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DYNAMIC ANALYSIS OF PERIODIC STRUCTURES IN BEAMS AND BARS WITH IDENTIFICATION OF DAMAGE

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Abstract. *Events related to the failure in structures have shown the importance of maintenance. It is known that the vibration control is a very relevant field of study within Mechanical Engineering that seeks an attenuation and inspection of vibrations of a system. The competitiveness of industries is directly linked to investment in process and equipment monitoring. Structural integrity is essential for the analysis of a system to be correct and valid when the vibration analysis has an unexpected response; there is probably a change in this structure. Thus, the primary purpose of monitoring is to previously identify damage such as cracking and corrosion, reducing the risk of catastrophic failure. To detect and quantify the damage, the study of reflected waves and incidents present a good result. Currently, periodic structures are in evidence. They consist of a series of repeated elements connected to the same joints and are present in aircraft, buildings, bridges and even in critical structures. In this context, the current work has the purpose of locating the fault and quantifying it, using the transfer-matrix method that aid in the representation of periodicity.*

Keywords: *SHM, periodic structure, damage detection, band gaps.*

1. INTRODUCTION

The major focus of engineering is to seek more effective solutions to problems. Inman *et al.* (2005) obtaining innovative responses leads to challenges related to all engineering segments, especially projects. During the development phases of projects, dynamic behavior must be known. The lack of knowledge of the levels and characteristics of the dynamic response can lead to stress levels in the non-predicted materials, generating a possibility of failure. These can be caused by the application of repetitive loads as well, which are directly involved with fatigue, crack propagation, and accumulation of structural damage. Mechanical components tend to react to stimuli, such as stress or vibration. When increasing the number of components of a system, more complicated is the analysis of this, needing to use means more detailed than linear static analysis.

In general, all types of structures are subjected to various internal and external factors that cause damage and malfunction due to corrosion wear, deterioration due to lack of quality control or an extreme situation of an accident or an influence of the environment. Parallel to material changes, such as geometric properties, a damage identification system is essential, detecting anomalies in maintenance time, reducing accident risks, and reducing costs.

The Intelligent Structures, according to Farrar *et al.* (2006), can equip bridges, aircraft, automobiles, and ships with sensors and actuators that enable these vehicles to control vibrations, promoting the improved performance. Such structures are inspired by nature, where they seek to reproduce the adaptive characteristics of natural systems; that is, these structures must respond to external and internal excitations appropriately.

Over time, this improvement on the detection of vibrations is increasingly demanded by the market, with the objective of reducing fatigue failures and improving the dynamic behavior in order to achieve stability of the system. The study of intelligent structures has been encouraging structural optimization techniques, sensors, and controllers. The combined use of these generates a system for better control of external excitations. According to the Farrar and Worden (2007) reference, the failure prognosis process evaluates the current state of the structure, estimates the operating conditions to which this structure will be subjected and, based on simulations, the remaining useful life is predicted.

The Structural Integrity Monitoring is a prognostic failure phase, evaluating the present state of the structure. The SHM seeks to perform measurements of the dynamic response of a system continuously, from sensors installed in the structure. The extracted data become sensitive to the presence of the damage, altering values in the measurements obtained; these are analyzed statistically for the determination of the current state of the system in relation to its integrity. Thus, Doebling *et al.* (1998), the purpose of the monitoring system should be to accumulate enough information about the damage that occurred to take appropriate actions to restore the correct performance of the structure or at least ensure security.

Knowing that the growth of damages in structural elements is a significant problem, the present work studies the analysis of waves in structures with periodicity. The periodic structure is a set of equal structural components that repeat themselves in the same way. The combination of these sets of components leads to improved physical properties that apply in many fields, such as trains and bridges. The elements in periodicity present band gaps, which are frequency bands in which there is no propagation of the waves by the structure, thus aiding in the control of noise and vibrations. For Mead (1996), the periodic structure is a structure composed of many structural components, which is a continuous structure, such as solar panels, airplane wings and fuselages, oil pipelines, train tracks, and many others.

The flow and propagation of waves are very important aspects for periodic structures since they allow a reduction in the time of calculation, accuracy in all frequency bands and easy combination with other substructures. The dispersion analyzes of the periodic structures are concentrated mainly in this area. Tassilly (1987) and Liang *et al.* (2017) studied the dispersion relationship between the number of waves and the frequency of a class of non-uniform periodic elements. In the field of vibrations, energy band analysis and dynamic responses of structures with periodicity have been highlighted in large studies.

The Impedance method was studied by Ungar (1966), who represented the vibration of a uniform beam allowing the analysis of the structures with loads. Later, Gupta (1970) presented an analysis for the periodic beams introducing the concept of cell and transfer matrix. The propagation and attenuation parameters are now used in the study of periodicity in structures. The incident wave in a medium with different characteristics, such as geometry, is known to be segmented into the propagated wave and reflected wave. If the periodicity varies, the boundaries of the passband and stopband regions are mixed. So if there is an irregularity in the structure, a part of the wave does not propagate, characterizing the destructive interference of the passband.

Mead (1975) presented the damping effect on the periodic structure. While Langley (1994) used the damped periodic structures to locate the wave. The localization of a source has always been in focus in the context of mechanical wave propagation; the transfer method was introduced by Kissel (1991) to identify them using random disorder.

The present work studies the periodic bar and beam, analyzing the characteristics of the response by this model. In this paper, the first results are from a cell with no damage, to validate the method. After this, the technique will be implemented for a periodic structure with more cells with structural damage.

2. PERIODIC STRUCTURE MODELLING

Figure 1 shows the top view of two beams with changes in the cross section area. The change in the cross section area occurs in a periodic fashion, such that the structure can be characterized as a periodic system with N cells. The cells are unit parts of the main structure that are arranged in repetitive way. The interest in this paper is to analyze the vibration by longitudinal wave propagation and the vibration due to bending motion of the structure. The longitudinal vibration is studied using rod model and the Euler-Bernoulli beam theory. A three-dimensional finite element model is also developed to verify the limits of the high frequency behavior of the Euler-Bernoulli model. The repeating part of the structure is known as the cell. And in this study, the cell is represented by three parts as illustrate in the bottom of Fig. 1. Figure 2 shows the top view of each cell which are symmetrical with its center, therefore the first and third parts of the cell have the same length L_1, the part with area reduction has length L_2. It is this change in the area that promotes the periodicity of the structure. It is important to mention that although the examples have different cross section area, they can be represented by the same model for longitudinal or bending vibration. The parts of the cell have different cross section areas, S_1 and S_2, respectively.

2.1 Transfer Matrix Method

In this study, the relationship between the transfer matrix and the concept of propagation and attenuation waves will be analyzed to describe the dynamic characteristics of bars and beams. The transfer matrix is the relationship between two ends of a structural element connected by nodes, showing the iteration of forces and displacements. This method consists of a transfer matrix for each substructure. Thus, the dynamic response of the complete structure is the multiplication of the substructure transfer matrices. From mathematical arrangements, it was possible to define the transmissibility matrices, which provide displacement and force outputs from the last bar. This method, studied by Rubin (1967), will help to model a periodic structure, the matrix system is further simplified because the structures are identical, and the matrix is repeated.

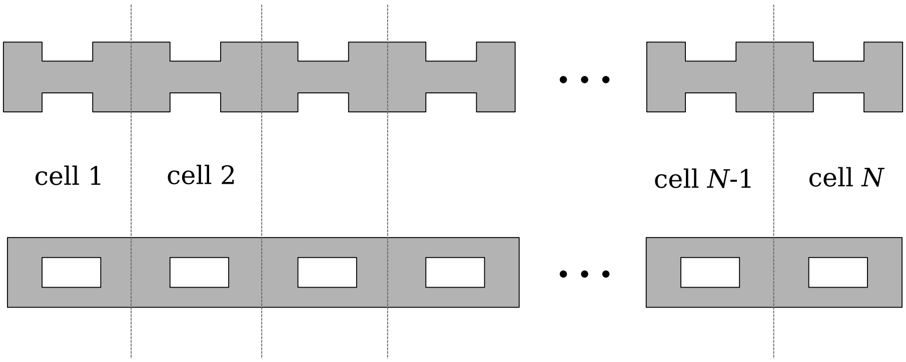


Figure 1. Schematic representation of periodic structure with N cells.

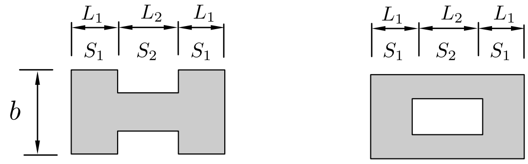


Figure 2. Schematic representation of the cell of periodic structure.

$$\mathbf{T}_{total} = \mathbf{T}_1\mathbf{T}_2 \cdots \mathbf{T}_N \tag{1}$$

where N is the number of periodic units in the structure. The matrix $\mathbf{T}_{total}$ represents the whole structure relating the left and right hand side of the beam. The dynamic stiffness representation of dynamic structures can be used to obtain the transfer matrices, using structural elements such as beams and plates distributed with mass and rigidity. The response of a structure can be expressed in terms of its mobility or impedance. Frequently, the displacement can be written: $c(t) = Xe^{j\omega t}$, where X is the complex amplitude. Similarly, the force is written $f = Fe^{j\omega t}$. For the representation of a periodic structure, the impedance matrix for a bar with with cross-section area S can be defined as:

$$\begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix} = \begin{bmatrix} D_{11} & D_{12} \\ D_{21} & D_{22} \end{bmatrix} \begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix} \tag{2}$$

where, $D_{11} = D_{22} = ESk\cot(kL)$, $D_{12} = D_{21} = -ESk/\sin(kL)$. The area S and the length L is replaced, according to the bar cross section. The impedance representation can be transformed into a transmission matrix by the method shown by Rubin (1967), relating the force state vectors and velocities at each end by:

$$\begin{Bmatrix} X_2 \\ F_2 \end{Bmatrix} = \begin{bmatrix} -D_{11}D_{21}^{-1} & -D_{11}D_{21}^{-1}D_{22} + D_{12} \\ -D_{21}^{-1} & D_{21}^{-1}D_{22} \end{bmatrix} \begin{Bmatrix} X_1 \\ F_1 \end{Bmatrix}, \text{ or given in compact form: } \psi_2 = \mathbf{T}\psi_1 \tag{3}$$

Thus, with the transfer matrix it is possible to obtain the matrix of the unit cell, for the model proposed in this study, the transfer matrix of one cell can be obtained by:

$$\mathbf{T}_{cell} = \mathbf{T}_1\mathbf{T}_2\mathbf{T}_1 \tag{4}$$

where $\mathbf{T}_1$ is the transfer matrix for the section of the cell with length L_1 and cross section S_1 and $\mathbf{T}_2$ is the transfer matrix of the section of the cell with length L_2 and cross section S_2.

3. INFLUENCE OF DAMAGE IN THE STRUCTURE

According to Mead (1973), while simpler structures transmit vibrational energy by one type of wave motion, others transmit in bending and longitudinal combinations. By finding a discontinuity in the periodic structure, the waves interact and become another type of wave, showing how the process of propagation of waves in periodic structures works, Fig. 3.

Brennan (1994) shows the relation between state vector and the waves at the junction. To represent a discontinuity in the structure, the bar/beam area was varied. Equation (5) is used to represent the beam and the damage.

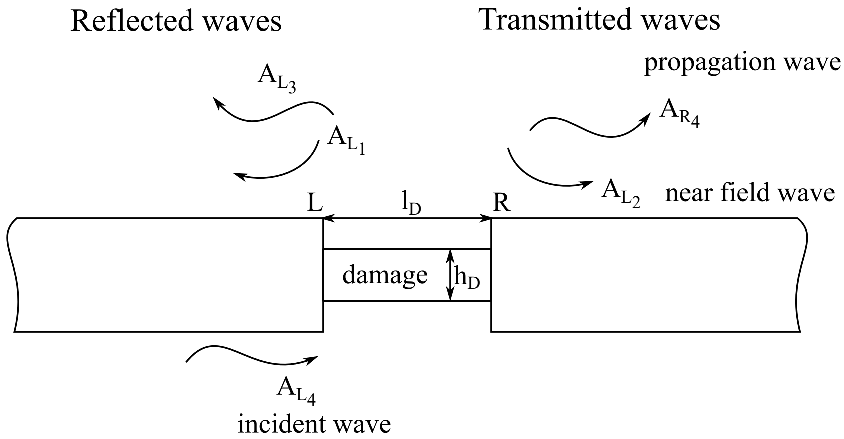


Figure 3. Diagram of the scattering waves.

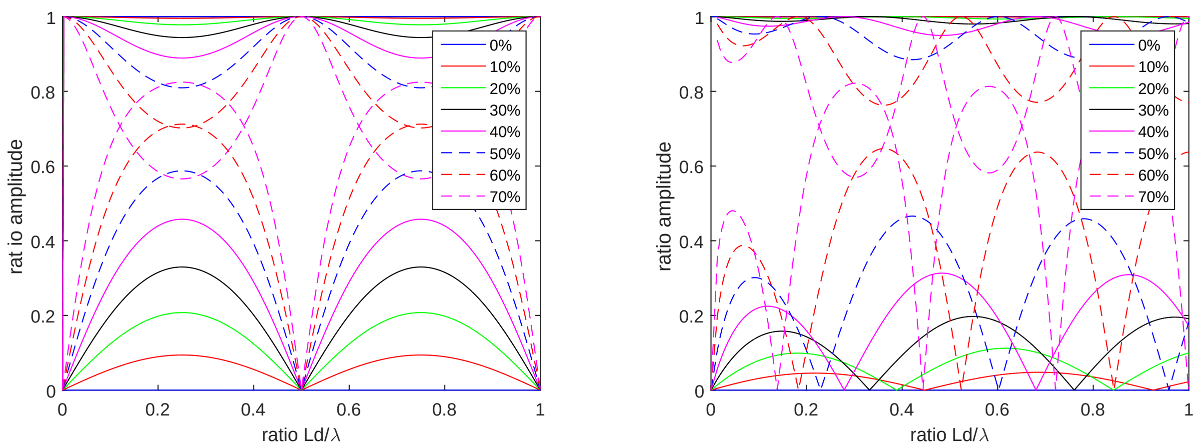
Ayala Castillo (2015) applied the balance of forces and boundary conditions, where the state vectors of the no-harm structure are equal to the state vectors of the damaged structure, as in Eq.(5).

$$\begin{Bmatrix} u \\ F \end{Bmatrix} = \begin{Bmatrix} u \\ ESu' \end{Bmatrix} = \begin{bmatrix} 1 & 1 \\ iESk_l & -iESk_l \end{bmatrix} \begin{Bmatrix} A_l \\ A_r \end{Bmatrix} \tag{5}$$

Thereby, the simulation shows the decrease in stiffness and mass of the damage causes an effect on the amplitude of reflection and wave transmission. The transmission ratio of the incident wave (A_{Rr}/A_{Lr}) and the reflection ratio of the incident (A_{Ll}/A_{Lr}) wave were analyzed, considering the total incident wave (L_D/λ), where L and R are the wave motion from left to right. Note that the wave reflection values do not vary with frequency, even varying the size of the damage.

The flexural waves are different from the longitudinal waves because the phase velocity depends on material properties and frequency, shows in Fig. 4. Using the beam rotation, θ, internal bending moment, M and internal out-of-plane force, Q, is possible to simulate the behavior of flexural waves, as in Eq.(6).

$$\begin{Bmatrix} W \\ \theta \\ M \\ Q \end{Bmatrix} = \begin{Bmatrix} W \\ W' \\ EIW'' \\ EIW''' \end{Bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ k_f & jk_f & -k_f & -jk_f \\ EIk_f^2 & -EIk_f^2 & EIk_f^2 & -EIk_f^2 \\ EIk_f^3 & -jEIk_f^3 & -EIk_f^3 & jEIk_f^3 \end{bmatrix} \begin{Bmatrix} A_1 \\ A_2 \\ A_3 \\ A_4 \end{Bmatrix} \tag{6}$$



(a) (b)

Figure 4. (a) Longitudinal waves reflected and transmitted due to a longitudinal propagation incident for different percentages of damage and flexural waves reflected and (b) transmitted due to a longitudinal propagation incident for different percentages of damage.

It is known that the two wave types depend on frequency. The damage size does not change the longitudinal waves if it is on the same frequency, as in Fig. 4a. However, for flexural waves, Fig. 4b, they change when the size of the damage varies. It is seen that with the decrease of the thickness of the damage, the amplitude of the wave begins to diminish. When the percentage of damage becomes larger, the values of L_D/λ decrease, that is the value of the reflected wave is changed.

4. NUMERICAL SIMULATION

In this section, numerical simulations are performed longitudinal and bending vibration on a periodic beam consisting of 15 cells. The beam properties are presented in Tab. 1.

Table 1. Beam Properties used in the Numerical Simulations

Property	Value
Total Length L_T	1.5 m
Number of cells N	15
Cell length L_{cell}	0.01 m
Thickness h	4.3 $\times 10^{-3}$ m
Width b	13 $\times 10^{-3}$ m
Young's Modulus E	70 GPa
Density	2710 kg

The identification of possible damages in a periodic structure is idealized in the Fig. 5. For this, a numerical analysis was performed to obtain the frequency response of the structure with and without damage, for longitudinal and bending vibration.

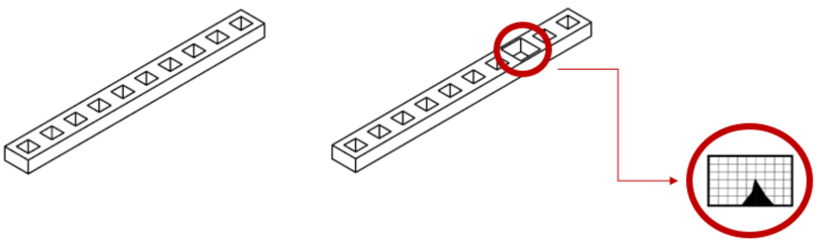


Figure 5. Idealization of a possible damage in a periodic structure

According to (Mead, 1999), the eigenvalues of the transmission matrix for a unit cell are related to the propagation constants and phase, which indicate the frequency regions where stop and pass band occurs.

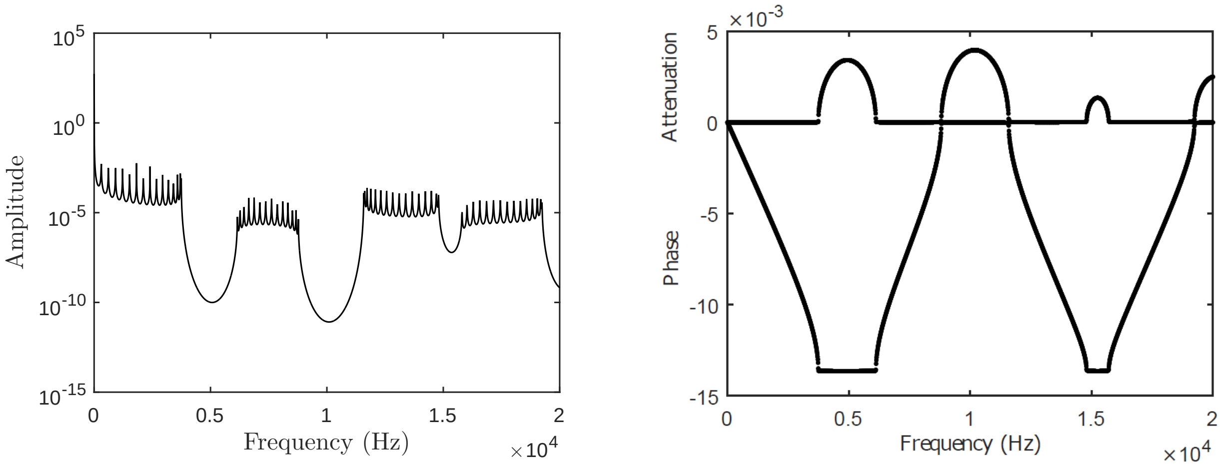
In the zones of attenuation and propagation zones, the movement in a component is equal to $e^{-\delta \pm i\varepsilon}$ times the movement in the next component. The complex propagation constant of the wave motion at the periodicity is $e^{-\delta \pm i\varepsilon} = \mu$, δ is the attenuation constant and ε is the phase constant. For Mead (1999), there are two parts that vary with frequency, Fig. 6 and 7, where only the negative values of ε are presented. In the propagation zones, the attenuation constant is zero and the phase constant varies with frequency. In the attenuation zones, the phase constant is zero or π while attenuation constant varies.

The three-element cell is made of only one material, Aluminum. Both the first and third elements have the same length, L_1, this varies only with the number of cells. However, the central element varies concerning the percentage of the fixed element length, L_1. The holes have a fixed thickness of $8mm$. The forced response of the bar and beam was analyze considering a free free boundary condition.

4.1 Longitudinal Response

To understand the way a damage affects a periodic structure, a variation of one hole size was analyzed in for different positions. In this first analysis, a harmonic axial force of magnitude 1 N was applied at one end of the beam and the longitudinal displacement was measured at the other end. The size of of hole was varied considering a normal distribution with standard deviation being 20 % of the hole size. To obtain the statistical results defining the frequencies responses, the simulation was executed 1000 times

The results shown in Fig. 8 illustrates the results for this analysis. Below the figures, there is a illustration showing the position of a possible damage. Figure 8(a) and (d) with damages in the first and last cell produces very similar results, there is almost no frequency shifts in the pass band region between the two stop bands. As the damage moves from to the middle of the bar, the pass band range is more affected by the changes in the



(a) (b)

Figure 6. (a) Frequency response of the periodic bar with 15 cells (Longitudinal vibration) (b) Phase and attenuation of the periodic bar.

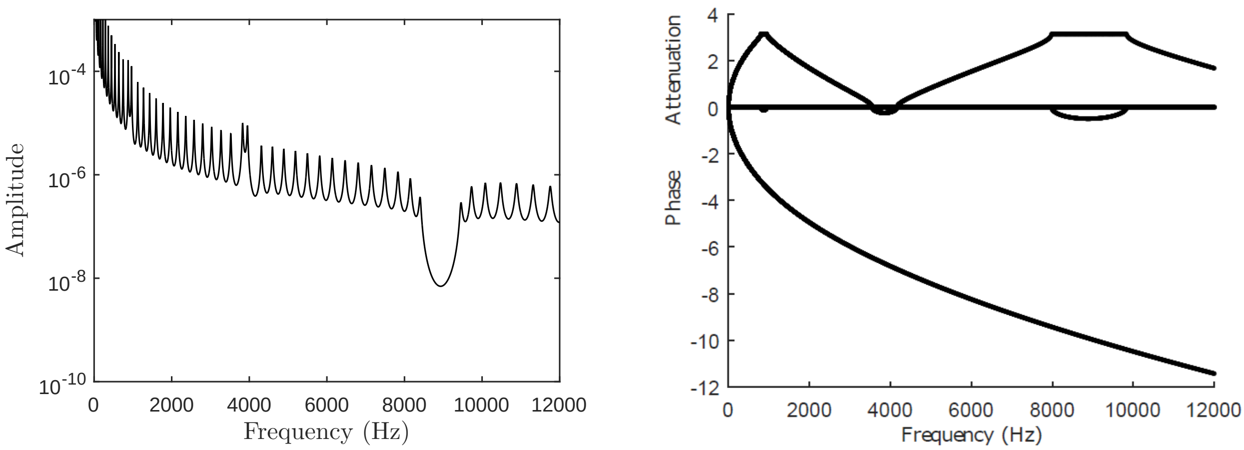


Figure 7. (a) Frequency response of the periodic beam with 15 cells (Bending vibration) (b) Phase and attenuation of the periodic beam.

cross section, as seen in Fig. 8(b) and 8(c). Both the frequency and amplitudes are affect by the change in one cell of the bar.

4.2 Bending Response

A similar analysis based on the procedure described in the last subsection was performed for bending vibration. In this case, the results are shown in Fig. 9. In this case, the results shown that bending vibration behaves differently from the longitudinal vibration. Changes in the first cell produce large variation in the frequency response than changes in the middle of the beam.

To better observe this effect, the Fig. 10, showing the effects on the stop band in the range 6-12 kHz.

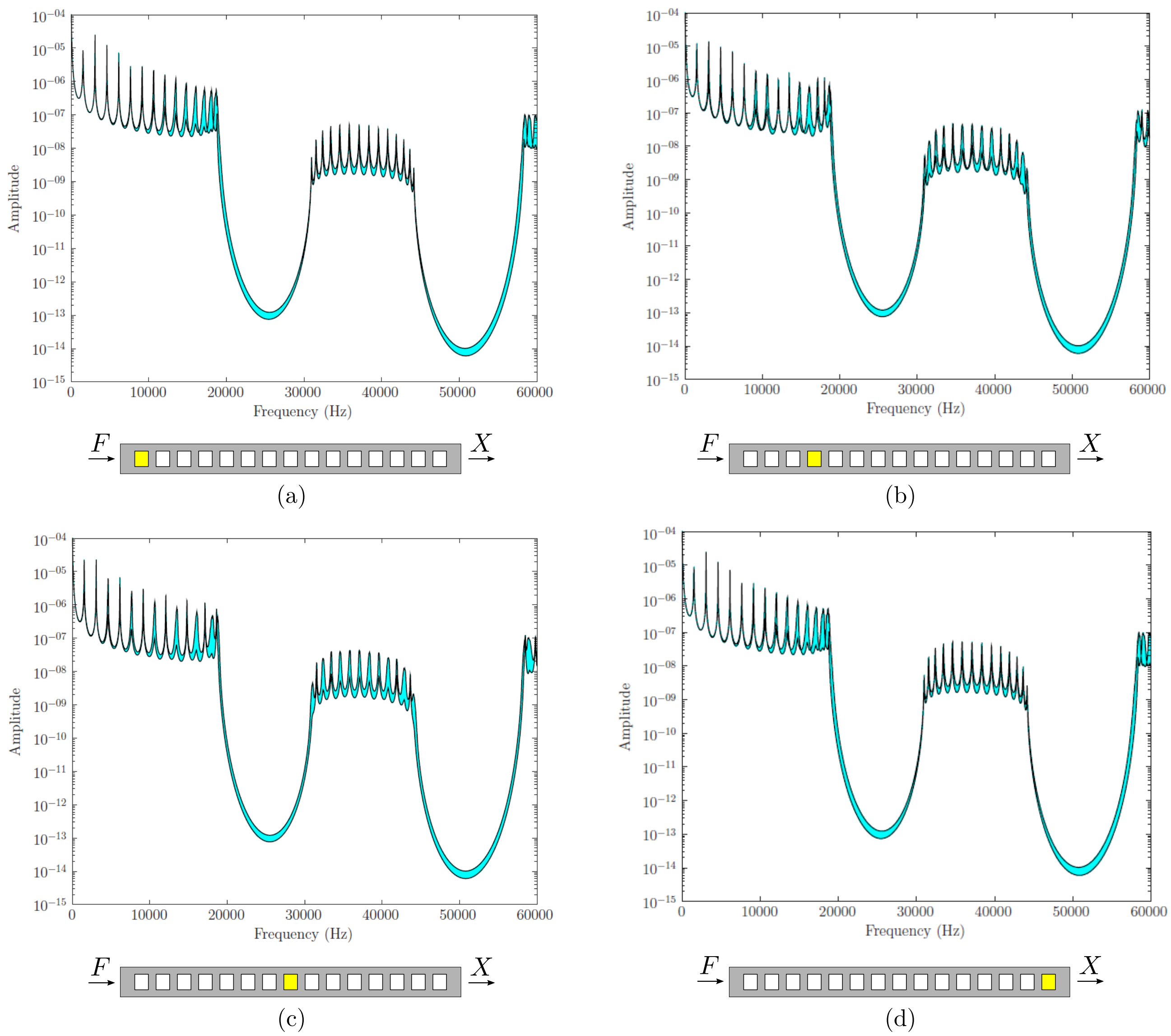
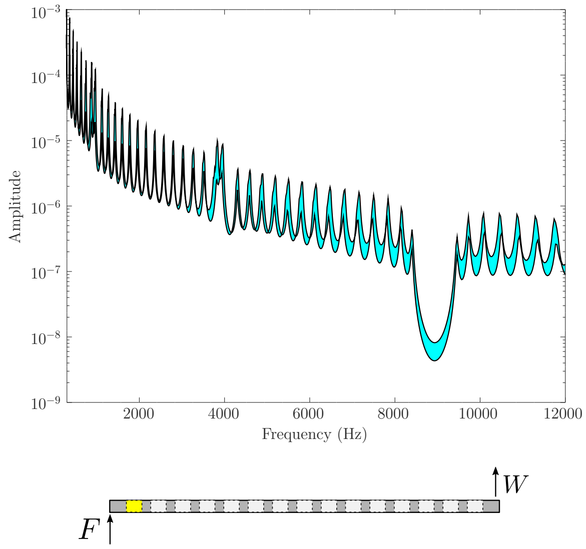
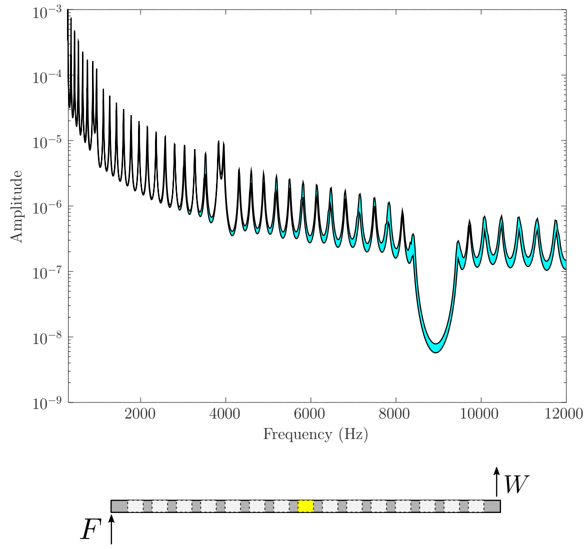


Figure 8. Frequency response comparison for different damage positions (Longitudinal vibration). The upper and lower curves correspond to percentile 90 and 10, respectively. From Fig. (a) to (d), the damage position was in the first, fourth, middle and last hole, varying with a normal distribution and standard deviation corresponding to 20% of the nominal hole area.

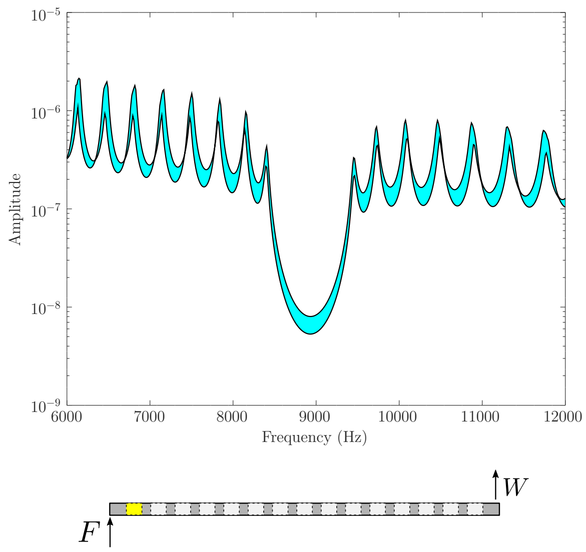


(a)

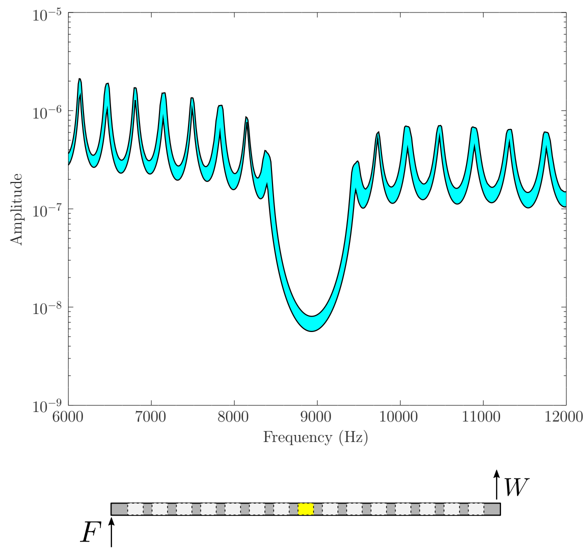


(b)

Figure 9. Frequency response comparison for different damage positions (Bending vibration). The upper and lower curves correspond to percentile 90 and 10, respectively.



(a)



(b)

Figure 10. Detail of frequency response comparison for different damage positions (Bending vibration). The upper and lower curves correspond to percentile 90 and 10, respectively.

5. CONCLUSIONS

In this study, considering a periodic structure with 15 cells, the band gap region is formed at lower frequencies in longitudinal vibration, around 3 kHz, than in bending vibration, which has the first significant band gap near 8 kHz. In bending vibration, it is also observed that the attenuation regions increase with increasing frequency. It is concluded that for longitudinal vibration, the position of the damage interferes when it is near the middle of the bar, showing variation in both amplitude and frequency. However, for bending vibration, the largest interference in frequency response is identified when damage changes are made in the first cell than in the middle of the beam.

6. ACKNOWLEDGEMENTS

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8. RESPONSIBILITY NOTICE