COBEM UBERLÂNDIA 2019

ABCM Associação Brasileira de Engenharia e Ciências Mecânicas

25th ABCM International Congress of Mechanical Engineering
October 20-25, 2019, Uberlândia, MG, Brazil

NUMERICAL STUDY OF A PLANE SURFACE-HALF SPHERE UNDER TWO DIFFERENT APPROACHES: HERTZIAN CONSIDERATION AND NON-LINEAR

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Abstract. *This paper aims to evaluate a device whose purpose is to obtain superficial parameters of bodies in contact. A numerical study with the Finite Element Method is used to evaluate the contact between the two bodies that make up the cited device. Two different cases are analyzed, first an evaluation within the elastic domain of the material, where the obtained results can be confronted with the analytical solution proposed by Hertz. Subsequently a more complex case is conducted, where the strain-strain correlation is non-linear, a configuration that does not have an established analytical solution.*

Keywords: *Contact Mechanics, Hertz, Finite Element Method*

1. INTRODUCTION

There are many physical problems in the real world involving some type of mechanical contact. Specifically, in mechanical engineering, contact can occur, for example, in the loading transfer between structures in solids or even in the fabrication of structural parts.

A first understanding of Contact Physics between two bodies dates back to the earliest civilizations of ancient Egypt, circa 1880 BC. Some millennia later, the subject of contact between bodies was best studied by several scientists such as: Leonardo da Vinci (1400), Guillaume Amontons (1699), Leonhard Euler (1748) and Charles-Augustin Coulomb (1785). However, it was only in 1882 the physicist Heinrich Rudof Hertz, known as the father of Contact Mechanics, presented analytical solutions for certain configurations of contact between two bodies, formulations that are better described in the subsequent section.

With the advancement of numerical formulations, there are now several methods, such as the Finite Element Method, which make possible to study contact configurations, both Hertzian, that is, problems with known analytical solutions, and more complex contacts, where there is no known solution.

Due to the occurrence of mechanical contact in a large part of the studies, mainly those that make use of the Finite Element Method within the sub-area of Solid Mechanics, there are several research projects and papers dealing with this topic. The work presented by Chen-En Wu et al. (2015), for example, presents an analytical and numerical treatment of the contact between a sphere and a plane under large deformations. In this same line of research, that is, studies dealing with the application of Hertzian cases along with numerical treatment, like for example, studies by Herk et al. (2006), Ghaednia et al. al (2018), Santos et al (2004) and Kadam (2018).

The main objective of this study is to propose a numerical representation of the torsion and traction machine used in experimental studies to analyze the mechanics of contact between bodies (Figure 1). The contact output parameters refer to the stress levels obtained for a contact configuration between a semi-sphere and a parallelepiped body.

Figure 1. MTS torsional and Traction Machine (Lins, G. and Amaral, T. Projeto e Fabricação de Dispositivo para Ensaios de Contato entre Sólidos com Geometria Hertziana).

This first analysis consists on the first step of a numerical study of wear between surfaces due to rotational displacements of the semi-sphere around its axis of symmetry.

As a first study, a force equivalent to 1 and 2 N will be applied in order to compare the results obtained by the numerical model adopted with the analytical results presented by Hertz, considering the study of a Hertzian case.

The second study concerns the application of 1 KN along with a rotation of 2 degrees on the *z* axis. This case constitutes a non-linear study of the material and the geometry, considering the presence of stress levels higher than the yield strength of the respective material. This study can-not be validated by Hertz's analytical model.

The construction of the numerical model was based on a simplification of the equipment shown in Figure 1. For this purpose, it was considered a parallelepiped-shaped body that will serve as a base along with a semi-sphere. The CAD model is shown in the Figure 2.

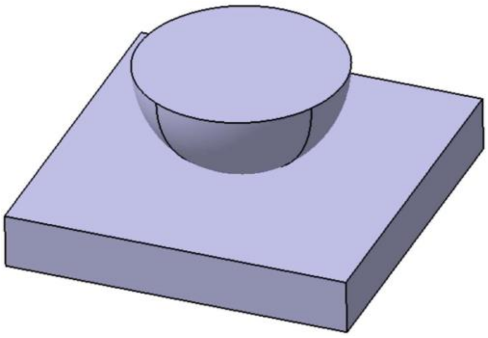

Figure 2. Semi-sphere and plane surface CAD model.

The commercial software Catia© was used to create the CAD model, while the Hypermesh© was used for simplification in the model, in order to facilitate the creation of structured meshes. The use of the Abaqus© software took place in all the stages related to the FEM application.

The properties referring to the geometry of the bodies that make up the numerical model, as well as the properties of the materials are presented in the table 1. It should be noted that both components are made up of PC/ABS 60:40.

Table 1. Geometric and material properties

Radius of the semi-sphere	7.5 mm
Length and width of the rectangular holder	14 mm
Thickness of the rectangular holder	4 mm
Longitudinal Modulus of Elasticity	1.1 GPa
Poisson’s ratio	0.35

Yield stress	55 MPa
Density	1000 Kg/m³

2. ANALYTICAL STUDY OF A HERTZIAN SPHERE-PLANE CONTACT

In order to solve the problem of contact between two bodies of revolution, the Hertz theory is applied under three main hypotheses: The material of the solids in contact must have an isotropic and linear elastic behavior, according to the law proposed by Hook; the surfaces of the solids in contact are continuous; the imposed load must be purely normal, without the occurrence of tangential forces, that is, they are surfaces without the presence of friction (Johnson, K. L., 1953).

Regarding local displacements, Hertz considered that the solids can be treated as elastic semi- spaces submitted to a purely normal request, since the region in contact is much smaller than the other dimensions of the solid, that is, the stress levels in the contact zone are not critically dependent on the geometric shape of the regions distant from them.

Figure 3 shows the Hertz study for the case of a point contact between two spherical bodies.

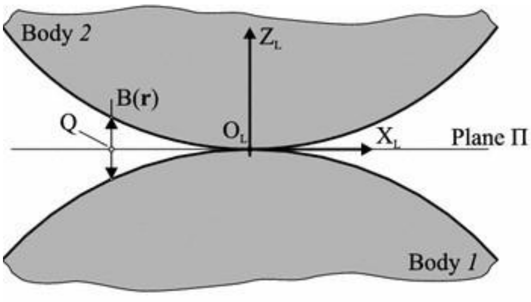

Figure 3.Point contact between two spherical Bodies (Ghaednia, H. A review of Elastic-Plastic Contact Mechanics, 2017).

Based on the analytical formulations proposed by Hertz for the study of circular point contact, contact category applied in this study, it is possible to show that the circular contact radius is defined by:

$$A = B = \frac{\pi}{2E^*}\frac{P}{(A+B)} \tag{1}$$

Where, Po represents the maximum Hertz pressure, A and B are the equivalent curvatures and E* the equivalent modulus of elasticity.

The penetration value can be determined by:

$$\delta = \frac{\pi}{2}\frac{aP_0}{E^*} \tag{2}$$

The parameters of equivalent curvature A and B can be determined by the equation shown below. For this study, considering the presence of a spherical body and another plane, it is necessary to apply the limit of R_2 to infinity.

$$A = B = \frac{1}{2}\left(\frac{1}{R_1} + \frac{1}{R_2}\right) \tag{3}$$

From the formulations presented for the case of point contact between two spheres, along with Figure 4, it is possible to verify that as the applied normal load is added, both the contact radius and the penetration between the bodies increases. The preliminary determination of such parameters is of paramount importance for numerical studies in order to optimize the appropriate mesh refining for the regions that will actually be in contact.

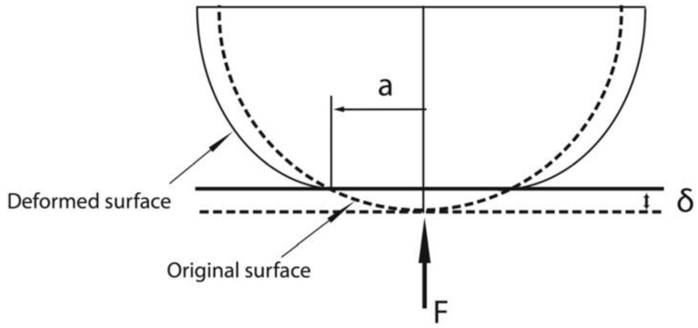

Figure 4. Sphere – plane surface parameters (Ghaednia, H. A review of Elastic-Plastic Contact Mechanics, 2017).

The use of Boussinesq's Theory, based on the assumption postulated by Hertz that the contact solids behave as elastic half-spaces, allows us to obtain the state of stress within the solids in contact.

Taking the basis of the coordinate axis shown in Figure 4 and assuming that in the case of a solid subjected to a Hertzian contact, the stress along the axis OZ are main stresses due to both geometric symmetry and contact request symmetry that occurs on this axis.

An analysis of the formulations proposed by Hertz for the determination of stress levels allows to conclude that the principal stresses σ_{xx}, σ_{yy}, and σ_{zz} reach their maximum value on the surface, decreasing as it enters the body.

The principal stress of greater magnitude is the one present in the direction of load application, that is, in the OZ axis. The determination of such stress can be obtained from the equation presented below.

$$\sigma_{max} = C_\sigma \sigma_{ref} \tag{4}$$

Where,

$$C_\sigma = \frac{3}{2}\frac{k}{\pi}\frac{1}{C_a^3} \tag{5}$$

$$\sigma_{ref} = a(A + B)E^* \tag{6}$$

For the study carried out, the eccentricity k is equal to 1, that is, a circular contact region. The term Ca is approximately equal to 0.91, and can be obtained by using the equation given below.

$$Ca = \sqrt[3]{\left(\frac{3a}{2\pi b}\right)\int_0^{\frac{\pi}{2}} \sqrt[2]{1-\left(1-\frac{a^2}{b^2}\right) sin\,(\alpha)^2 d\alpha}} \tag{7}$$

The determination of the shear stresses can be performed using the following equation:

$$\tau_{max} = C_\tau \sigma_{ref} \tag{8}$$

Where C_τ is approximately equal to *2.35* and can be determined by the equation shown:

$$C_\tau = \frac{3a}{2b}\int_0^{\frac{\pi}{2}} \frac{d\alpha}{\sqrt[2]{1-(1-\frac{a^2}{b^2}) sin\,(\alpha)^2}} \tag{9}$$

Finally, Table *2* presents the maximum magnitudes of normal and shear stress for the cases of 1 N and 2 N from the application of the formulations presented in the course of this section.

Table 2. Hertz Contact stresses for 1 and 2 N.

Force (N)	Normal Contact Stress (MPa)	Shear Stress (MPa)
1	14.51	4.33
2	18.28	5.46

3. COMPUTATIONAL CONTACT MECHANICS

According to Piedade Neto, 2011, the *Signorini* Problem is a classic example of a formulation for the elastic investigation in solids mechanics and can be applied as a basis for several generalizations. This problem describes frictionless contact between a linearly elastic body with a rigid surface. (Laursen, T. A., 2003)

Signorini's contact problem in a solid domain Ω subjected to a force of body *f* and prescribed displacements Γ_σ and Γ_u in its outline is described by the variational inequality below.

$$\int_{\Omega} \delta\boldsymbol{\varepsilon}:\boldsymbol{\sigma} d\Omega \leq \int_{\Omega} \delta\boldsymbol{u}.\boldsymbol{f} d\Omega + \int_{\Gamma_\sigma} \delta\boldsymbol{u}.\bar{\boldsymbol{t}} d\Gamma + \int_{\Gamma_c} \delta\boldsymbol{u}.\boldsymbol{t}_N d\Gamma, \forall \frac{\delta\boldsymbol{u}}{\delta\boldsymbol{u}} = \mathbf{0} \; on \; \Gamma_u; \; \delta\boldsymbol{u} \geq 0 \; on \; \Gamma_c \tag{10}$$

Where δ_ε and δ_u represents fields of deformation and compatible virtual displacements and σ represents the stress field in the solid.

The third integral to the right side of the inequality represents the virtual work of the contact and can occur in the region defined by Γ_c. Finally, the term t_N represents the force between the surfaces in this region in contact.

The problem presented by *Signorini* states that the study of contact between bodies must satisfy the equations and inequalities presented below.

$$\begin{cases} u_i n_i = 0 \\ \sigma_{ik} n_i n_k \geq 0 \\ \sigma_{ik} n_i \tau_k = 0 \end{cases} \quad \begin{cases} u_i n_i > 0 \\ \sigma_{ik} n_i n_k = 0 \\ \sigma_{ik} n_i \tau_k = 0 \end{cases} \tag{11}$$

The first system of equation shows that if there is not a gap between the bodies analyzed, ie, the bodies are in contact, necessarily the normal stress level present in the contact is equal to or greater than zero (considering the compression as positive). The second system of equations shows that if this gap between the bodies is greater than zero, that is, the contact between the bodies is nonexistent, there is no presence of normal stress in the contact zone.

As additional points for ratifying the occurrence of contact between bodies, Kuhn-Karush-Tucker's conditions establish the relations of penetration and force in contact.

4. ADOPTED NUMERICAL MODEL

In this study, the adopted FEM numerical model has a structured mesh formed by hexagonal elements in the contact zones and in the adjacent regions. The contact radii presented in section 2 served as a parameter for defining this zone. For the regions farther from the zone of interest, a free mesh of tetrahedral elements was used.

In the zones of interest, areas of possible contact, the elements have an average size equal to 30 micrometers, becoming larger as it moves away from this region. Figure 5 presents the finite element model of the analyzed regions.

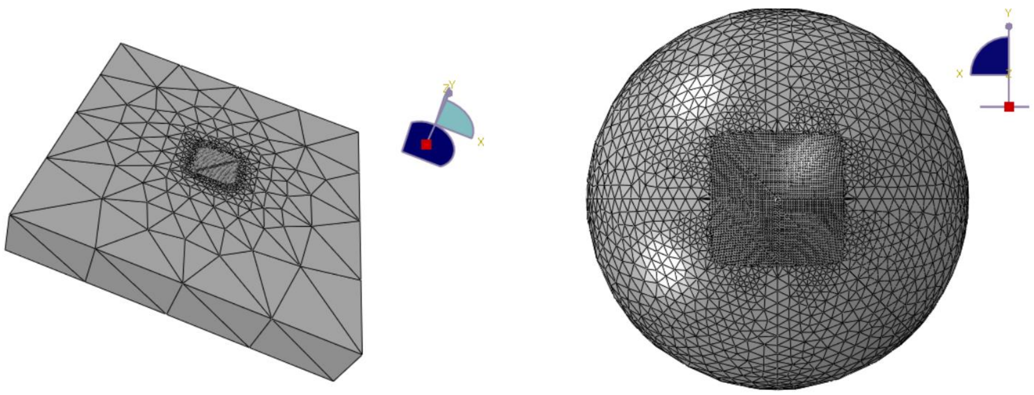

Figure 5. Finite Element Model for semi sphere – plane surface.

Regarding the boundary conditions and loads applied to the structures, the nodal displacements of the *x* and *y* directions of the nodes belonging to the symmetry axis of the semi-sphere and all displacements and rotations of the support body's base were restricted.

The application of the load on the *z* axis occurred on the entire flat surface of the semi-sphere by using a tool called *coupling* at a pre-set reference point. In the study where there is application of a rotation of the semi-sphere, that occurred on the *z* axis.

For the consideration of the contact between the bodies, the surface-to-surface method was defined, considering the contact conditions of the analyzed objects. The property of the tangential contact adopted was without friction, whereas for the normal one the penalty method and stiffness factor equal to 1 was used.

The direct solution method with Newton complete integration was used for all studies. In the specific case of applying 1 KN of force and rotation around its main axis, in addition to enabling effects of non-linearities due to large displacements, a stabilization method with a damping factor equal to 0.0002 was made.

In the study of the application of 1 KN of force, it was necessary to input the strain-strain curve of the material, because the stress levels are higher than the material's yield strength, that is, the ratio tangent modulus varies as strain is increased. Thus, the adopted model was the multilinear isotropic hardening.

5. RESULTS AND DISCUSSION

The first study concerns the application of 1 and 2 N as previously treated. The results of stress, strain and displacement can be ascertained from the Hertz analytical solution, since the problem presents the necessary considerations for this purpose. Figure 6 shows the curve of the contact stress in relation to the distance, having as reference the contact center.

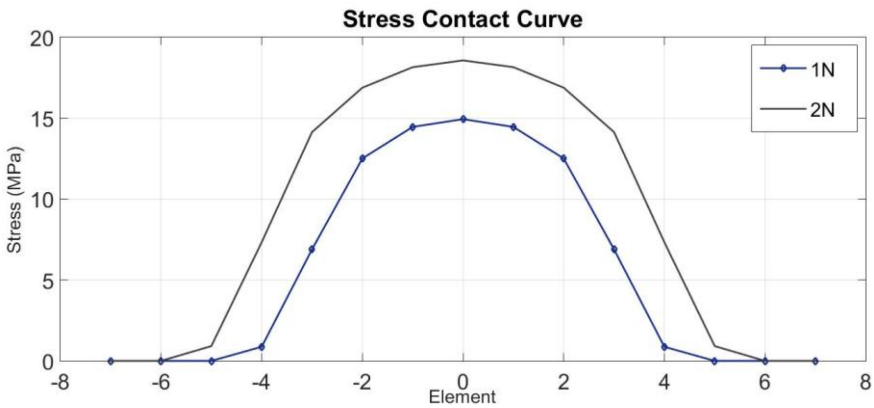

Figure 6. Stress Contact Curve for 1 and 2 N.

The stress levels are presented in table 3 along with a reference to the values obtained analytically.

Table 3. Comparison between the analytical and numerical.

Numerical results (MPa)	Analytical results (MPa)	Percentual difference (%)
14.94	14.51	2.96
18.57	18.28	1.58

The second study concerns the application of a normal compression force equivalent to 1 KN along with a rotation of 2 degrees around the z axis. As previously stated, this configuration can-not be validated with the analytical results, since this case study runs away from the elastic zone of the material. It should be noted that the application of rotation also escapes the considerations necessary for the application of Hertz theory.

Figure 7 represents the contact stress levels for the aforementioned study. In this case it is possible to verify that the stress levels are higher than the yield strength as expected, therefore, a plastification occurs in the contact zone.

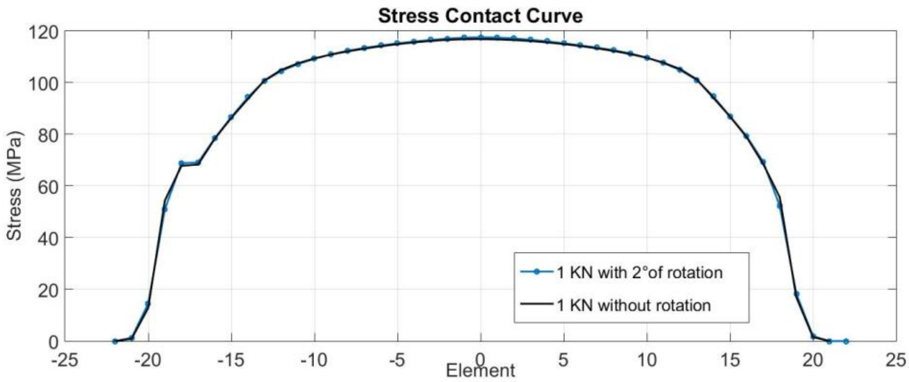

Figure 7. Comparative curve between the application of 1 KN with and without rotation.

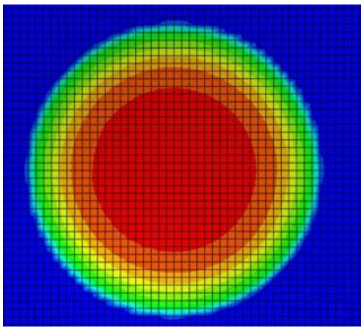

Figure 8. Contact stress gradient for 1 KN application with rotation.

In view of the figure 7 and 8 it is possible to verify that the application of a rotation of the semi-sphere around its main axis did not cause considerable variations in the stress contact. The maximum normal contact stress between both analyses varied in 0.75 MPa. The simulation without the application of rotation had a maximum contact stress of 116.78 MPa while the simulation with the application of 2 degrees of rotation showed 117.54 MPa.

If the study was conducted within the elastic limit of the material, where the mechanical contact behavior could be represented by the Hertz analytical formulation, the normal contact stress level would be equal to 154.1 MPa. The result of stress obtained considering the inelastic behavior of the material was inferior to this one that agrees with the expected one, since the decrease of the tangent module.

6. CONCLUSION

Considering the results presented in the first study, it is possible to verify that the adopted numerical model provides coherent results of contact stresses, according to the analytical model of Hertz. The values obtained presented percentage differences, in relation to the analytical solution, less than 3% for the two cases investigated.

It is also possible to affirm that the prior knowledge of analytical formulations of cases such as the one studied, where the analytical response is known, allows a better definition of parameters of paramount importance for the FEM application, such as the definition of the zone radius where it will have elements of smaller dimension, which is important for the establishment of contact.

Considering a numerical model able to evaluate the linear behavior of the contact between a semi-sphere and a plane surface, this one was used to evaluate a more complex case, where Hertz theory could no longer be applied. The presence of plastification and rotation in the sphere around its main axis resulted in a behavior of the contact pressure quite different from that found in the linear case. In this case, the stress levels of the points farther from the zone where the contact was established grow more abruptly. As it approaches the place of contact, stresses increase less abruptly.

In relation to the application of a rotation of the semi-sphere around its main axis, the result of normal contact stress shows that the increment is too small to lead to great variations. For the study carried out to date, this rotation does not entail considerable variations, since only one rotation cycle was applied. The study of the conditions established in this work under several cycles, added by some pertinent theories, will enable further investigation of superficial contact wear

In short, the definition of a model able to evaluate the presence of plasticity from the application of a normal compression load along with a rotation of the semi-sphere around its main axis is presented as the first step for future studies, where the superficial wear of the material used in this work will be evaluated through Archard formulation and the dissipated energy.

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