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MATHEMATICAL / NUMERICAL MODELING OF A FREE PISTON ELECTRIC GENERATOR

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Abstract. *Subject of several academic research and some applications in the industry, linear machines have great potential to replace conventional internal combustion engines given their potential to minimize losses and maximize power. The direct combination of a free piston engine with a linear generator is called free-piston engine generator (FPEG). It is considered an alternative non-conventional electric power generator. This work aims to develop a mathematical and numerical model of a linear free-piston generator in mechanical resonance. The modeling consisted of the development of a thermodynamic model, which gives the pressure inside the combustion chamber, and a dynamic model of a spring mass system, which defines the force due to the pressure inside the combustion chamber as excitation. The coupling of both models allowed the study and simulation of the thermal cycle, as well as the analysis of the following parameters: geometric parameters of the engine, pressure, loads, forces and mass / spring assembly. The results obtained were satisfactory when compared to data obtained in the scientific literature.*

Keywords: *resonance, free piston engine, free-piston engine generator, numerical modeling*

1. INTRODUCTION

Originally proposed by Pescara in 1928, free piston engines developed rapidly between the 1930s and 1960s. During this period, other entrepreneurs, such as the Junkers in Germany, also worked on free piston engines. Subsequently, a large number of patents on these machines or related were published. The original Pescara patent describes a single-piston spark-ignition air compressor, but it only sought to protect a large number of applications using the free-piston principle (Mikalsen and Roskilly, 2007a).

According to Li et al. (2010a), the simple structure and low friction made the engine popular at the time. Over time, engines based on the Otto and Diesel cycle were gaining maturity, at the same time the limitations of the engine were exposed, such as: difficult engine synchronization, low efficiency under various types of loading and difficulty of departure. Thus, the employment of free piston engines was gradually abandoned in the 1960s.

However, in recent years, with the effects of global warming and the shortage of fossil fuels, several groups of researchers have regained interest in free piston engines. The research is dedicated to the exploration of new energy conversion devices and use of ecological fuels (Li et al., 2010b).

As pointed out by Lim et al., (2013), the main advantages of the free piston engine include: simplicity in mechanical structure with few moving parts, low friction losses and high operational flexibility.

According to Mikalsen and Roskilly (2007b), one of the applications of these machines is the free-piston engine generator (FPEG), which integrates a linear combustion engine with a linear electric generator into a single unit. The principle of operation consists of withdrawing energy from the fuel in a manner similar to that of a conventional car engine. However, the linear motion of the pistons directly converted into electricity directly.

According to Xiao et al., (2010a), several researches on the use and construction of the FPEG have been approached in recent years. Most researchers tend to adapt ideal and simplified models of conventional motors to simulate the linear piston-free generator.

Researchers at West Virginia University describe the development of a twin-piston engine with ignition. A motor prototype obtained an output of maximum electrical power of 316 W at 23.1 Hz, with a bore of 36.5 mm and a maximum stroke of 50 mm (Atkinson et al., 1999).

Papers published recently show almost all the global FPEG researches are in the initial phase. Different types of tools are used to study this new emerging engine.

Some researchers aim to model and simulate the dynamic operation of an FPEG in order to obtain an understanding of the relation between the characteristics of the piston movement and the necessary thermal energy. In a similar way, we can find in literature researches such as:

Xiao et al., (2010b) established a numerical model of the FPEG. The natural frequency of the oscillation system was obtained from their model. A simulation program was developed in Matlab / Simulink to solve these mathematical equations, and simulation results showed that the movement of the FPEG was a forced vibration system with variable coefficient of damping and stiffness.

Hansson et al.,(2006) investigated the resonant behavior of an FPEG. They linearized the system after expanding the equation around an equilibrium point. Finally, the approximations of the oscillation characteristics of the free piston were reached. However, only compressive pressure forces were calculated in their model, and the pressure increased by the release of heat from the gaseous fuel mixture was not considered.

Thus, this work aims to explore the resonant characteristics of mechanical systems. For this, the dynamic system of the linear free-piston generator will be represented as a damped mass-spring system operating in mechanical resonance.

It is intended to develop a dynamic and thermal model that describes the characteristics of movement and release of heat. The model will then be linearized so that the system is represented by a forced vibration equation with viscous damping. In addition, the approximations of the spring constant and the natural frequency can be obtained. Based on this, the vibration equation and the solution for the displacement can be derived.

2. LINEAR FREE PISTON ELECTRIC GENERATOR

The free piston linear motor consists of two opposing cylinder / piston assemblies connected by a rod. The combustion of the air-fuel mixture happens alternately in each cylinder. The expansion of the gases in one chamber provides compression in the other, thus promoting the linear movement of the piston.

The linear electric generator is composed of permanent magnets that are coupled to the piston rod. Thus, with their linear and alternating motion, the magnets move inside a set of coils, generating electricity. Fig. 1 schematizes a linear free piston electric generator.

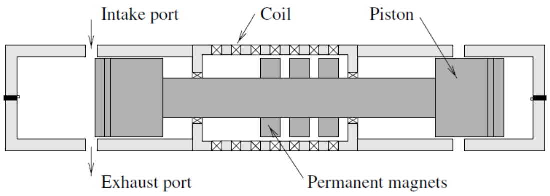

Figure 1. Prototype illustration of the free piston engine generator developed at the University of West Virginia. Adapted from: Mikalsen and Roskilly (2007)

3. MATHEMATICAL MODEL

3.1 Dynamic Model

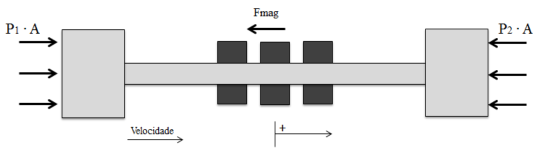

Figure 2. FPEG free body diagram

The forces acting on the system, as shown in Fig. 2, are: the force of the gases in the each cylinder (F_r and F_l), the magnetic force of the linear generator (F_{mag}), and the force of friction (F_f.) As the motor movement is linear, the lateral loads applied on the piston are minimal, in addition to being insignificant when compared to the other forces. Thus, neglecting the force F_f, we get to three main acting forces, the ones in each piston are given by the surface area and the pressure inside the chamber, which is a function of the rod position obtained in the thermodynamic model.

The applied magnetic force is a function of the generator parameters such as resistance, mass and operating velocity, and it acts opposing movement, thus it is assumed it is proportional to the motor velocity:

$$F_{amort} = F_{mag} = C_{mag} \cdot \dot{x} \tag{1}$$

where x is the rod displacement from neutral position, C_{mag} is a coefficient of proportionality. Therefore, Newton's second law can be expressed by:

$$m\ddot{x} = \vec{F}_l(x) + \vec{F}_r(x) + \vec{F}_{amort}(\dot{x}) \tag{2}$$

Eq. 2 describes a nonlinear system of one degree of freedom. Based on such information, the dynamic equation of the free piston motor generator (FPEG) can be linearized to a forced vibration system with viscous damping, which allows for a comparison to a mass-spring-damper-system.

3.2 Thermodynamic Model

The FPEG thermal cycle can be represented in a simplified way by a process of compression, combustion and expansion. A control volume is used to describe the thermodynamic process and it is assumed that there is no mass loss through the piston rings. It is considered that the pressure variation in the cylinders is influenced by the heat release from the fuel combustion, denoted p_{comb} as well as from the compression strokes (p_{vol}). Thus, the pressure (p) in each cylinder can be written as:

$$P_{r,l} = p_{vol} + p_{comb} \cdot \sigma_{r,l} \tag{3}$$

The unit function σ is introduced to activate / deactivate the influence of the heat release for both cylinders, and acts as a step function dependent on movement orientation.

$$\sigma_l = \begin{cases} 1, & \dot{x} < 0 \\ 0, & \dot{x} < 0 \end{cases} \tag{4}$$

$$\sigma_r = \begin{cases} 0, & \dot{x} < 0 \\ 1, & \dot{x} < 0 \end{cases} \tag{5}$$

According to Jia et al., 2016, if no heat transfer to the cylinder walls is considered in the thermodynamic model and there is no gas leakage through the piston rings, the ideal FPEG operating cycle can be described by two adiabatic processes connected by a process of heat release at constant volume.. Thus, the resulting force in each cylinder can be expressed by:

$$F_{r,l} = A \cdot \left(p_0 \left(\frac{V_0}{V}\right)^{\gamma} + \Delta p_{cm} \left(\frac{V_c}{V}\right) \cdot \sigma_{r,l} \right) \tag{6}$$

where p_0 is the pressure in the cylinder in the equilibrium position, assumed to be equal to the ambient pressure, V_0 is the cylinder volume in the equilibrium position, V is the cylinder volume, γ is the polytropic exponent (assumed constant at first), Δp_{cm} is the pressure increase during the heat release process at constant volume, the value is the same for both sides, and V_c, is the clearance volume.

3.3 Forced Vibration Equation

For the equation the motion of the linear free-piston generator can be represented by a mass-spring-damper system it is necessary to linearize the nonlinear expressions present in the equation of the forces imposed on each cylinder.

According to Jia, et al., 2016, a Taylor series can be used as a form of linearization and it is expanded around the equilibrium point of the system. After linearization Eq. (6) can be written as:

$$F_{r,l}(x) = \left(p_0 + \Delta p_{cm} \left(\frac{L_c}{L_s}\right)^{\gamma} \cdot \sigma_{r,l} \right) A - \left(\frac{\gamma p_0}{L_s} + \Delta p_{cm} \frac{\gamma L_c^{\gamma}}{L_s^{\gamma+1}} \cdot \sigma_{r,l} \right) A \cdot x \tag{7}$$

Substituting Eq. (2) and Eq. (3) in Eq. (7) and further expanding some terms, Eq(8) is obtained:

$$m\ddot{x} + (C_{vis} + C_{mag})\dot{x} + \left(\frac{2\gamma p_0 A}{L_s} + \frac{\Delta p_{cm} A \gamma L_c^{\gamma}}{L_s^{\gamma+1}} (\sigma_l + \sigma_r) \right) x = \Delta p_{cm} A \left(\frac{L_c}{L_s}\right)^{\gamma} (\sigma_l - \sigma_r) \tag{8}$$

Where the subscript *l* and *r* indicate the values for the left and right cylinders and C_{mag} and C_{vis} are constants of proportionality. The damping force of the system is expressed by the magnetic force, which converts the kinetic energy into electric and the viscous damping force which though the resistance of the air to the motion dissipates energy. These forces are considered through constants of proportionality C_{mag} and C_{vis}.

As the combustion occurs alternately in each cylinder, as already presented, the σ function from each cylinder will be a rectangular wave function over time. According to Jia et al., (2016), the rectangular wave of excitation can be described by a series of Fourier. Thus, considering the first mode of the series we get to the dynamic model equation:

$$mx'' + (C_{vis} + C_{mag}) \cdot x' + \left(\frac{2\gamma p_0 A}{L_S} + \frac{\Delta p_{cm} A \gamma L_C^{\gamma}}{L_S^{\gamma+1}}\right) x = \Delta p_{cm} A \left(\frac{L_C}{L_S}\right)^{\gamma} \left(\frac{4}{\pi} sen\omega t\right) \tag{9}$$

Then the FPEG dynamic model is linearized to the same form with the single degree-of-freedom forced vibration system with viscous damping. Where k is the stiffness of the air-spring system; c is the damping coefficient; the excitation $F_0 sen\omega t$ is a continuous force whose magnitude F_0 varies sinusoidally with time.

We can then define the system's natural frequency and the required ignition frequency to maintain the FPEG in resonant behavior.

4. NUMERICAL MODEL

The Matlab software was used to solve the coupling between the thermal and the dynamic models. The Equation 25 was rewritten as a matrix, as presented in Eq. 32, and the displacement and velocity of the system can be obtained using the Runge-Kutta integration method.

$$[mc0m]\{\ddot{x}\dot{x}\} + [0k - m0]\{x'x\} = \{\dot{x}x\} \tag{10}$$

The input parameters used have been taken from the literature as presented by Li et al. (2008b). Moreover, it assumes a constant value for the damping coefficient, represented by the load force coefficient. The coefficient of the load force is a function of the operating parameters of the linear generator. These values are shown in table 1:

Table 1. Parameters of the prototype presented by Jia, et al., (2016)

Parameters	Value
Bore [mm]	52,5
Maximum total stroke [mm]	70,0
Stroke [mm]	35
Moving mass [kg]	5
Coefficient of the load force $[N/(ms^{-1})]$	395
Ambient temperature [°C]	25
Inlet pressure [kPa]	101,325

Data such as fuel, slack length, compression ratio, air-fuel ratio (AFR), heat capacity ratio and combustion efficiency are set as input variables and the values were assumed according to varied literature.

Table 2. Parameter assumed

Parameters	Value
Clearance length [mm]	2,7
Compression ratio	13
AFR	14,7
Ratio of heat capacities	1,3
Combustion efficiency	1

The above method accounts for an ideal situation being both adiabatic and isentropic, however in a real situation there is some chance in the polytrophic coefficient n that relates pressure and volume chances in a given system:

$$P \cdot V^n = cte \tag{11}$$

Real processes are neither fully isothermal nor adiabatic, thus, it is expected for n to be between one and the ratio between specific heats γ:

$$\frac{C_p}{C_v} = \gamma \approx 1{,}37 \tag{12}$$

In this new model, the net force is calculated as the result of the pressure difference in each cylinder, and is inputted in the forced vibration equation. However, the polytrophic coefficient, which was a constant, is now recalculated at each step to account for volume, pressure and temperature changes. Values were taken from the thermal database HOT for Matlab (Martin (2010)), considering atmospheric air.

For the sake of comparison, the same initial condition were imposed and once again the Ode 45 function was used to solve the differential equation.

5. RESULTS

The FPEG natural frequency is directly related to the inlet pressure and piston stroke. They are variables that change with excitement. The natural frequency map of the FPEG is drawn in Fig. 1 for a better understanding. All variables are adjusted to healthy ranges, where the intake pressure is between 1.2 and 2 KPa and the piston stroke is between 0.03 and 0.036 m.

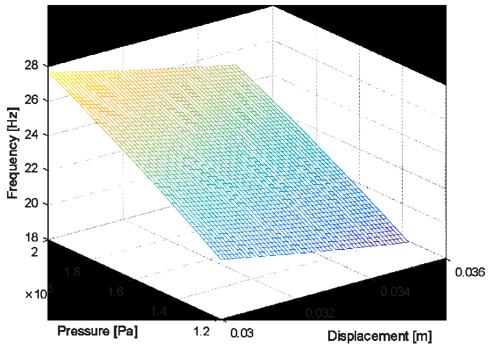

Figure 3. Natural frequency characteristic of FPEG.

For each initial pressure and set stroke, the system will oscillate at a natural frequency. Thus, it can be ensured that the FPEG works in resonance or not, therefore, by changing some input parameter and maintaining uniform pressure and stroke, the frequency of the system will be changed to another that does not match the natural one.

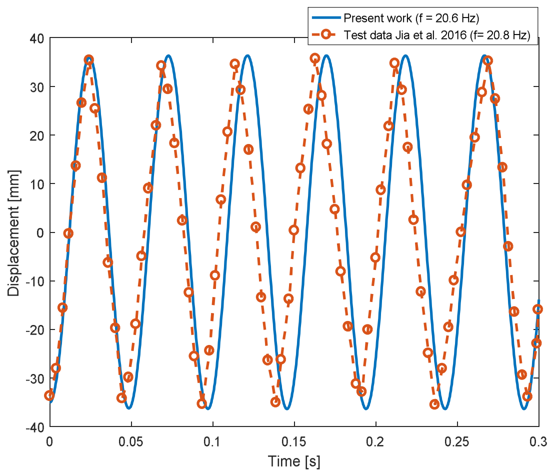

Figure 4. Comparison of simulated model (f= 20,6 Hz) and test data by Jia et al. 2016 (f=20,8 Hz)

Figure 2 shows the simulation of the displacement of the piston as a function of time during the combustion process compared to the prototype data presented by Jia et al. (2016d), in the same operating conditions.

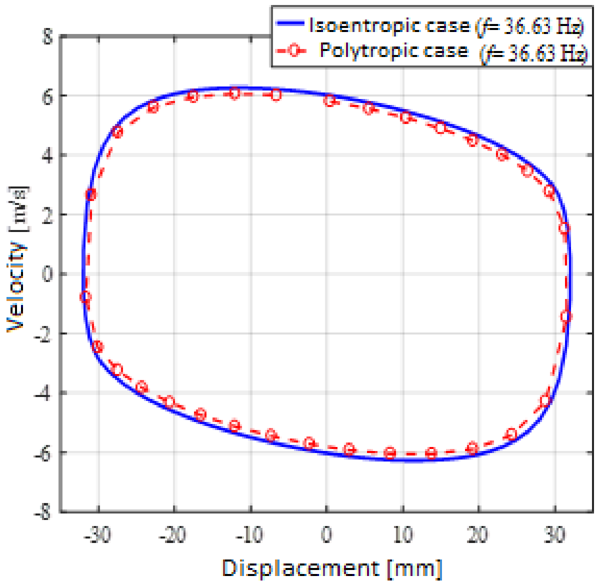

Figure 5. Figure 1. Velocity x displacement graph for variable polytrophic coefficient model (red) against the isentropic model (blue)

When comparing the isentropic case against a more real one, as expected, it shows higher energy conversion potential, which is reflected in both higher maximum velocity as well as maximum amplitude. The curve stays similar in both cases in a way that fits the FPEG alternating movement as well as the resonance frequency, which is unaltered.

6. CONCLUSION

It can be concluded that the combustion process controls the final operation and convergence of the FPEG. According to the mathematical model and based on literature data, the combustion frequency can be adjusted by changing the ignition time, mass of fuel, pressure of admission, etc. Thus, all these methods could be used to change the engines operating frequency. The frequency map shows the possible values of the natural frequency of the system for input data previously established, so it is possible to explore the resonance effect.

The comparison between the data of the literature and those developed in this work presented satisfactory results. The initial discrepancy between the frequency of the simulated model and the prototype is due to simplifications of the thermal model and linearization of the exiting force. The difference in amplitude is due to the values adopted in the simulation, which may be different from those actually used in the work that was taken as reference.

As not all adopted parameters are coincident with that of the literature, a greater breadth of displacement was obtained than the reference data. However, it is perceived that the system converges to a value over a short response time. No matter the initial state, its trajectory always reaches the limit under periodic excitation. Therefore, maintaining an excitation is crucial to this mechanisms good functioning.

Also, the proposed models allow for easy parametrization as a way of finding optimal values intended to build a prototype for further studies.

7. ACKNOWLEDGEMENTS

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