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### SMALL SIZE HELICOPTER DYNAMICS IDENTIFICATION

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**Abstract.** *Flight testing of an instrumented model-scale Flybarless unmanned helicopter (Goblin 380) was conducted for the purpose of dynamic model identification. The time domain Output Error Method technique was applied in order to identify the bare airframe helicopter dynamics model based on flight test data. The model structure considers the helicopter and all its components as a lumped system. The linear state-space model was derived for the hover condition. The application of the identification method resulted in an adequate curve fitting for the closed loop system, airframe and stabilization system. The stability derivatives for the lateral, longitudinal, directional and heave dynamics were obtained. The resulted model can be used to drive the development of classic and modern control strategies, as well, navigation and guidance for the unmanned helicopter vehicle.*

**Keywords:** *Unmanned Helicopter, Helicopter Dynamics, System Identification*

#### 1. INTRODUCTION

The field of Unmanned Aerial Vehicles (UAVs) has advanced significantly in the last years. The demand of the use of such technology, since military and civilian operations, also increased rapidly. Each application demands for specific aerial vehicles characteristics. The helicopters are very versatile, they can take off and land vertically, hover and fly forward, backward and laterally. These characteristics allow the helicopter to be deployed in congested or isolated areas where the fixed wing cannot. In order to apply these impressive flying capabilities in an UAV platform a flight control system is needed. The design of such control system starts from the plant model knowledge. Helicopters are highly non-linear, coupled, unstable and presents fast dynamics. Because of the high complexity of the helicopter flight dynamics, retrieve a complete model by the first principle involving all the flight dynamics phenomena can be a very hard task rather results in uncertainties. However the development of the flight control system requires a minimum flight dynamics modeling accuracy otherwise the response performance achievement becomes inaccessible to most control design methods.

In order to overcome these issues the system identification is carried out in the selected aerial platform. System identification is a procedure by which a mathematical description of vehicle dynamic behavior is extracted from flight test data. System identification can be thought of as an inverse of simulation. Simulation requires adoption of a priori engineering assumptions for the formulation of model equations. These simulation models allow the prediction of aircraft motion. In contrast, system identification begins with measured aircraft motion and "inverts" the responses to extract a model that accurately reflects the measured aircraft motion, without making a priori assumptions.

Small radio controlled helicopters are initially suitable to research and development programs of rotary wing UAVs. Rather than full scale helicopters, radio controlled helicopters have its heads almost totally rigid, i.e., the rotor head does not contain the flapping hinge, referred as hingeless rotor head, and so the blades can't move upwards and they are too stiff to flap significantly. The aero-elastic flapping motion is in most cases the primary stabilizing method in full-scale helicopters as the blades are free to flap, or springs are mounted on the rotor to increase the stability of the system. The hingeless system is adopted to decrease the control time and give a better sensation of control to the pilot, however as the model size decreases the inherent instability of the system increases.

To overcome the lack of stabilization of the hingeless system in radio controlled helicopters, a stabilizer bar is added to the rotor system. This "flybar" is usually orientated at a 90 degrees offset angle to the main rotor blades. The stabilizer bar acts as a lagged rate feedback in the pitch and roll axes, reducing the bandwidth and control sensitivity to cyclic lateral and longitudinal inputs. This type of rotor head is called Bell-Hiller, see Trevisan.

When no stabilizer bar is used, in a "Flybarless" rotor head, also called Bell rotor head, there is no dampening and no self correction offered from a flybar. As mentioned earlier, this makes the the rotor system very sensitive, too sensitive in most cases for smaller size radio controlled helicopters leading to instability. Such type of rotor system requires a System Augmentation and Control System (SCAS) to stabilize the helicopter. In the radio controlled world

this type of stabilization system is called Flybarless System (FBL System). The FBL System basically consists in a Flight Control System (FCS) based on angular rates feedback to PID controllers and a feed forward path in order to provide stability and precision control to pilot inputs. Additionally a mixing unit provides the signal to the swash plate servos. And so, basically, the block diagram of a FBL System is shown in Fig. 1. However the exact control structure is unknown and changes from manufactures to manufacturer.

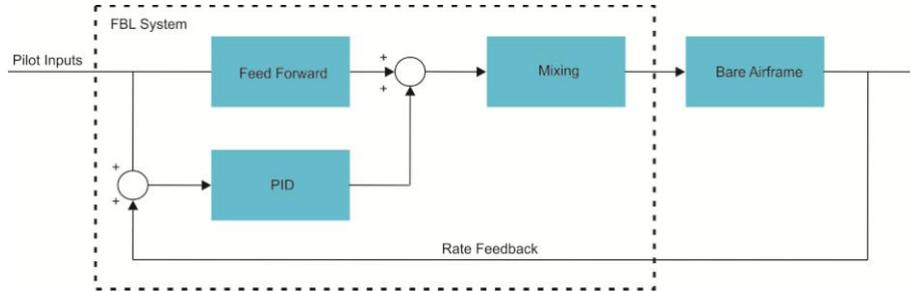


Figure 1. Flybarless System

Necessary to mention, even in a Flybared or Flybarless helicopter the same fashion rate feedback is needed in order to stabilize the tail. The FBL System incorporates this feature, however in the case of a Flybared helicopter an external gyro must be installed.

There are many advantages of the Flybarless helicopter equipped with a FBL System: more stable hovering; enhanced maneuverability while still being stable; the rotor head is simpler with fewer parts; and fewer parts means lower weight and less drag, which translates into more power. However, they are more expensive compared to a flybared one; there is more load on the cyclic servos and more difficult to setup. In this research a Flybarless helicopter was selected to system identification. The helicopter is equipped with a Commercial Off-The-Shelf (COTS) FBL System in order to resolve the inherent instability of the Flybarless head.

The goal of this paper is to identify the helicopter hover dynamics in order to drive the flight control system development. The first approach is to identify the closed loop system, i.e., the helicopter bare airframe and the FBL System dynamics, this is called Indirect Identification. The Indirect scheme identifies a closed-loop system by using the reference signal and the response signal, National Instruments (2013), see Fig. 2.

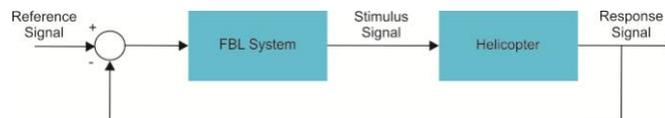


Figure 2. Indirect Identification Scheme

In this paper, a time domain system identification method, Output Error Method (OEM) is applied to the unmanned helicopter hover flight condition. Since its introduction in the 1960s, the OEM is the most widely applied time-domain method to estimate aircraft parameters from flight data, Jategaonkar (2006). The outcome are the stability derivative values for the state equations that matches the input/output time domain data set for this point of flight envelope driving the development of classic and/or modern control strategies for a helicopter unmanned vehicle.

## 2. VEHICLE DYNAMICS

The model structure for our small-size helicopter is largely based on the model structure used for the identification of full-size helicopters. The model structure specifies the order and form of the differential equations which describe the dynamics. Typically, the dynamics of the helicopter are represented as rigid-body (airframe dynamics, 6 degrees of freedom), which can be coupled to additional dynamics such as the rotor or engine/drive-train dynamics. For control purposes, as well as for flying qualities studies, simpler linear models are often sufficient. Linear models have been used extensively and successfully for rotorcraft aircrafts. At a precise operating point, and even within a certain region around that point, linear models accurately capture the essential affects of the vehicle, Mettler and Tischler (1999). In this paper the derivation of the model structure considers the helicopter bare airframe and its FBL as a lumped system. The complete linear model structure is obtained by collecting all the differential equations in the matrix differential equation Eq. (1):

$$\dot{x} = Fx + Gu \tag{1}$$

The state vector and control vector are given by Eq. (2) and Eq. (3):

$$x = [u \ v \ w \ p \ q \ r \ \varphi \ \theta \ \psi]^T \quad (2)$$

$$u = [\delta_{A1} \ \delta_{B1} \ \delta_{ped} \ \delta_{col}]^T \quad (3)$$

The lateral dynamics for system identification are given by the equations from Eq. (4) to Eq. (6):

$$\dot{p} = L_p p + L_{A1} \delta_{A1} \quad (4)$$

$$\dot{\varphi} = p \quad (5)$$

$$\dot{v} = Y_v v + g\theta + Y_p p + Y_{A1} \delta_{A1} \quad (6)$$

The longitudinal dynamics for system identification are given by the equations from Eq. (7) to Eq. (9):

$$\dot{q} = M_q q + M_{B1} \delta_{B1} \quad (7)$$

$$\dot{\theta} = q \quad (8)$$

$$\dot{u} = X_u v + X_q q + X_{B1} \delta_{B1} \quad (9)$$

The directional dynamics for system identification are given by equations Eq. (10) and Eq. (11):

$$\dot{r} = N_r r + N_{ped} \delta_{ped} \quad (10)$$

$$\dot{\psi} = \frac{r\varphi}{\theta} \quad (11)$$

The heave dynamics are given the equation by Eq. (12):

$$\dot{w} = Z_w w + Z_{col} \delta_{col} \quad (12)$$

### 3. FLIGHT TESTS

For these tests, the aircraft was instrumented with an Attitude Heading Reference System, for attitude, angular rates and acceleration measurements. A consumer off-the-shelf FBL system was installed and configured in the aircraft to deal with the rates stabilization. Additionally, an onboard computer equipped the aircraft for data logging. See Fig. 3.

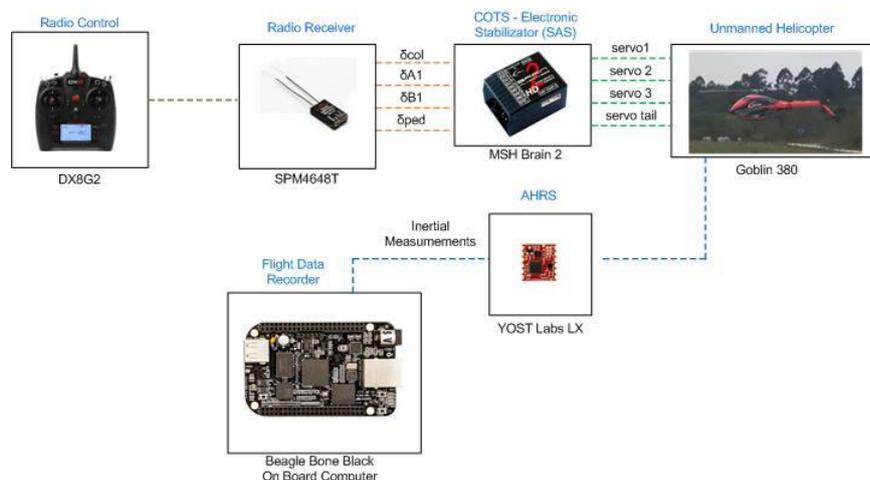


Figure 3. Unmanned Helicopter System Architecture

In order to perform the close loop system identification, the cyclic commands, the collective command and the tail rotor command were calibrated to  $\pm 100\%$  excursion and its measurements are also available in the onboard computer log file.

### 3.1 Aerial Platform Characteristics

The helicopter selected for the system identification is the Goblin 380. It is a small-size two-bladed Flybarless helicopter originally designed for entertainment. Its specification is shown in Tab. 1.

Table 1. Goblin 380 Specification

Overall Length	704mm
Overall Width	130mm
Overall Height	224mm
Overall Weight <sup>(1)</sup>	1.500Kg ~
Main Rotor Diameter	856mm
Main Blade Length	380m
Tail Rotor Diameter	192mm
Tail Blade Length	70mm
Gear Ratio	5:1
Rotor Speed	2600RPM
Engine Type	Electric 890RPM/V
Flight Autonomy	8 min

<sup>(1)</sup> Including rotor blades, battery and avionics

## 4. ESTIMATION PROCESS

### 4.1 Estimation Algorithm

In this section, the parametric identification scheme applied to the 6DoF motion helicopter dynamics is described. The Output Error Method is one of the most used estimation methods in aircraft identification and aerodynamic parameter estimation. It has several desirable statistical properties, including its application to nonlinear dynamical systems and the proper accounting of measurements noise. The model structure is supposed to be known, as it is discussed in the previous sections, and the identification process consists in determining the parameter vector  $\Theta$ , which gives the best prediction of the output, signal  $z(t)$ , using some sort of optimization criteria. The attainment of an estimate through optimization of a cost function based on the prediction error of the plant requires, usually, the minimization of a nonlinear functional. Thus, the Levenberg-Marquadt method is used here as the optimization algorithm. The cost function to be minimized involves the prediction error Eq. (13)

$$e(k, \hat{\Theta}) = z(k) - y(k, \hat{\Theta}) \quad (13)$$

Where  $y(k, \hat{\Theta})$  is the output prediction based on the actual estimate  $\hat{\Theta}$  of the parameter vector  $\Theta$ . See, Jategaonkar (2006) for a more detailed explanation relating this type of identification process. The parameter vector for the lateral, longitudinal, directional and heave dynamics is given by Eq. (14), Eq. (15), Eq. (16) and Eq. (17) respectively,

$$\theta = [L_p \ L_{A1} \ Y_v \ Y_p \ Y_{A1} \ \Delta_p \ \Delta_\phi \ \Delta_v]^T \quad (14)$$

$$\theta = [M_q \ M_{B1} \ X_u \ X_q \ X_{B1} \ \Delta_q \ \Delta_\theta \ \Delta_u]^T \quad (15)$$

$$\theta = [N_r \ N_{PED} \ \Delta_r \ \Delta_\psi]^T \quad (16)$$

$$\theta = [Z_w \ Z_{COL} \ \Delta_w]^T \quad (17)$$

Consider a dynamic system, identifiable, with model structure  $M(\Theta)$  defined and output  $y$ . Suppose that  $p(y|\Theta)$  is the conditional probability Gaussian distribution of the random variable  $y$  with dimension  $m$ , mean  $f(\Theta)$ , and covariance  $R$ , with dimension  $m \times m$ .  $p(y|\Theta)$  is known as the likelihood functional, and in [10] the authors attribute its name due to the fact that it is a measure of the probability of occurrence of the observation  $y$  for a given parameter  $\Theta$ .

The Maximum Likelihood Estimate is defined as the value of  $\Theta$  which maximizes this functional, in such a way that the best estimate of  $\Theta$ , according to the MLE criteria is

$$\hat{\Theta} = \text{ArgMax } p(y|\Theta) \quad (18)$$

thus, the likelihood functional is

$$\hat{\Theta} = \text{ArgMax } p(y|\Theta) \quad (19)$$

$$p(y|\Theta) = \frac{1}{(2\pi)^{m/2}|R|^{n/2}} \exp\left\{-\frac{1}{2}\sum_{k=1}^n [e(k, \Theta)]^T |R|^{-1} [e(k, \Theta)]\right\} \quad (20)$$

whose maximization is equivalent to the minimization of

$$J(\Theta) = \sum_{k=1}^n \frac{1}{2} \{[e(k, \Theta)]^T |R|^{-1} [e(k, \Theta)] + \ln|R|\} \quad (21)$$

since, in the optimization process,  $J(\Theta)$  is equivalent to  $-\ln(p(y|\Theta))$ , except for a constant term.

The identification algorithms based on the Gauss-Newton method is of second order. This method, although complex, is suitable for a quadratic cost function, and is expected to converge quickly. First, we approximate  $J(\Theta)$  by a parabolic function  $J_L(\Theta)$  under the condition  $\Theta_L$  (retaining only the 3 first Taylor series terms),

$$J_L(\Theta) \cong J(\Theta_L) + (\Theta - \Theta_L)^T \nabla_{\Theta}^T J(\Theta_L) + \frac{1}{2} (\Theta - \Theta_L)^T [\nabla_{\Theta}^2 J(\Theta_L)] (\Theta - \Theta_L) \quad (22)$$

The optimization condition is obtained when,

$$\nabla_{\Theta} J(\Theta^*) = 0 \quad (23)$$

Applying (22) to equation (21), results, for  $\Theta$  close to the local minima  $\Theta^*$ ,

$$\nabla_{\Theta} J_L(\Theta) \cong \nabla_{\Theta} J(\Theta_L) + (\Theta - \Theta_L)^T [\nabla_{\Theta}^2 J(\Theta_L)] = 0 \quad (24)$$

which can be used to find the minima of the original cost function through the recursion,

$$\Theta_{i+1} = \Theta_i - [\nabla_{\Theta}^2 J(\Theta_i)]^{-1} \nabla_{\Theta}^T J(\Theta_i) \quad (25)$$

The complexity in the calculation of the Hessian matrix,  $\nabla_{\Theta}^2 J(\Theta_L)$  in (25) is avoided through the Gauss-Newton method, which uses the approximation,

$$\nabla_{\Theta}^2 J(\Theta) \approx \sum_{k=1}^n [\nabla_{\Theta} \hat{y}_k(\Theta)]^T |\hat{R}|^{-1} [\nabla_{\Theta} \hat{y}_k(\Theta)] \quad (26)$$

where the terms involving the second derivative are discarded. The gradient of the estimated output,  $\nabla_{\Theta} \hat{y}_k(\Theta)$ , is called Sensibility Function.

The Levenberg-Marquadt algorithm is an extension of the Gauss-Newton [11]. The idea is to modify (26) to

$$\nabla_{\Theta}^2 J(\Theta) \approx \sum_{k=1}^n [\nabla_{\Theta} \hat{y}_k(\Theta)]^T |\hat{R}|^{-1} [\nabla_{\Theta} \hat{y}_k(\Theta)] + \lambda I \quad (27)$$

and the inversion in (25) is not performed in an explicit manner, i.e., typically the original equation

$$[\nabla_{\Theta}^2 J(\Theta) + \lambda I] \Delta \hat{\Theta} = \nabla_{\Theta}^T J(\Theta_i) \quad (28)$$

is solved via SVD.

The inclusion of  $\lambda I$  in (28) solves the problem of an ill conditioned approximated Hessian. The Levenberg-Marquadt algorithm can be interpreted in the following way: for small values of  $\lambda$  it behaves like Gauss-Newton algorithm, while for high values of  $\lambda$  it behaves like the gradient method. More details about the Levenberg-Marquadt method can be found in Jategaonkar (2006).

## 4.2 Identification Results

An experienced pilot manually commanded the aircraft exciting the lateral, longitudinal, directional and heave motion dynamics independently, i.e., exciting each axis at a time, the FBL System kept the helicopter stable in the case of coupling effects, same fashion way encountered in Adipravita (2007). The doublet maneuver was elected to be executed by the pilot in the beginning of the research. The system identification goal is to identify the hover flight condition, however this flight condition is hard to be maintained and the pilot did its best prior executing the maneuver.

The time histories of input and output variables of the system were recorded in the onboard computer for post processing. The linear velocities,  $u$ ,  $v$ , and  $w$ , were reconstructed based on gyro and accelerometer readings in the same fashion way encountered in Ivler and Tischer (2008).

The Output Error Method and the Levenberg-Marquadt algorithms were obtained from Jategaonkar (2006). The routines were implemented in Matlab environment, by reading the time histories to the same environment, the bare airframe system identification of the Flybarless helicopter succeeded.

The flight test results for the lateral dynamics and its parameters convergence estimates obtained from the OEM are shown in the Fig 4. The system identification results can be seen in Tab. 2.

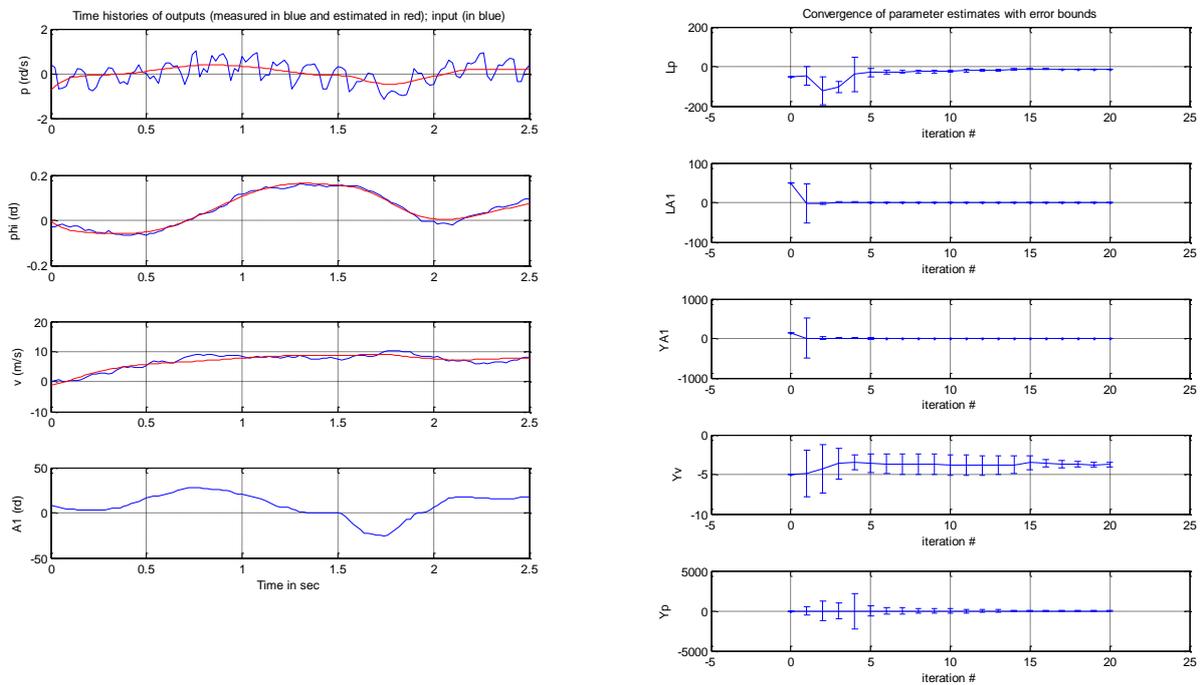


Figure 4. Time histories for the lateral dynamics response for doublet maneuver input and its parameter convergence estimates with error bounds

Table 2. Identification results for lateral dynamics.

Derivative	Identified Value	Relative STD%
$L_p$	-1.2861e+001	5.63
$L_{A1}$	2.3188e-001	5.41
$Y_{A1}$	-6.1559e-001	25.31
$Y_v$	-3.7615e-001	9.22
$Y_p$	3.6884e-001	22.58
$\Delta_p$	-7.48611e-001	14.84
$\Delta_\phi$	-4.08410e-003	158.84
$\Delta_v$	-1.11732e+000	45.87

The flight test results for the longitudinal dynamics and its parameters convergence estimates obtained from the OEM are shown in the Fig 5. The system identification results can be seen in Tab. 3.

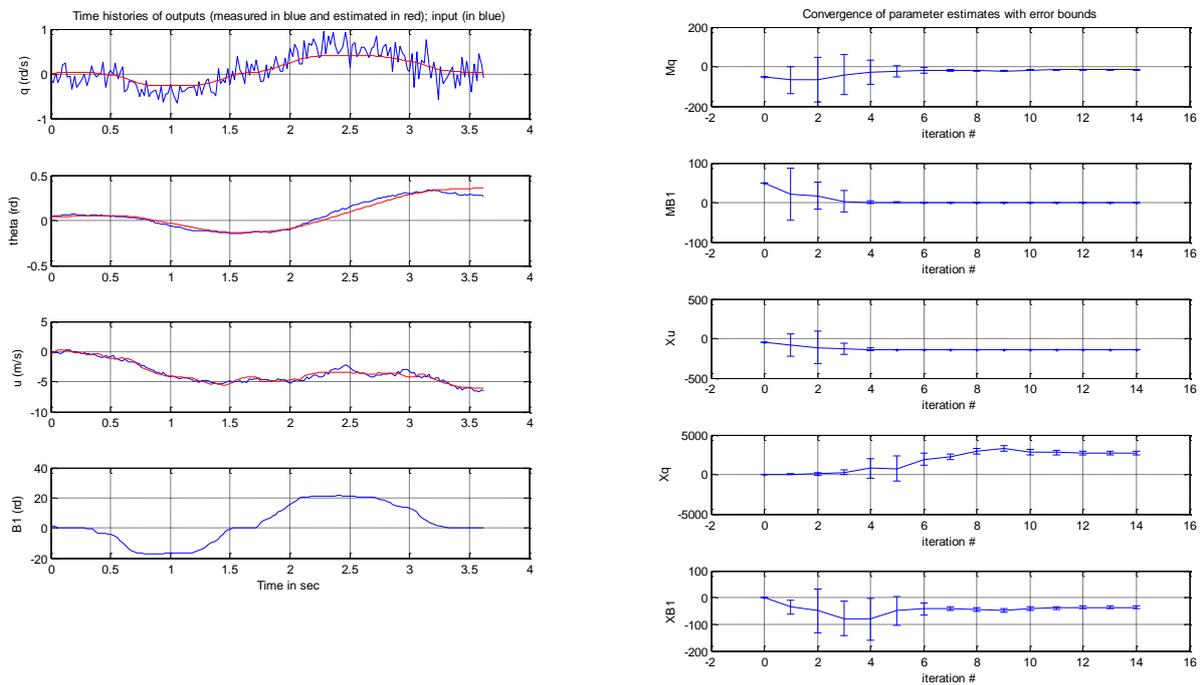


Figure 5. Time histories for the longitudinal dynamics response for doublet maneuver input and its parameter convergence estimates with error bounds

Table 3. Identification results for longitudinal dynamics.

Derivative	Identified Value	Relative STD%
$M_q$	-1.48839e+001	11.66
$M_{B1}$	2.62332e-001	11.73
$X_u$	-1.38877e+002	0.03
$X_q$	2.68945e+003	9.81
$X_{B1}$	-3.65561e+001	12.87
$\Delta_q$	-6.18461e-003	397.69
$\Delta_\theta$	3.92554e-002	14.80
$\Delta_u$	-5.19704e-001	54.90

The flight test results for the directional dynamics and its parameters convergence estimates obtained from the OEM are shown in the Fig 6. The system identification results can be seen in Tab. 4.

Table 4. Identification results for directional dynamics.

Derivative	Identified Value	Relative STD%
$N_r$	-6.99519e+001	2.74
$N_{ped}$	2.21125e+000	2.75
$\Delta_r$	3.04606e-001	33.30
$\Delta_\psi$	5.87985e-001	1.37

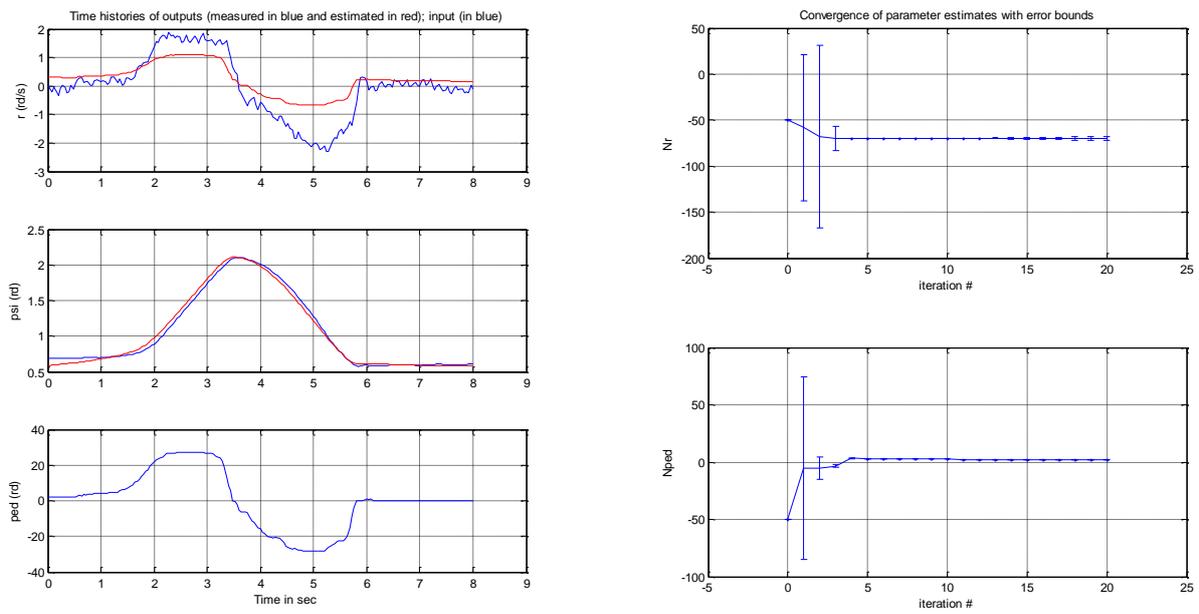


Figure 6. Time histories for the directional dynamics response for doublet maneuver input and its parameter convergence estimates with error bounds

The flight test results for the heave dynamics and its parameters convergence estimates obtained from the OEM are shown in the Fig 7. The system identification results can be seen in Tab. 5.

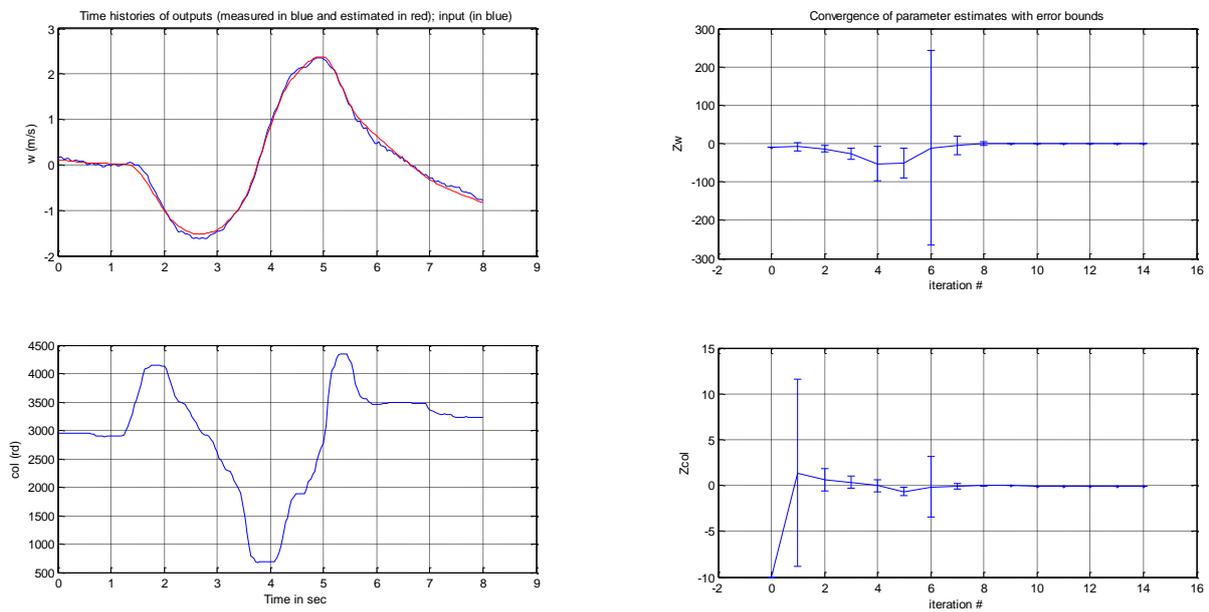


Figure 7. Time histories for the heave dynamics response for doublet maneuver input and its parameter convergence estimates with error bounds

Table 5. Identification results for heave dynamics.

Derivative	Identified Value	Relative STD%
$Z_w$	-1.61190e-001	3.00
$Z_{\text{col}}$	-9.04814e-002	0.46
$\Delta_w$	1.09125e-001	11.46

## 5. CONCLUSIONS

System identification time domain techniques were applied to a small-size unmanned helicopter. The closed loop stability derivatives were obtained and the transfer functions for the lateral, longitudinal, directional and heave dynamics can be obtained from the state space representation. The results showed an adequate curve fitting for the attitude equations and linear velocities. It is important to mention that the linear body velocities  $u$ ,  $v$  and  $w$  were obtained by the reconstruction of the accelerometer readings and the initial condition for the doublet maneuver is uncertain, i.e., the helicopter was drifting for one side or another at the beginning of the doublet maneuver rather than completely stopped. As mentioned earlier, to maintain the pure hover condition is a very hard task to the pilot. The curve fitting is not so adequate for the angular states because of the inherent vibration of the helicopter. One or more approaches can be used to overcome the problem in the future, e.g., filtering techniques or the use of dampers when mounting the inertial unit. The results can be improved by repeating the maneuvers and taking an average, however the overall results are adequate, and sufficient in order to design outer loop control laws.

Same architecture can be used to the Direct System Identification, i.e. use the stimulus and response signals to identify the dynamic system model as if the dynamic system is in an open-loop system. However it is advisable to use the Frequency Domain Identification, as this is very common when dealing with rotary wing aircrafts in order to identify the bare airframe dynamics. In this case it is very advisable to build an automatic maneuver generator, as the sinusoidal sweep can be very difficult to be executed by the pilot in the ground.

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