system to asymptotically stabilize GR of helicopter considering actuators faults in the actuators. The proposed approach can suppress this instability in all rotor speed range by using only one fault-tolerant control gain. The Lyapunov stability criteria is used to design the controller based on Linear Matrix Inequalities (LMI). A polytopic representation of the GR boundary and the actuators faults are used. Using convex optimization a fault-tolerant control gain is computed and the all unstable rotor speed range is stabilized. A numerical simulations are carried out to demonstrate the effectiveness of this methodology. The results confirm the viability of the proposed approach to design active fault-tolerant controllers to suppress the GR.*

Keywords: *Ground Resonance Helelicopter, Actuators Faults, Polytopic Representation, Linear Matrix Inequalities*

1. INTRODUCTION

Ground Resonance (GR) in helicopters is potentially catastrophic instability commonly caused by the couple of the regressive rotor mode with the fuselage in-plane motion in certain rotor speed ranges (Coleman and Feingold, 1956). A classical analysis of this phenomenon was developed by Coleman and Feingold (1956) using a simple linear mechanical model. Despite its relative simplicity, the method accurately predicts the GR boundaries of instabilities for isotropic rotors (i.e., all blades similar among them). In addition, different studies involving linear and nonlinear GR phenomenology can be found in Flowers and Tongue (1990); Kunz (2002); Sanches *et al.* (2012); Gourc *et al.* (2016) and references therein. GR occurrence implies to excessive levels of vibrations in helicopter's structure which can lead a catastrophic destruction of this, if they are not reduced on time.

Traditionals approaches to mitigate this instability involve the installation of lag dampers on the blade root. These passive lag dampers are designed to work under particular conditions and their performance can be substantially reduced when they are applied on different situations. Besides that, this kind of approach commonly not predicted faults of the actuation devices, which is essential to design reliable control systems. This is an important limitation for practical engineering applications. In another hand, the use of semi-active and active control strategies is an attractive option to suppression GR since they are capable of adjusting their dissipation levels for each rotor operational condition (Panda *et al.*, 2004) and also they are capable to assure the stability, performance indexes, robustnees and tolerant of actuators faults in all range of GR boundary.

In this context, this article proposes the use of linear matrix inequalities (LMI) to compute only one fault-tolerant control gain to stabilize a rotor speed range in which occurs ground resonance of helicopter. LMIs have been used to solve mechanical and aeromechanical problems, some of these presented in Johnson and Erkus (2002); Silva and Jr. (2006); Bueno *et al.* (2014), and these formulations are commonly based on the well known Lyapunov's theory (Boyd *et al.*, 1994). The typical solutions of these methodologies involve convex optimization which can be efficiently solved by using a powerful class of computational algorithms (Gahinet *et al.*, 1995).

The rotor speed and the actuator fault are represented by a convex polytopic hull in a GR boundary. The LMI version of the LQR (i.e., linear quadratic regulator) controller with decay rate restriction is used. Results and discussions based on the analytical developments and numerical simulations are introduced to show the proposed strategy to stabilize the GR considering actuators faults.

2. GROUND RESONANCE DYNAMIC MODEL

The mechanical model used in this work captures the essential features of GR phenomenon. It is assumed that the helicopter on its landing gear can be represented by effective parameters applied at the rotor hub, as done in Coleman and Feingold (1956). The fuselage on the landing gear is considered isotropic, which can be represented by a rigid block with equivalent mass m_f linked to linear springs with linear stiffness k_f along the x, y directions. It is further considered only inplane motions of the rotor hub, on the longitudinal x and lateral y directions, described by the x_f (i.e., associated to pitch angle) and y_f (i.e., associated to roll angle) degrees of freedom, respectively.

The rotor-blades system is considered isotropic (i.e., all blades are similar among them), operating through constant angular rotor speed (Ω). It is composed by four blades (N_b= 4) symmetrically mounted on the rotor hub. Each blade presents the lead-lag motion ξ_k (i.e., the blade in-plane motion) and its location is determined by the azimuthal angle ψ_k = $\Omega t + \frac{\pi}{2}(k-1)$ with x-axis (Coleman and Feingold, 1956). The blades are considered uniform and rigid, also including the same mass m_b, mass moment S_b, and mass moment of inertia I_b around the z-axis in relation to the lag rotating center (i.e., point A in Figure 1b). They have a root linear torsional spring k_b on the lag hinge. An illustrative scheme of this system is shown in Figure 1, where e is the lag hinge offset, b is the location of the blade's center of mass (C.M.) - the point B in Figure 1b measured from the lag hinge. This illustration is an important scheme to simplify the understanding of the physical meaning behind the equations presented herein.

For this problem the aerodynamic effects are not considered. Also, the fuselage and the blade structural damping are neglected in order to capture all the system's instabilities, as done in other similar self-excited vibration control problems, as for an instance, the aeroelastic flutter (Bueno *et al.*, 2014).

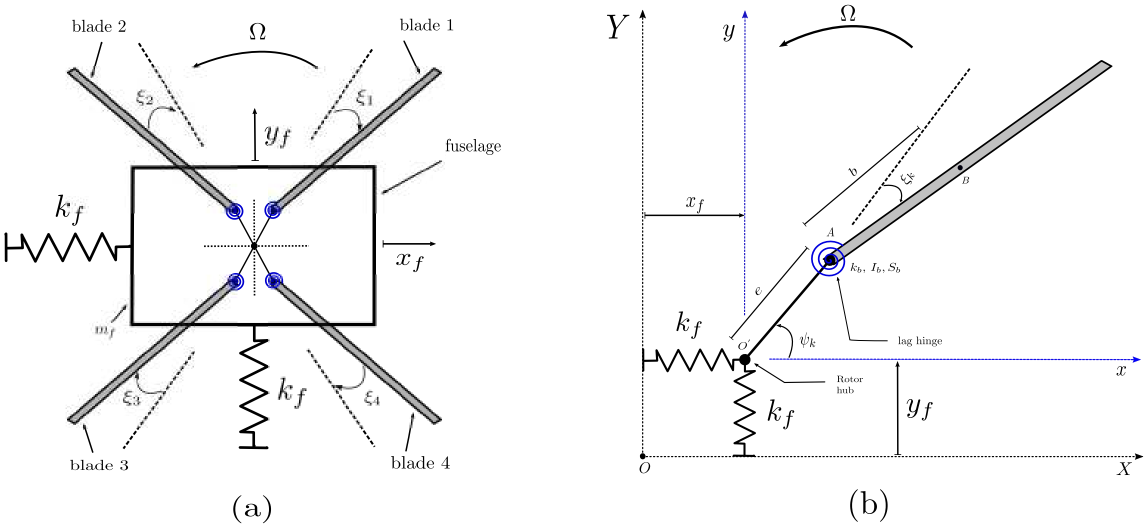

Figure 1. Mechanical model of a four-bladed helicopter rotor for GR analysis, (a) Top view from the model, (b) Detailed representation of the rotor-blade for a k-th blade.

Ground Resonance Model (GRM) is a system of equations that can obtained through the direct application of the Lagrange's equations. A linear GR model is considered in this work and to simplify the design of the controller, it is used a nondimensional form of the equations of motion by dividing these by Ω^2, that results in

$$\begin{aligned}
&\xi_k'' + Sx_f''\sin(\psi_k) - Sy_f''\cos(\psi_k) + \left(\frac{\omega_b^2}{\Omega^2} + eS\right)\xi_k = 0\\
&x_f'' + \sum_{k=1}^{N_b} S_f\xi_k''\sin(\psi_k) + 2\sum_{k=1}^{N_b} S_f\xi_k'\cos(\psi_k) - \sum_{k=1}^{N_b} S_f\xi_k\sin(\psi_k) + \frac{\omega_f^2}{\Omega^2}x_f = 0\\
&y_f'' - \sum_{k=1}^{N_b} S_f\xi_k''\cos(\psi_k) + 2\sum_{k=1}^{N_b} S_f\xi_k'\sin(\psi_k) + \sum_{k=1}^{N_b} S_f\xi_k\cos(\psi_k) + \frac{\omega_f^2}{\Omega^2}y_f = 0
\end{aligned} \tag{1}$$

where $\tau = \Omega t$ is nondimensional time; $()' = d()/d\tau = d()/\Omega dt$ and $()'' = d^2()/d\tau^2 = d^2()/\Omega^2 dt^2$ indicate the first and second nondimensional time derivatives; $S = S_b/I_b$ is the nondimensional blade mass moment; $S_f = S_b/M_f$ is the nondimensional fuselage inertia; $\omega_b^2 = k_b/I_b$ is the non-rotating lag blade frequency; $\omega_f^2 = k_f/M_f$ is the fuselage frequency, with $I_b = \bar{I}_b + b^2 m_b$ and $M_f = m_f + N_b m_b$.

2.1 COLEMAN TRANSFORMATION

GRM contains periodic terms that depend on the azimuthal angle ψ_k of each k^{th} blade. Coleman Transformation (Coleman and Feingold, 1956) is used to transforms the system of equations from the rotating frame (i.e., physical domain), where the system is linear time periodic, to a non-rotating frame (i.e., Coleman domain). In this second domain, the system is Linear Time Invariant (LTI) and represented in terms of Coleman coordinates. For a four helicopter rotor-blades, the blade lag angle (ξ_k) can be only written in terms of the cosine (ξ_{1c}) and sine (ξ_{1s}) components of the MBC cyclic modes, respectively (Johnson, 2013). The resulting system contains four degrees of freedom (dof), i.e., the longitudinal and lateral hub dofs and the sine and cosine ones. In this case, these can be conveniently compacted in a matrix form given by:

$$\mathbf{M}_{nr}\mathbf{q}''_{nr} + \mathbf{G}_{nr}\mathbf{q}'_{nr} + \mathbf{K}_{nr}\mathbf{q}_{nr} = \mathbf{0} \tag{2}$$

where $\mathbf{q}_{nr} = \{x_f \quad y_f \quad \xi_{1s} \quad \xi_{1c}\}^T$ is the new nondimensional vector of displacements (described in the non-rotating coordinate system); $\mathbf{M}_{nr}$, $\mathbf{G}_{nr}$ and $\mathbf{K}_{nr}$ are respectively the mass, gyroscopic and stiffness new matrices given by:

$$\mathbf{M}_{nr} = \begin{bmatrix} 1 & 0 & 2S_f & 0 \\ 0 & 1 & 0 & -2S_f \\ S & 0 & 1 & 0 \\ 0 & -S & 0 & 1 \end{bmatrix}, \; \mathbf{G}_{nr} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -2 \\ 0 & 0 & 2 & 0 \end{bmatrix}, \; \mathbf{K}_{nr} = \begin{bmatrix} \frac{\omega_f^2}{\Omega^2} & 0 & 0 & 0 \\ 0 & \frac{\omega_f^2}{\Omega^2} & 0 & 0 \\ 0 & 0 & \frac{\omega_b^2}{\Omega^2} + eS - 1 & 0 \\ 0 & 0 & 0 & \frac{\omega_b^2}{\Omega^2} + eS - 1 \end{bmatrix} \tag{3}$$

Based on the above matrices the following state-space realization can be used

$$\mathbf{x}'_{nr} = \mathbf{A}_{nr}\mathbf{x}_{nr}, \qquad \mathbf{A}_{nr} = \begin{bmatrix} \mathbf{0}_{4\times4} & \mathbf{I}_{4\times4} \\ -\mathbf{M}_{nr}^{-1}\mathbf{K}_{nr} & -\mathbf{M}_{nr}^{-1}\mathbf{G}_{nr} \end{bmatrix} \tag{4}$$

where $\mathbf{x}_{nr}(\tau) = \{\mathbf{q}_{nr}(\tau) \quad \mathbf{q}'_{nr}(\tau)\}^T \in \mathbb{R}^{2n\times1}$ is the state vector; $\mathbf{A}_{nr} \in \mathbb{R}^{2n\times2n}$ is the dynamic matrix and its eigenvalues allow to identify the GR for each steady Ω in the operational helicopter envelope (Coleman and Feingold, 1956), and $n = 4$ is the number of dofs in this new coordinate system.

3. CONTROLLER DESIGN TO SUPPRESS GR

3.1 ACTUATOR PLACEMENT ON EACH BLADE

To provide the control action is considered a hypothetical actuator mounted on each blade. The interaction of the control force (F_{a_k}) with the actuator horn (See Figure 2) results in a control moment (T_{a_k}) applied just in lag direction that is considered opposite to blade lag angle. A schematic illustration for the actuation attachment mechanism is shown in Figure 2. Therefore, the blade equation of motion defined in Eq. 1 can be rewritten including the control action $-T_{a_k}$ in its left hand side (i.e., instead of using zero).

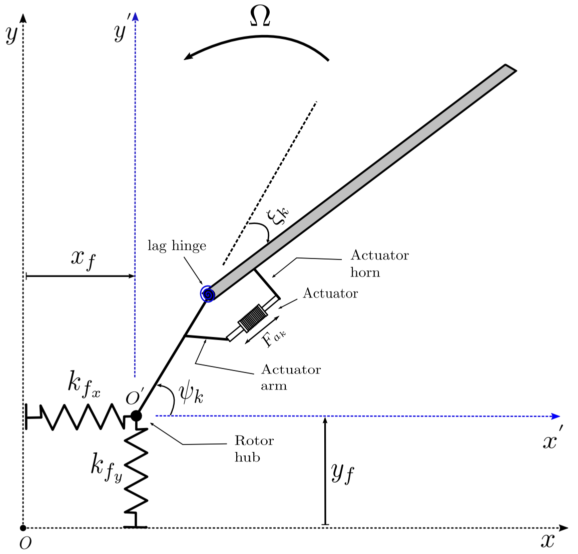

Figure 2. Attachment of the actuation's mechanism for GR mitigation.

To write the control force vector in the non-rotating coordinate system is necessary to consider the following MBC sine and cosine components of the control force

$$T_{a_{1s}} = \frac{2}{N_b}\sum_{k=1}^{N_b} T_{a_k}\sin(\psi_k), \quad T_{a_{1c}} = \frac{2}{N_b}\sum_{k=1}^{N_b} T_{a_k}\cos(\psi_k) \tag{5}$$

In this case, the system of equations (Eq. 2) can be rewritten also including the actuators force vector as

$$\mathbf{M}_{nr}\mathbf{q}''_{nr} + \mathbf{G}_{nr}\mathbf{q}'_{nr} + \mathbf{K}_{nr}\mathbf{q}_{nr} = -\frac{1}{I_b\Omega^2}\mathbf{T}_{a_{nr}} \tag{6}$$

where $\mathbf{T}_{a_{nr}} = \{0 \;\; 0 \;\; T_{a_{1s}} \;\; T_{a_{1c}}\}^T$ is the vector of control forces described in the non-rotating frame.

3.2 ACTUATOR FAULTS MODELING

Its considered that all the actuators are vulnerable to faults in response to the issued control signals. Thus, to account this kind of failure, the control input vector $\mathbf{u}_{nr}$ is replaced by $\rho\mathbf{u}_{nr}$. Based on the values of ρ, differents fault scenarios can be described. For instance, if $\rho = 0$, the actuator has completely failed; if $\rho = 1$ there is no fault in the actuator and this is totally effective; if $0 < \rho < 1$ there exists partial fault in the actuator. This actuator fault model was successfully used in the works Li *et al.* (2012), Wu *et al.* (2018), Song and Sun (2019).

To design the fault-tolerant controller its considered that ρ is bounded by its minimum value $\underline{\rho}$ and its maximum value 1, i.e., $\rho \in [\underline{\rho}, 1]$. Thence, based on the adopted fault model, a state space realization of Eq. 6 results in

$$\mathbf{x}'_{nr} = \mathbf{A}_{nr}\mathbf{x}_{nr} + \mathbf{B}_{nr}\mathbf{u}_{nr}, \text{ with,} \qquad \mathbf{B}_{nr} = \frac{\rho}{I_b\Omega^2}\begin{bmatrix}\mathbf{0}_{4\times 2} \\ -\mathbf{M}_{nr}^{-1}\tilde{\mathbf{B}}\end{bmatrix}, \quad \tilde{\mathbf{B}} = \begin{bmatrix}0 & 0\\0 & 0\\1 & 0\\0 & 1\end{bmatrix}, \tag{7}$$

where $\mathbf{B}_{nr} \in \mathbb{R}^{2n\times m}$ is the control input matrix; m is the number of control inputs.

This actuator fault model can be easily described by polytope convex hulls and included in LMI formulations for controllers, as reported in Li *et al.* (2012).

3.3 LMI FORMULATION TO LQR PROBLEM

To design the fault-tolerant controller, an LMI based LQR problem is implemented. Details of this formulation can be found in Johnson and Erkus (2002). In order to improve the transient response of the controller, a decay rate (θ) restriction it is considered following Boyd *et al.* (1994). Consider a LTI system described by Eq. 7, the LQR-LMI controller problem consists to obtain a control gain $\mathbf{F} \in \mathbb{R}^{m\times 2n}$ such that $\mathbf{u}_{nr} = \mathbf{F}\mathbf{x}_{nr}$, that minimize the quadratic performance index

$$J = \int_0^\infty (\mathbf{x}_{nr}^T\mathbf{Q}\mathbf{x}_{nr} + \mathbf{u}_{nr}^T\mathbf{R}\mathbf{u}_{nr})\, d\tau \tag{8}$$

subject to $V'(\mathbf{x}) + 2\theta V(\mathbf{x}) < 0$ (Boyd *et al.*, 1993), where $\mathbf{Q} = \mathbf{Q}^T > \mathbf{0} \in \mathbb{R}^{2n\times 2n}$ and $\mathbf{R} = \mathbf{R}^T > \mathbf{0} \in \mathbb{R}^{m\times m}$ are symmetric positive matrices that weighs the states and control energies, respectively (Anderson and Moore, 2007), and the pair $[\mathbf{A}_{nr}, \mathbf{B}_{nr}]$ must be controllable. The solution of LQR-LMI controller problem is introduced by Theorem 1.

Theorem 1 *The system described by Eq. 7 is asymptotically stable, in the sense of Lyapunov's theory, by a state feedback control law:* $\mathbf{u}_{nr} = \mathbf{F}\mathbf{x}_{nr}$ *and guaranteed cost* J*, if exists the matrices* $\mathbf{W} = \mathbf{W}^T \in \mathbb{R}^{2n\times 2n}$*,* $\mathbf{Z} \in \mathbb{R}^{m\times 2n}$ *and* $\mathbf{X} \in \mathbb{R}^{2n\times 2n}$*, such that satisfy the following convex minimization problem:*

$$\begin{gathered} \textit{min} \quad Tr(\mathbf{QW}) + Tr(\mathbf{X}) \\ \textit{subject to} \\ \mathbf{A}_{nr}\mathbf{W} + \mathbf{B}_{nr}\mathbf{Z} + \mathbf{W}\mathbf{A}_{nr}^T + \mathbf{Z}^T\mathbf{B}_{nr}^T < \mathbf{0}, \quad \begin{bmatrix}\mathbf{X} & \mathbf{R}^{1/2}\mathbf{Z} \\ \mathbf{Z}^T\mathbf{R}^{1/2} & \mathbf{W}\end{bmatrix} > \mathbf{0}, \quad \mathbf{W} > \mathbf{0}\end{gathered} \tag{9}$$

where the Lyapunov matrix and the state feedback gain are respectively given by $\mathbf{P} = \mathbf{W}^{-1}$ *and* $\mathbf{F} = \mathbf{W}^{-1}\mathbf{Z}$.

Proof: See Johnson and Erkus (2002). The matrices $\mathbf{W}$, $\mathbf{X}$ and $\mathbf{Z}$ in Ineq. 9 are obtained by solving the convex minimization problem, where $Tr()$ denotes the matrix trace.

3.4 POLYTOPIC REPRESENTATION OF GR BOUNDARIES OF INSTABILITY

To achieve an asymptotically stable system behavior in all rotor speed range $\left[\underline{\Omega}, \overline{\Omega}\right]$ that defines a GR boundary of stability using an unique control gain, a polytopic representation of this is performed. The strategy is to consider the rotor speed Ω as an uncertain parameter and define the associated polytopic convex hull. However, once the system of equations contains a quadratic term ($1/\Omega^2$, Eq. 7) involving the rotor speed, an algebraic manipulations must be previously used to write a LMI. In this sense, it is introduced a new uncertain parameter $\phi = 1/\Omega^2$ and it is considered the associated system $[\mathbf{A}_{nr} = \mathbf{A}_{nr}(\phi), \mathbf{B}_{nr} = \mathbf{B}_{nr}(\phi)]$ which depends on this new linear uncertain parameter limited through the minimum ($\underline{()}$) and maximum ($\overline{()}$) values, i.e., $\phi \in [\underline{\phi}, \overline{\phi}] = [\overline{\Omega}^{-2}, \underline{\Omega}^{-2}]$.

The design of the fault-tolerant controller to supress the GR involves the consideration of two individual polytopes associated to: (i) polytopic description of GR boundaries, (ii) actuator fault model, where each convex hull is defined in function of parameters ϕ and ρ, respectively. These polytopes must be combined to the controller design, however, an algebraic manipulations must be previously used to describe the model in affine form. Let $\varrho = \rho\phi$ and using the sector nonlinear method, results in $\varrho \in [\underline{\varrho}, \overline{\varrho}] = [\underline{\phi}\,\underline{\rho}, \overline{\phi}]$.

In practice, the admissible values of ϕ and ϱ are constrained in a polytope in the parameter space $\mathbb{R}^{n_p}$ with $n_v = 2n_p$ vertices (i.e., in this case $n_p = 2$). This polytope is built by the convex combination of each vertex (Boyd *et al.*, 1994). For this problem, the polytope of uncertain parameter is a rectangle with four vertices, $n_v = 4$, in parameter space $\mathbb{R}^2$ that can be expressed by

$$\mathcal{P} = \{\phi, \varrho\} \in Co\ \{[\phi, \varrho]_i, \ldots, [\phi, \varrho]_{n_v}\} \equiv \left\{ \sum_{i=1}^{n_v} \lambda_i [\phi, \varrho]_i,\ \sum_{i=1}^{n_v} \lambda_i = 1,\ \lambda_i \geq 0 \right\},\ i = 1, \ldots n_v \tag{10}$$

Based on these values it is possible to define the polytopic system by

$$[\mathbf{A}_{nr}, \mathbf{B}_{nr}](\phi, \varrho) \in Co\ \{[\mathbf{A}_{nr}, \mathbf{B}_{nr}]_i, ..., [\mathbf{A}_{nr}, \mathbf{B}_{nr}]_{n_v}\} \equiv \left\{ \sum_{i=1}^{n_v} \lambda_i \left[\mathbf{A}_{nr}, \mathbf{B}_{nr}\right]_i,\ \sum_{i=1}^{n_v} \lambda_i = 1,\ \lambda_i \geq 0 \right\},\ i = 1, \ldots, n_v \tag{11}$$

To achieve an asymptotically stable system behavior in all rotor speed range $\left[\underline{\Omega}, \overline{\Omega}\right]$ using an unique fault-tolerant control gain it is necessary to write the LMI in Eq. 9 for each vertex of the polytope defined in Eq. 11, as given by the following LMI:

$$\begin{gathered} \min\left[Tr(\mathbf{QW}) + Tr(\mathbf{X})\right] \quad \text{subject to} \\ \mathbf{A}_{nr_i}\mathbf{W} + \mathbf{B}_{nr_i}\mathbf{Z} + \mathbf{W}\mathbf{A}_{nr_i}^T + \mathbf{Z}^T\mathbf{B}_{nr_i}^T < \mathbf{0}, \quad \begin{bmatrix} \mathbf{X} & \mathbf{R}^{1/2}\mathbf{Z} \\ \mathbf{Z}^T\mathbf{R}^{1/2} & \mathbf{W} \end{bmatrix} > \mathbf{0}, \quad \mathbf{W} > \mathbf{0},\ i = 1, \ldots, n_v \end{gathered} \tag{12}$$

4. NUMERICAL APPLICATION

To illustrate the proposed approach, numerical simulations are carried out considering the helicopter's parameters defined in Table 1. The GR stability analysis is performed to find the GR boundaries using the Coleman method (Coleman and Feingold, 1956) and the fault-tolerant control design is done by using the introduced formulation.

Table 1. Helicopter parameters used in the numerical application (Sanches *et al.*, 2012).

Parameter	**Value**
Fuselage mass, m_f	2,902.9 kg
Blade mass, m_b	31.9 kg
Moment of inertia of blades, I_b	259 kg m^2
Offset, e	0.2 m
First blade mass moment, S_b	75 kg m
Blade non-rotating frequency, ω_b	3 π rad/s
Fuselage frequency, ω_f	6 π rad/s

4.1 GROUND RESONANCE ANALYSIS

An eigenvalue analysis is initially performed using the matrix $\mathbf{A}_{nr}$ (Eq. 4) to determine the frequencies ($\omega_i, \forall\, i = 1, ..., 4$) and modal damping ($\zeta_i, \forall\, i = 1, ..., 4$) for each system mode. The Coleman and the modal damping diagrams (Figures 3b and 3a) are used to characterize the stability of the rotor/fuselage system in the context of ground resonance.

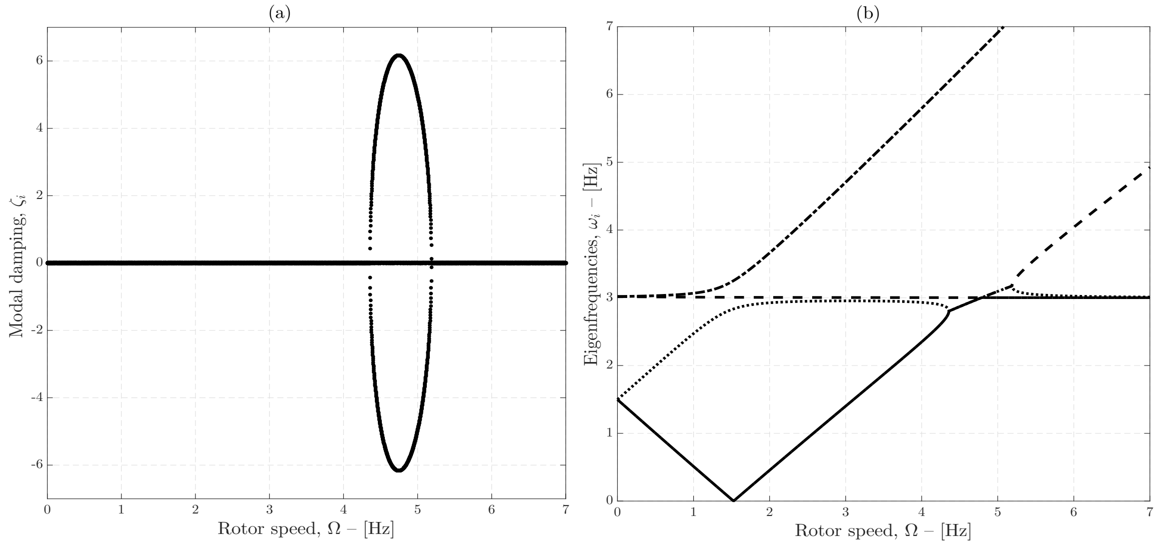

Figure 3. Ground Resonance analysis, (a) Modal damping diagram, (b) Coleman diagram.

Figure 3a shows the damping behavior from which one system instability is detected (i.e., the range of negative relative modal damping). Figure 3b shows the Coleman diagram, which presents the frequencies evolution through the rotor speed. By comparing these results is possible to identify the system instability in the coalescence of regressive blade mode (ξ_{1s}) with the fuselage modes (x_f and y_f). In this case one boundary of instability is observed because the fuselage is considered isotropic. Based on these results the unstable range of rotor speed is $\Omega \in [\underline{\Omega} = 4.33, \overline{\Omega} = 5.19]$ Hz.

4.2 DESIGN AND APPLICATION OF THE FAULT-TOLERANT CONTROLLER

To design the controller, its used a polytope to comprise the GR boundary of instability, considering a level of 30% (i.e., $\underline{\varrho} = 0.7$) for actuators fault. Thus, $\Omega \in [4.33, 5.19]$ [Hz], $\rho \in [0.7, 1]$ and based on these values the system can be represented by

$$(\mathbf{A}_{nr}, \mathbf{B}_{nr})(\phi, \varrho) = Co \left\{ \; (\mathbf{A}_{nr}, \mathbf{B}_{nr})(\underline{\phi}, \underline{\varrho}) \quad (\mathbf{A}_{nr}, \mathbf{B}_{nr})(\overline{\phi}, \underline{\varrho}), \quad (\mathbf{A}_{nr}, \mathbf{B}_{nr})(\underline{\phi}, \overline{\varrho}) \quad (\mathbf{A}_{nr}, \mathbf{B}_{nr})(\overline{\phi}, \overline{\varrho}) \; \right\}, \tag{13}$$

and the stabilization gain is computed by $\mathbf{F}$.

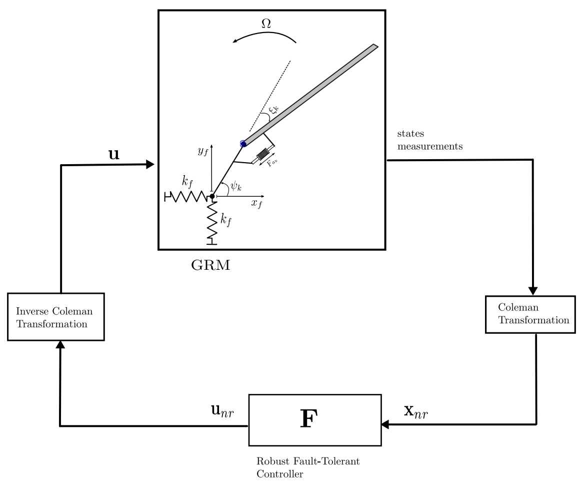

Figure 4. Feedback scheme for active suppression of GR phenomenon.

To compute the control gain is considered the new coordinate system obtained from the Coleman transformation. Based on this gain, the control forces $\mathbf{u}_{nr}$ are defined in the non-rotating frame and they need to be converted to the physical coordinates for practical applications. In this case, an inverse MBC transformation is applied on the control inputs as follows (see Figure 4):

$$\mathbf{u}(\tau) = \mathbf{T}_{nr}(\tau)\mathbf{u}_{nr}(\tau), \quad \mathbf{T}_{nr}(\tau) = \begin{bmatrix} \sin(\psi_1) & \cos(\psi_1) \\ \sin(\psi_2) & \cos(\psi_2) \\ \sin(\psi_3) & \cos(\psi_3) \\ \sin(\psi_4) & \cos(\psi_4) \end{bmatrix} \tag{14}$$

where $\mathbf{u} = \{T_{a_1} \;\; T_{a_2} \;\; T_{a_3} \;\; T_{a_4}\}^T$ is the physical control input vector and $\mathbf{T}_{nr}$ is the inverse MBC coordinate transform matrix. The resulting feedback gain is periodic once it depends on the azimuthal angle of blades.

For this numerical application, the control gain is computed by using $\theta = 0.1$, $\mathbf{Q} = 0.07 \times \mathbf{I}_{8\times8}$ and $\mathbf{R} = 0.001 \times \mathbf{I}_{2\times2}$, where $\mathbf{I}$ is the identity matrix. These values were defined arbitrarily.

Figure 5 shows the longitudinal (x_f) and lateral (y_f) fuselage time response at differents rotor speeds taked along the GR boundary. The results demonstrate the assured asymptotic stability in all these rotor speeds (i.e., the equilibrium position is reached in approx. 1.7 seconds) for the case with and without faults on actuators. It is observed a small degradation in the response of controller where the faults in actuators are present, therefore, these are not dangerous in function of the controller robustnees.

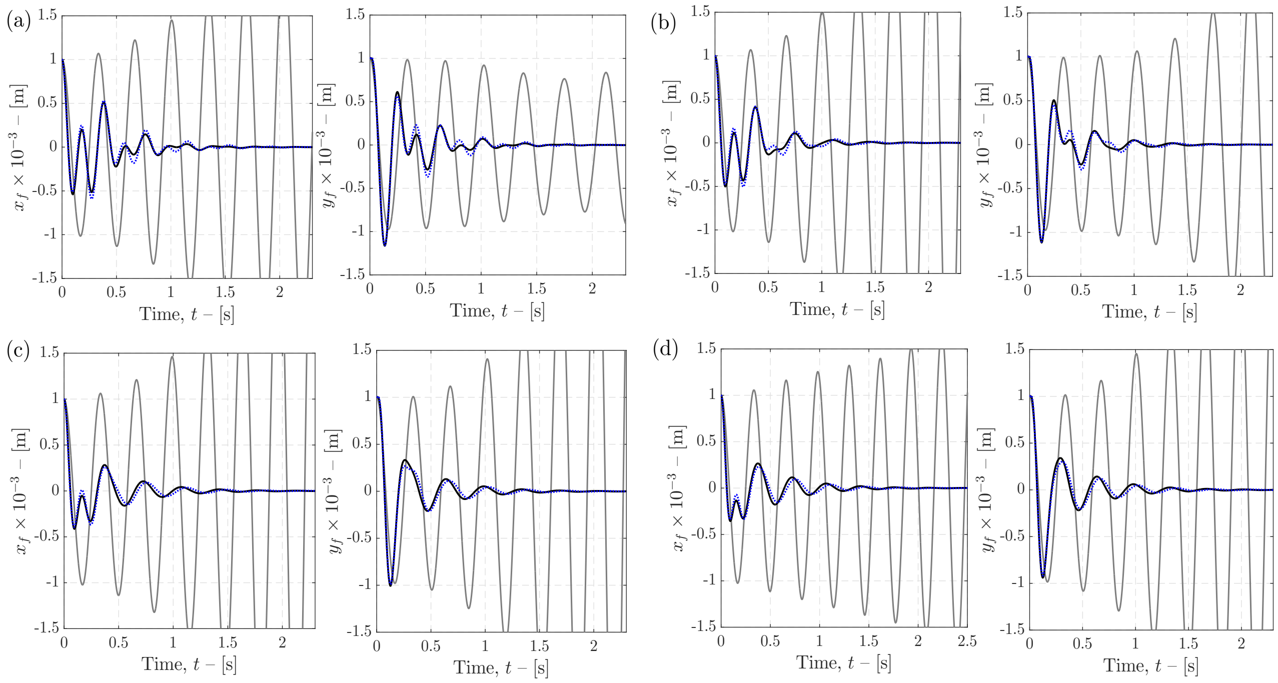

Figure 5. Time response of fuselage, (a) $\Omega = 4.33$ Hz, (b) $\Omega = 4.5$ Hz, (c) $\Omega = 4.9$ Hz, (d) $\Omega = 5.17$ Hz.

The control efforts developted by the actuators placed on the blades 1 and 2 are shown in Figure 6, respectively. For these blades the maximum control moments are 6.5 kN.m for blade 1 and 4.7 kN.m in blade 2.

Based on these results it is possible to conclude that the proposed control approach can be used to stabilize the GR phenomenon and tolerate a possible fault on the actuators.

The control efforts are calculated for each torque T_{a_i}, $i = 1, ..., 4$, and they can converted to control forces by using the geometric actuator arm (Zhao *et al.*, 2004). A maximum force level of 13 kN is obtained and it is representative, once a typical hydraulic actuator for helicopter blades supplies ∼3200 lbf, i.e., ∼14.3 kN (Bauchau and Liu, 2006). Also, a modern semi-active MR lag damper supplies ∼2369-4737 lbf, which corresponds to ∼10.6 - 21.1 kN (Zhao *et al.*, 2004) and a semi-active lag damper with friction effects can develop ∼6000 lbf, which corresponds to ∼26 kN (Bauchau *et al.*, 2010).

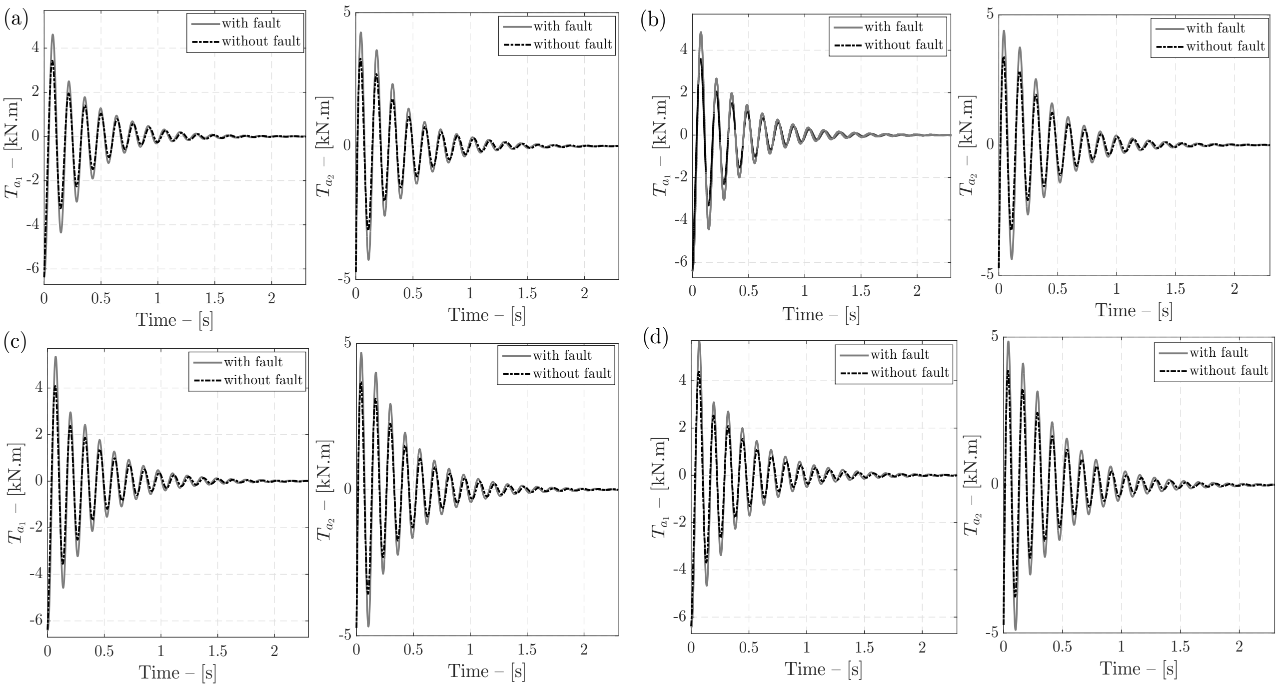

Figure 6. Control moment on the blades 1 (T_{a_1}) and 2 (T_{a_2}), (a) $\Omega = 4.33$ Hz, (b) $\Omega = 4.5$ Hz, (c) $\Omega = 4.9$ Hz, (d) $\Omega = 5.17$ Hz.

5. CONCLUSIONS

In this work an alternative approach to stabilize the GR phenomenon in helicopters using active control techniques considering actuators faults is presented. The classical mechanical model for GR analysis is employed.

The main contribution of this work consists to represent the GR boundaries and the actuators faults by convex polytopic hulls. A LMI version of the LQR controller with a restriction on decay rate is employed. The quadratic robust criteria ensures that all previously unstable rotor speed range becomes asymptotically stable by using only one fault-tolerant control gain. Also, this representatin is able to keep commom levels of control force in all GR boundary.

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