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Level Set Method Optimized with the Euclidean Distance Transform

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Abstract. In petrophysics the great scale of the problems requires robust algorithms for numerical simulations. In this article an adaptation of the well known Level Set Method is proposed with the goal of finding a well resolved Signed Distance Function. A new method of initialization and reinitialization is used, it involves the use of Euclidean Distance Transform (EDT). The EDT is also used as a way of finding a velocity function that, accurately, represents the steady state flow through a rock in cases where pressure driven forces are more relevant than surface forces. The results of simulations involving the EDT velocity function will be compared with other simulations that used velocity functions obtained with more well known algorithms, like Lattice Boltzmann Method's (LBM) and the Maximum Subscribed Sphere (MSS).

Keywords: Level-Set, Petrophysics, Simulation, Numerical Methods

1. INTRODUCTION

The Level Set is a surface tracking method introduced by Osher and Sethian (1988). It is largely used in Image Processing and Computational Fluid Dynamics because its capacity of dealing with complex surfaces.

The problem starts with a generic surface $\phi(\vec{x}(t), t)$ under the influence of forces that make it move normal to itself with an velocity F . The surface is the solution of:

$$\phi(\vec{x}(t), t) = 0 \quad (1)$$

Differentiating Eq. 1 leads to:

$$\frac{\partial \phi}{\partial t} + F |\nabla \phi| = 0 \quad (2)$$

Starting with an distribution $\phi_0 = \phi(\vec{x}, t = 0)$ it is possible to solve the initial value problem of the PDE (Eq. 2).

The Level Set Method is built such that it handles two phases naturally according to the signal of ϕ .

1.1 SIGNED DISTANCE FUNCTION AND RENORMALIZATION

An important characteristic of the method is that $|\nabla \phi|$ should be as close to 1 as possible in order to guarantee numerical stability. Even if correctly initialized, as time passes ϕ will eventually diverge from a signed distance function because of numerical errors introduced by several algorithms iterations. Chopp (1993) suggests that to deal with this problem one should reinitialize the function until $|\nabla \phi|$ is close enough to 1. However this process is extremely expensive computationally.

Sussman *et al.* (1994) proposes that for $|\nabla \phi| > 1$ the following PDE should be solved until $|\nabla \phi| = 1$,

$$\frac{\partial \phi}{\partial t} + S(\phi)(|\nabla \phi| - 1) = 0 \quad (3)$$

where $S(\phi)$ is a function defined by:

$$S(\phi) = \frac{\phi}{\sqrt{\phi^2 + |\nabla \phi|^2}} \quad (4)$$

Eq. 3 is solved by the same methods as Eq. 2. A new, and simpler, method is proposed where the current ϕ is substituted by a version initialized with the EDT of the pore space. This method has the advantage of being more efficient than solving Eq. 3 until the gradient condition is satisfied, since it can be done with two runs through the matrix and guarantee with precision $|\nabla \phi| = 1$.

1.2 EUCLIDEAN DISTANCE TRANSFORM

The Euclidean Distance Transform (EDT) is a map of values where for each foreground pixel the euclidean distance is given for the nearest background pixel. In this work we use the method proposed by Saito and Toriwaki (1994). Among the advantages of using the EDT is it's velocity as the algorithm runs on $O(n)$ and the certainty that for any initial surface will be possible to establish a signed distance function.

1.3 VELOCITY FUNCTION

One of the great problems of using the Level Set as a way to simulate flow through petroleum rocks is the need for a velocity function that represents the whole domain at all the simulation steps. Having to find a new velocity function at each iteration would cause the algorithm to become impracticable.

As a way of finding a reasonable efficient function, that represents the capillary regime, some tests where made using the EDT of the pore space as a velocity function. The ideia was that when capillary forces where dominant the velocity of the flow would be more dependent of the geometry of the rock. It would be slower in narrow pores as faster in open ones, witch should lead to a preferential path.

Something to keep in mind is that with the EDT, we have an approximation of a two phase flow with a one phase flow velocity field. It might be interesting for some cases and this approximation will be more discussed on following sections.

2. IMPLEMENTATION

2.1 INITIALIZATION

The initial distribution of fluids inside the rock is given as parameter for the program. Than the EDT is applied in the porous region, creating a map that we'll use as velocities. At the same time the EDT is applied to find the distances between the fluids inside the rock, creating two masks. The masks are them combined into a new image where one of the fluids is represented by negative values and the other by positive ones.

Since the initialization is calculated using the EDT, based on the distance between pixels, the distance variation will always be equal or very close to 1, since $\nabla r = \hat{r}$, granting the property $|\nabla\phi| = 1$.

2.2 VELOCITY MAP

One of our objectives is the possibility of finding a consistent velocity map based only in the initial fluid distribution and the rock geometry. The EDT is used to do so, but it has some limitations and some hypothesis are necessary to use it:

- It must be possible to approximate a two phase flow with a single phase velocity field.
- Capillary pressure must be greater than surface forces.

As a way of comparing the precision of the EDT velocity map approximation we compare it with maps obtained using LBM and the MSS. The three maps can be seen in the following figure:

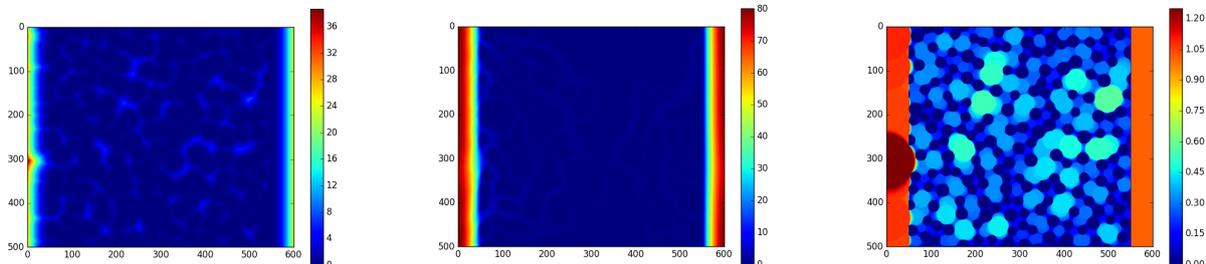


Figure 1. Comparison between three velocity maps. From left to Right EDT, LBM and MSS.

Fig. 1 shows a great difference between the maps. From the way the EDT is obtained is easy to see the pores where the velocity is greater (the bigger ones), this gives the impression on preferential paths, but it has a visible difference when compared with the map obtained with LBM. This happens in parts because LBM has a physical base behind it, while the EDT map is purely mathematical, it only "sees" the geometry and how pore and solid are distributed.

An advantage of using the Level Set, with the EDT velocity map, is the fast way it finds its velocity map (during the initialization process), but this map is an approximation and shows some serious problems, witch will be discussed in the

following sections. The LBM is a very famous method with great and consistent results. MSS is interesting and shows some results similar to LBM.

2.3 SOLVING THE EQUATION

Once the problem is initialized it is necessary to solve it to steady state. Our simulations run until 75% of the pore space was being filled with invading fluid.

A variation of Runge-Kutta Method of third order known as TVD (Total Variation Diminishing) is used to solve the time terms while Upwind Method is used to solve the space derivatives. To enhance algorithm efficiency Eq. 2 is not solved in solid parts since $F = 0$ makes the solution constant in time.

As we discussed in subsection 1.1, from times to times is necessary to renormalize the original ϕ function to grant numerical stability. In our tests two renormalization methods were used. The first is suggested by Sussman *et al.* (1994) and used by Prodanovic *et al.* (2013). They solve Eq. 3 until its stationary state. This method is the most popular in literature, although is very slow.

We suggest a new, faster, method for reinitialization of ϕ that does not needs solution of Eq. 3, and requires only two runs through the matrix. A process similar to the initialization is used, where two masks are generated with the EDT, but based on the signal of ϕ in that position instead of its value. With the two masks a new equidistant ϕ is created. The advantages of this method are the guarantee that $|\nabla\phi| = 1$ with only two runs and a great numerical efficiency, Nevertheless it shows some problems when F is too small, which will be discussed in section 3.

3. RESULTS AND DISCUSSIONS

To discuss the method the results of simulations in a 2D image of dimensions 500x601 pixels with a $\Delta t = 0.005$ are shown.

This article aimed to validate the Level Set as an useful tool for petrophysics projects. To do so it was expected that the method get similar results to algorithms used as reference in the area (LBM and MSS).

3.1 LBM VELOCITY MAP

The idea behind this test was to see how the Level Set would "behave" when a velocity map from an already validated algorithm was used.

A simulation from a one phase flow was run using LBM, when stationary state was obtained its velocity map was gathered (central map in Fig. 1).

From the image is possible to see that the velocities inside the rock come close to zero, wich was expected, but is still possible to see the preferential paths where fluid should flow more easily. Once the fluid had come to the other side from one of these paths the other ones would not be invaded, causing the rock to not be filled entirely.

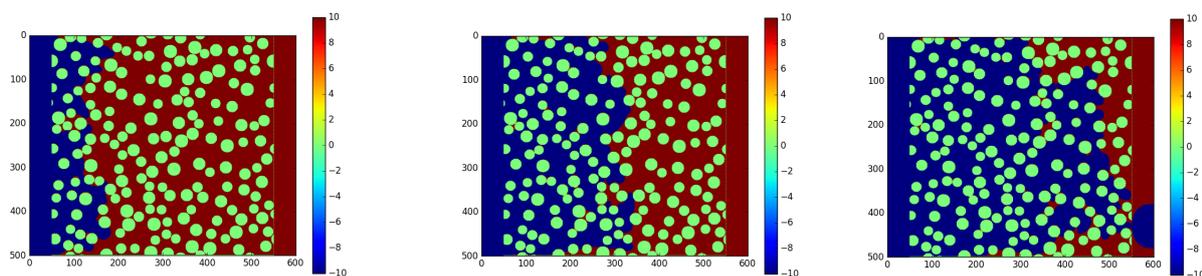


Figure 2. Invasion using the velocity map obtained with LBM in three moments: 25%, 50% e 75% of the rock filled.

Fig 2 shows the invasion when the velocity map obtained with LBM is used. Even having preferential paths, they are not followed during the simulation, what is seen is a front propagating as one through the rock. This happens even in areas where the flow should be more restrict and the paths where velocity is greater in the maps does not seem to make any real change in the flow.

3.2 MSS VELOCITY MAP

With the MSS the results are almost the same as the ones obtained with LBM's simulation. Although they are not so clear, there are preferential paths in the MSS velocity map. But as it happened in Fig. 2, when simulated with the MSS map, an unified invasion front is obtained.

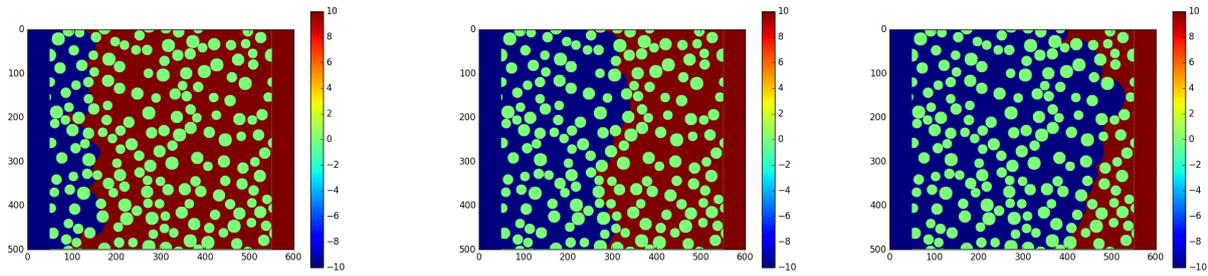


Figure 3. Invasion using the velocity map obtained with MSS in three moments: 25%, 50% e 75% of the rock filled.

Fig. 3 shows the invading front. The flow does not follow any preferential path, like the LBM simulation. But for the same conditions of renormalization and time step it does not follow the same path as the LBM simulation.

3.3 EDT VELOCITY MAP

The map obtained with the EDT is based more in an mathematical approximation than a physical one. When a narrowing occurs, the distance between the solid parts will be very small, so will be the velocity in that point. When a bigger pore is found the distance between the solid parts increases, generating greater velocities in those areas. It was expected that with these conditions would be possible to find preferential paths inside the rock in a simple and fast way.

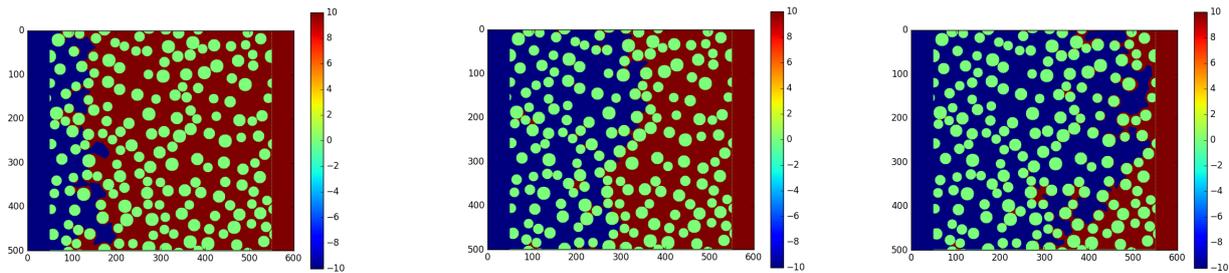


Figure 4. Invasion using the velocity map obtained with EDT in three moments: 25%, 50% e 75% of the rock filled.

An analysis of Fig. 4 shows that once again an unified invasion front is obtained, very similar to the ones obtained before.

3.4 PERFORMANCE ANALYSIS

As shown in the last sessions, the fluid has the tendency to invade the rock as an unified front saturating the hole rock almost uniformly, this behavior can be seen in the saturation curves in Fig. 5.

The relation between the curves shows that in all the maps the saturation of the rock occurs almost linearly. The great differences appear in the time it takes for each map to complete 75% of saturation. The simulation using the EDT velocity map was the fastest compared with the LBM and the MSS, this suggests a good performance. But it does not has a great effect when physical properties are not respected.

An important discussion involving this method is that, in petrophysics, problems tend to be very computationally expensive. Usually images vary between 1000^3 e 5000^3 pixels. Big images are need when you are trying to represent microporosities in a rock.

The Level Set performance is greatly affected by the way the method is build. It is necessary to reinitialize the function from time to time, as shown by Chopp (1993). In section 1.1 is briefly discussed that the most common method for reinitializing the function is to solve Eq. 3 for the whole domain to make the function a signed distance function again. However when the domain is too big, like the rocks commonly used for analysis, this process becomes extremely inefficient. Prodanovic *et al.* (2013) says that the reinitialization process is needed so that the gradient don't become neither too large or too small. So it suggests that when it starts to diverge from 1 the reinitialization process should occur. But in most implementations what happens is that this process occurs at the end of every iteration step, in an attempt to stop gradient divergences. In small images this isn't a great problem, but as the images grow the cost to reinitialize it at every step becomes way too high.

The idea of using the EDT of the pore space as a way to reinitialize the problem seemed interesting in a first moment, however it presented a great problem. As the function evolves it takes some iterations to a pixel that is positive to become

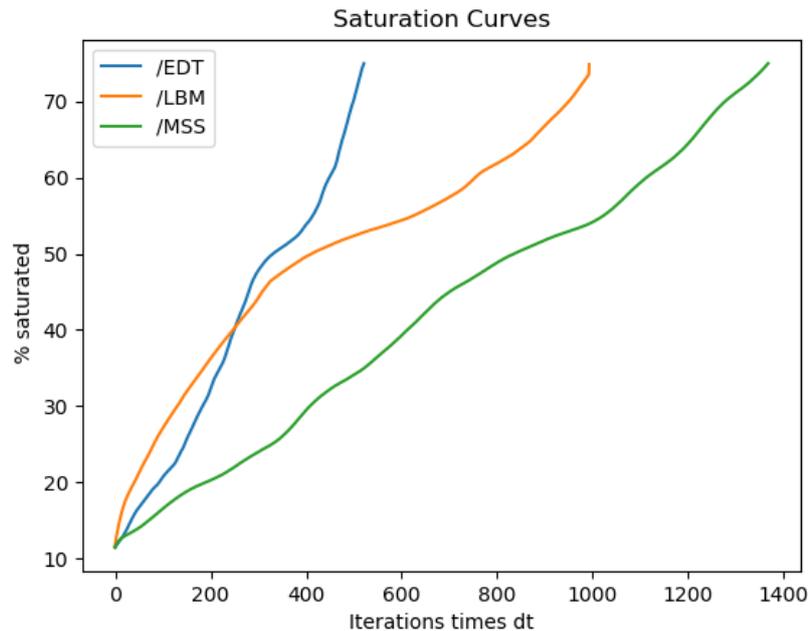


Figure 5. Saturation curves showing the percentage of rock filled with invading flow in the vertical axis, and the number of iterations times the time step in the horizontal axis.

negative (representing that the invading fluid had completely taken over that space), so when the function was reinitialized with the EDT if one pixel wasn't completely filled it would lose the "progress" of the invading fluid, causing a cut effect. So in areas where progress was way too slow (narrow pores) the function would never move, because it was constantly cut due to the EDT reinitialization.

4. CONCLUSIONS

The Level Set Method is interesting for Computational Fluid Dynamics, and is used in some petrophysics studies. However for this article's interests the method does not fit what was expected. The reinitialization process is extremely slow and inefficient, but it can't be ignored. The greater problem is to find a velocity function that represents the problem well and is efficient computationally. Using the EDT of the pore space as an approximation of the velocity function has a good mathematical base but the results don't match the expected. Even the maps obtained with other algorithms already tested, like LBM and the MSS, show bad results.

In the end there is no reason to use the Level Set Method instead of other, more precise and efficient, algorithms like LBM for the cases here studied.

5. ACKNOWLEDGMENTS

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6. REFERENCES

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