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## **ANALYSIS OF THE ACOUSTIC EMISSION OF A BUBBLE IN LIQUID MEDIUM**

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**Abstract.** *The sound emitted by a bubble of gas dissolved in a fluid medium is a phenomenon that has been studied since the late nineteenth century. In order to identify the frequency spectrum of this type of acoustic emission, different mathematical models have been proposed since then. In the present article, preliminary results of the computational simulation of the classic M. Minnaert and M. Strasberg models are presented. After the adequate validation of these models against experimental measurements of acoustic emission of air leaks generated in the test tank of the EPUSP Dynamics and Instrumentation Laboratory, these models will be used to generate a priori estimates of the acoustic properties of signals characteristic of gas leaks.*

**Keywords:** *bubble acoustics, signal processing, gas leakage*

### **1. INTRODUCTION**

The high concentrations of CO<sub>2</sub> in the atmosphere, the environmental problems, and the strong evidence that there is a relationship between global warming and an accelerated growth of industrial activity in the last two centuries, have resulted in different global actions that became part of international policies (IPCC, 2005; UNFCCC, 2008; Liverman, 2009; Knutti *et al.*, 2016) to prevent some catastrophic climate scenarios with potentially harmful effects on the ecosystem could be consolidated in a few decades.

To support the actions to eliminate greenhouse gases (CO<sub>2</sub>, CH<sub>4</sub>, NO<sub>2</sub>, etc) from the atmosphere, since the mid-1990s new technologies have been developed and adapted to capture and store carbon CCS, (Carbon Capture and Storage). One of the most common procedures adopted by the industries involved in these activities is to use natural salt deposits, either submarine or underground, formed by sedimentary rocks (Bacchu, 2008; Eccles *et al.*, 2009; Hosa *et al.*, 2011) to store CO<sub>2</sub> in a supercritical state. However, this procedure is not free of risks, and the possible leakage of gases, either during the injection or after filling the tanks, is one of the problems that most concern the industries involved in the activity, given the environmental prejudices that this can unleash.

Techniques for monitoring and identifying leaks in CO<sub>2</sub> deposits together with the development of mathematical models are topics that have been investigated in recent decades and are present in different works. Among the various techniques used, those based on the identification of acoustic signals emitted by gas leakages are among the most promising.

In the current article, we will present an analysis of two models of acoustic emission of bubbles widely cited in the literature, which were validated with some experiments carried out in our laboratory.

### **2. MATHEMATICAL MODELS**

Minnaert (1933) was the first to propose a mathematical model validated from laboratory experiments, capable of estimating the fundamental frequency of the acoustic signal emitted by a bubble immersed in liquid medium. In his experiments, a gas was injected under controlled conditions through a tube submerged in a tank filled with water in order to generate bubbles with diameters between 3 and 6 mm. The author observed that every bubble emitted an acoustic signal just as it separated from the tube; He also noted that the pitch of the sound was not constant, but exhibited irregular variations over time. From these observations, Minnaert proposed a mathematical model of acoustic emission based on the following hypotheses: 1<sup>st</sup>) the bubble is a sphere of variable radius  $r$  which oscillates around a reference value  $r_0$  according to a harmonic law; 2<sup>nd</sup>) an impulsive signal is generated at the moment the bubble closes; 3<sup>rd</sup>) the inertia of the surrounding water and the elasticity of the air within the bubble cause it to expand and contract periodically, thereby

producing a sound vibration that propagates through the water; 4<sup>th</sup>) the process of bubble formation can be considered adiabatic.

Applying the energy conservation equation to a control volume involving the bubble and all the liquid contained in the tank, and considering a compression/expansion cycle in which the volume of the bubble evolves from the reference  $V_0$  to a generic value  $V$ , yields

$$\Delta U = - \int_{V_0}^V (p - p_0) dV = \Delta K = \frac{1}{2} \int_r^\infty \left( \frac{dr}{dt} \right)^2 dm \quad (1)$$

where  $\Delta U$  and  $\Delta K$  are the variations of potential and kinetic energy in the control volume, respectively,  $p$  is the pressure inside the bubble when its volume is  $V$ ,  $p_0$  is the static pressure outside the bubble and  $dr/dt$  is the radial velocity of the liquid measured from the center of the bubble.

Assuming that  $r$  varies according to

$$r(t) = r_0 + a \sin \left( \frac{2\pi}{T} t \right) \quad (2)$$

and considering bubble periodic expansion and compression as an adiabatic process,

$$p - p_0 = \frac{3\gamma p(r - r_0)}{r} \quad (3)$$

for which  $\gamma = c_p/c_v$  is the heat capacity of the gas, the development of Eq. (1) leads to the expression of the fundamental frequency  $f_0$  of the signal emitted by the bubble, i.e.

$$f_0 = \frac{1}{T} = \frac{1}{2\pi r} \sqrt{\frac{3\gamma p}{\rho}} \quad (4)$$

where  $\rho$  is the density of the liquid.

The above expression shows that the tone frequency is inversely proportional to the radius of the bubble and that it is comparatively lower for higher dense liquids. It is important to emphasize that all the previsions derived from the model Eq. (4) could be confirmed by the series of experiments carried on by Minnaert. Prior to (Minnaert, 1933; Lamb, 1945) had already proposed a model in which the acoustic signal emitted by the bubble was represented by a summation of harmonic surfaces such that the spatio-temporal evolution of the bubble radius was expressed as

$$r(t, \theta, \varphi) = r_0 + \sum_{i=1}^n r_i S_i(\theta, \varphi) e^{i2\pi f_i t} \quad (5)$$

where  $\theta$  and  $\varphi$  are the spherical angular coordinates,  $S_i$  is the harmonic surface of order  $i$  and  $r_i$  is the respective amplitude of the oscillation in this mode.

Strasberg (1956) showed that, apart from the first mode, related to the temporal variation of volume without the corresponding shape variation, the contributions of all other modes Fig. (1) to the acoustic emission spectrum of the bubble are absolutely irrelevant.

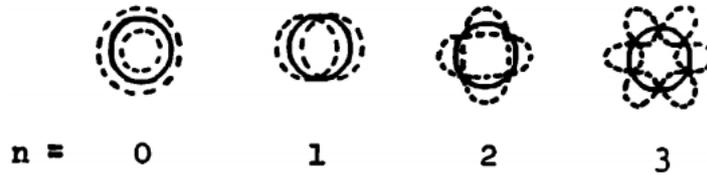


Figure 1. Shape of an oscillating bubble in several modes. The bubble walls are shown in section, with the reference configuration drawn in solid line and the extreme ones drawn in dashed lines. Extracted from (Strasberg, 1956).

Therefore, considering only the mode  $n = 0$  and admitting that the amplitude of oscillations is small, (Strasberg, 1956) suggests that the volume of the bubble can be described by a classic dynamic damped harmonic oscillator driven by an external force, as follows:

$$\frac{\rho}{4\pi r_0} \ddot{V} + c\dot{V} + \frac{\gamma P_0}{V_0} (V - V_0) = p_0 - p_e(t) \quad (6)$$

where  $V$  represents the bubble volume,  $c$  is a friction coefficient that must be determined experimentally,  $p_0$  is the static pressure when the bubble volume is  $V_0$  and  $p_e$  is the instantaneous external pressure of the liquid at the bubble site, assuming its absence.

It can be noticed that the undamped natural frequency of the system described by Eq. (6) corresponds exactly to the formula proposed by Minnaert Eq. (4). Hence, from Eq. (6), the dynamical model describing the instantaneous pressure of the sound emitted by the bubble at a given distance  $d$  is straightly derived, yielding

$$p_s(t) = \frac{\rho \ddot{V}}{4\pi d} \quad (7)$$

The variation of the external pressure of the liquid during the bubble formation process is a very brief phenomenon, so that the approximation  $p_0 - p_e(t) = 0$ , leading Eq. (6) to its homogeneous form, adequately describes the acoustic signal emitted by the bubble. Therefore, the solution of

$$\frac{\rho}{4\pi r_0} \ddot{V} + c\dot{V} + \frac{\gamma P_0}{V_0}(V - V_0) = 0 \quad (8)$$

provides the temporal evolution of the sound pressure  $p_s(t)$  radiated by the bubble, ie ,

$$p_s(t) = P_0 e^{-\pi\delta f_0 t} \cos(2\pi f_0 t - \varphi) \quad (9)$$

where  $\delta$  is a dissipative coefficient that, according to (Strasberg, 1956), can be calculated using the following empirical expression

$$\delta = 0,014 + 1,1 \times 10^{-5} f_0 \quad (10)$$

and  $P_0$ , the amplitude of the sound pressure, is given by

$$P_0 = \frac{\rho f_0}{2d} \sqrt{\dot{v}_0^2 + 4\pi^2 f_0^2 (v_0 - V_0)^2} \quad (11)$$

In Eq. (11),  $v_0$  and  $\dot{v}_0$  are the initial values of bubble volume and the bubble volumetric growth rate. Those values can be measured by shooting with a high-speed camera the evolution of the bubble over time, but the lack of appropriate instruments can make this solution unfeasible.

The initial value  $r_0$  of the radius of the bubble can be determined indirectly: firstly, Fourier analysis is applied to the acoustic signal in order to obtain an estimate  $\hat{f}_0$  of the fundamental frequency  $f_0$ ; Then, substituting  $\hat{f}_0$  into Minnaert's formula Eq. (4), gives the estimate value of  $r_0$ .

In the article of (Strasberg, 1956), a series of simplifying assumptions applied to Eq. (11) leads to the following expression for the  $P_0$  amplitude:

$$P_0 = \frac{r_0}{d} \sqrt{2\gamma p_0 p_+} \quad (12)$$

where  $r_0$  is the initial value for the radius of the bubble and  $p_+$  is a parameter called by the author as excess pressure, whose calculation or estimation method is not mentioned by the author at any point of his article. This omission of a method of calculation so essential for justifying the results obtained and experimentally validated by the author had already been highlighted by (Frizell, 1987), but may perhaps be explained by the fact that Strassberg was a researcher of the United States Navy and, at the time, it was investigating the submarine propellers noise generated by cavitation, a very sensitive topic indeed.

Thus, assuming that the excess pressure  $p_+$  is somehow determined, the set of Eq. (4), Eq. (9), Eq. (10) and Eq. (12) composes a simple analytical model capable of describing the acoustic emission of bubbles immersed in liquid medium.

In addition to the models presented and described in this article, there are other investigations that can be found in the literature related to this research field, such as the articles of (Deane and Czerski, 2008) and (Ainslie and Leighton, 2011). A very important reference is (Leighton, 1997), a book that addresses various aspects of the phenomenology of acoustic bubble emission.

Importantly, many of the recently published acoustic bubble emission models use original ideas from (Lamb, 1945), (Minnaert, 1933), and (Strasberg, 1956) which is why we have emphasized this bibliography in our article.

### 3. EXPERIMENTAL EQUIPMENT SETUP

Our experimental apparatus is composed of a data acquisition system, a leakage and pressure control system, and cylindrical tank water that is 3 meters deep and 3 meters in diameter as shown in Fig. 2,

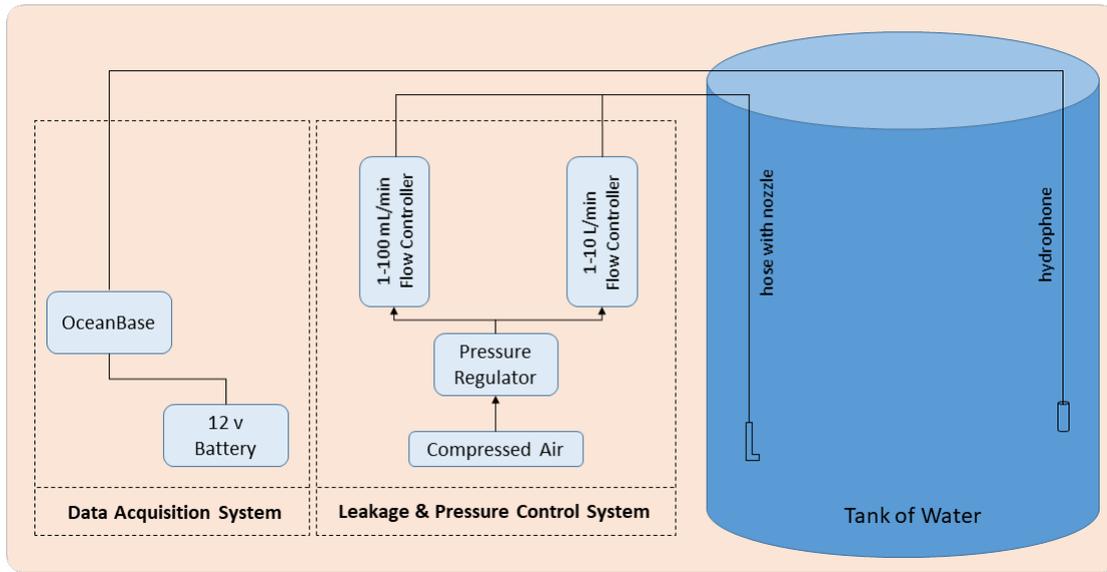


Figure 2. Sketch of our Experimental Data Acquisition System.

The leakage and pressure control system is composed of a pressure regulator that can allow a maximum pressure of 10 bar, a gas flowmeter capable of measuring flow rates from  $1,67 \times 10^{-8}$  to  $16,67 \times 10^{-5} \text{ m}^3/\text{s}$ , a polyurethane industrial hose with an adapter device that can be connected to nozzles of different diameters, and a compressed air system.

The data acquisition system is composed of an OceanBase<sup>TM</sup> device, which captures the audio signals through a hydrophone. This equipment is capable of recording up to 48 kHz sampling frequency in 32-bit WAV format.

In order to validate the models discussed in this article, the experimental apparatus was duly adjusted so as not to eject bubble plumes, but individual bubbles whose acoustic signals could be individually identified. Thus, the air flow was set at an extremely low value and the hose was coupled to a small diameter nozzle (6mm). The hose was submerged until the nozzle reached the bottom of the tank, that is, 3 meters deep; The hydrophone was located at a depth of 2m and 1m away from the nozzle.

#### 4. RESULT

Since we did not have a high-speed camera in our experimental set-up that would allow us to determine the initial radius of the bubble, we applied the Fourier transform Fig. 4 to the acoustic signal captured by the data acquisition system Fig. 3, thus identifying its fundamental frequency  $f_0$  (610Hz, for the Fourier transform peak value). Then, substituting this value in Minnaert's formula Eq. (4), we estimate the initial radius  $r_0$  of the bubble, which, in the case, was 5,4mm.

As already pointed out in Section 2, given that in Strasberg's article no mention is made of the method for calculating the excess pressure  $p_+$ , we estimate this parameter from experimental measurements. In the graph of Fig. 3 we considered the peak sound pressure  $P_0 = 0,03\text{Pa}$  and substituted this value in Eq. (12), ie, we estimated  $p_+$ , according to

$$p_+ = \frac{1}{2\gamma p_0} \left( \frac{P_0 d}{r_0} \right)^2, \quad (13)$$

so that  $p_+ = 1.16 \times 10^{-4}\text{Pa}$  is the necessary value for this parameter in order that the experimental measurements match the Strasberg's model.

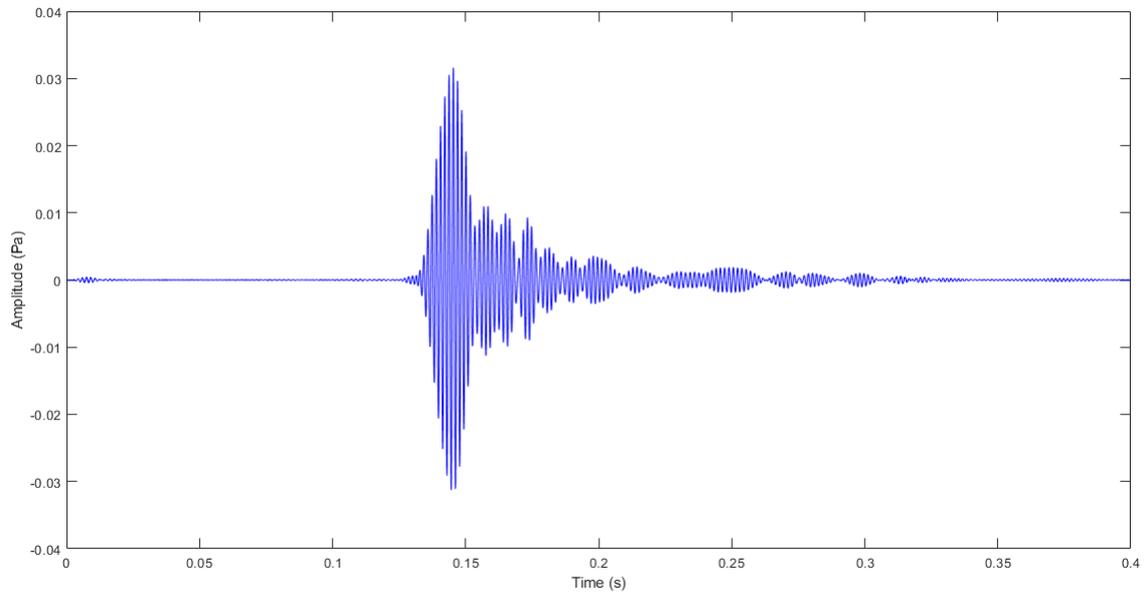


Figure 3. Acoustic emission captured by the data acquisition system.

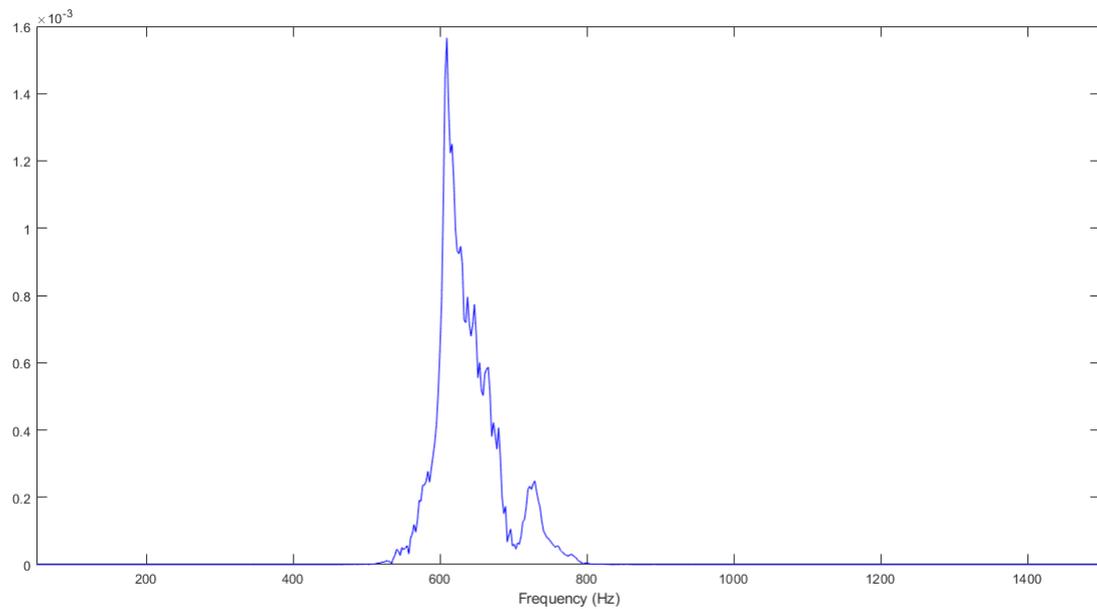


Figure 4. Fourier transform of the signal of Fig. 3

Then, adopting the estimate values for  $r_0$  and  $p_+$  and using Eq. (4), Eq. (10), Eq. (12) and Eq. (9), we arrive at a simple second order model representing the phenomenon of acoustic emission of bubbles in liquid medium. Fig. 5 shows the signal predicted by the Minnaert and Strasberg models, corresponding to the acoustic emission of a bubble under the conditions of the experiment performed in our laboratory.

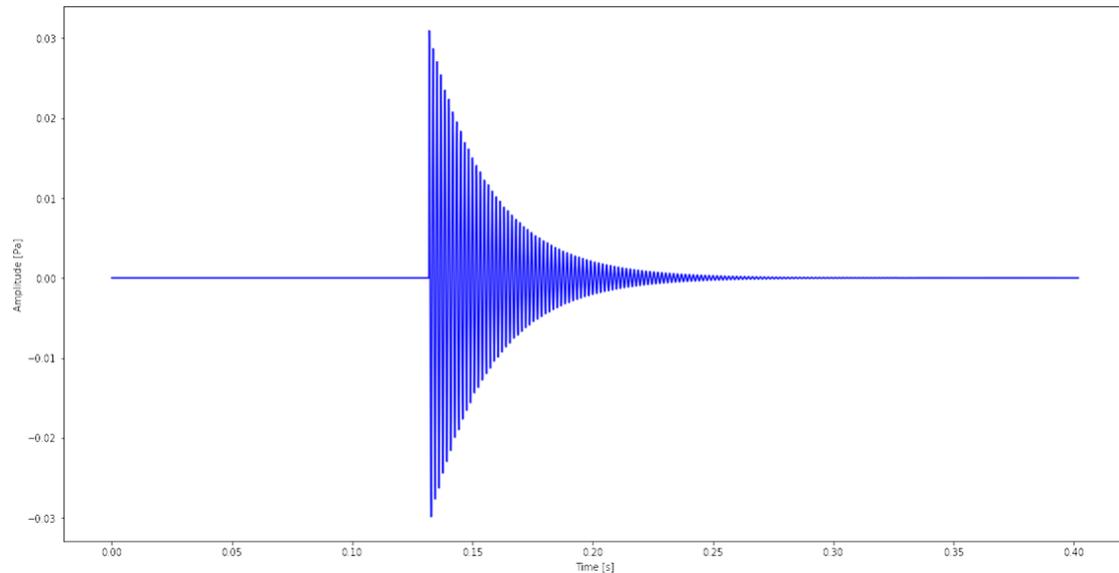


Figure 5. Audio signal of a bubble obtained in the simulations with radius: 5,4mm; frequency: 610Hz; amplitude: 0.03Pa

## 5. CONCLUSIONS

In this paper we analyzed two classic mathematical models of literature (Minnaert, 1933 and Strasberg, 1955), aimed at describing the acoustic emission of bubbles in liquid medium. We further show that, despite their mathematical simplicity, these analytical models can be used in combination with experimental measurements to estimate some important physical parameters concerning the focused phenomenon. It is also important to stress that, although there are more complex models in recent literature, they rely largely on ideas and methods previously presented in the articles of (Minnaert, 1933) and (Strasberg, 1956).

In the next step of our work we will use these two analytical models to construct a priori estimates of bubble plume acoustic emissions. This will be done by summing up a large number of analytical signals concerning individual bubbles of initial radii  $r_0$  distributed according to a suitable Gaussian distribution and with phases  $\varphi$  uniformly distributed in the range  $[0; 2\pi]$ .

Evidently, the synthesized acoustic signals thus estimated may serve as a reference for gas leak detection algorithms in an aquatic environment, that is the final object of this work.

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