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2D SPATIAL MODEL OF THE HUMAN GAIT SINGLE SUPPORT PHASE BASED ON PREDICT IVE DYNAMICS

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Abstract.
Simulation of human body motion is a valuable tool in different fields such as robotics and biomechanics. The simulation of human motion is a challenging problem from the physical and computational perspectives and several models have been proposed in literature. Using a methodology named predictive dynamics, this work aims to propose a 2D spatial model of human walking during the single stance phase (SSP), using a flat foot adaptation. The model is based on inverse dynamics in which the angular displacements are interpolated by 5th degree B-splines and the body kinematics is calculated by using the robotic formulation proposed by Denavit-Hartenberg (DH). The equation of motion is developed by the recursive Lagrangian formulation due to its computational efficiency. An optimization problem was set up in order to obtain the control points of the B-splines using as the objective function the dynamic effort. The constraints imposed on the movement are time-dependent, such as the torque/angle limits, and the dynamic stability criterion defined by Zero Moment Point, or time-independent. The results from the model are favorably compared to Winter's data, in particular the ground reaction forces.

Keywords: *Human Gait, Predictive Dynamics, Optimization, Robotics, Biomechanics*

1. INTRODUCTION

Several two-dimensional models have been proposed to simulate the human gait. There are generally two approaches for this dynamical system modelling: the inverse dynamics, in which is required the previous knowledge of kinematics variables; and the direct dynamics, in which acting torques and forces are the problem's input.

Here, a different approach has been employed to analyze and simulate human walking, it is called predictive dynamics. This method simulates models with several degrees of freedom and uses a optimization routine to find a real movement for model, defining an objective function related to the performance of human walking (Xiang, Arora, & Abdel-Malek, 2010).

Chao and Rim (1973) proposed one of the first models to use optimization for locomotion. They used a three degrees of freedom model to simulate a leg without trunk. The authors used the steepest descent method as the optimization method and the inverse dynamics' principles. They defined the objective function as the time integral of the difference between the laboratorial and calculated values of the angular displacements in order to obtain the moments related to those angular displacements. Koopman et al. (1995) used an 8-segment 3D model. Experimental data were used to determine the movement of the hip, knee and ankle. The design variables of the optimization problem in the inverse dynamics methodology were the joint profile of the pelvis and trunk discretized by Fourier series.

Xiang et al. (2009) presented a methodology for predicting human walking with a 55 degrees of freedom 3D model. The formulation uses a recursive method to calculate the kinematics and dynamics of the motion. The Lagrangian equations of motion and the ZMP stability criterion (Vukobratic & Borovac, 2004) were used as a constraint along with the physical constraints of the motion itself. However, it is extremely difficult to prescribe all characteristics of a model with a high number of degrees of freedom. A reduced order model capable of describing the human walking compatible with the available experimental data is of great value for the analysis of human walking.

Here a seven-degree of freedom 2D model with a flat foot model is proposed based on the methodology of Xiang et al. (2009) to address the simple support phase of human gait using the predictive dynamic methodology, due to its efficiency. Since the gait is symmetric and cyclic, it is only being considered half of the gait cycle. Furthermore, the SSP starts at 10% and ends at 50% of the gait cycle. The results compare favorably with available experimental results.

2. 2D MODEL OF SINGLE SUPPORT PHASE

Figure 1 shows the proposed model, where the circles represent the position of the center of mass of each link and the seven revolute degrees of freedom are denoted by θ_i (i=1, … , 7). The model has its origin at the toe of the stance leg, where two virtual prismatic degree of freedom (which are not showed on the Figure 1) with null displacement are considered. Its objective is to evaluate the ground reaction forces and to serve as the global reference system. Therefore, for the purpose of forces and torque calculations, the model will have 9 degrees of freedom, but the first two have no displacement. In this sense, the Figure 1 is only depicting 7 degrees of freedom. A link of negligible length is used between degrees of freedom 4 and 5.

The proposed analysis methodology is based on an optimization routine in which the imposed constraints are the ground penetration avoidance, equations of motion, dynamic stability (zero moment point), strength and joint limits based on experimental data. The joint profiles of angular displacement, velocity and acceleration are the design variables, which are interpolated by 5th degree B-splines (Biuzuner & Jesus, 2014). This methodology does not require numerical integration, increasing then the numerical efficiency.

The objective function is the dynamic effort, *f*. It is defined as the time integral of all joint torques (Xiang et al., 2009). The torque values are normalized by the maximum absolute value of the maximum joint torque, $|\tau|_{\max}$. It is written as

$$f = \int_0^T \left(\frac{\tau(\mathbf{q},t)}{|\tau|_{\max}} \right)^T \left(\frac{\tau(\mathbf{q},t)}{|\tau|_{\max}} \right) dt \,. \tag{1}$$

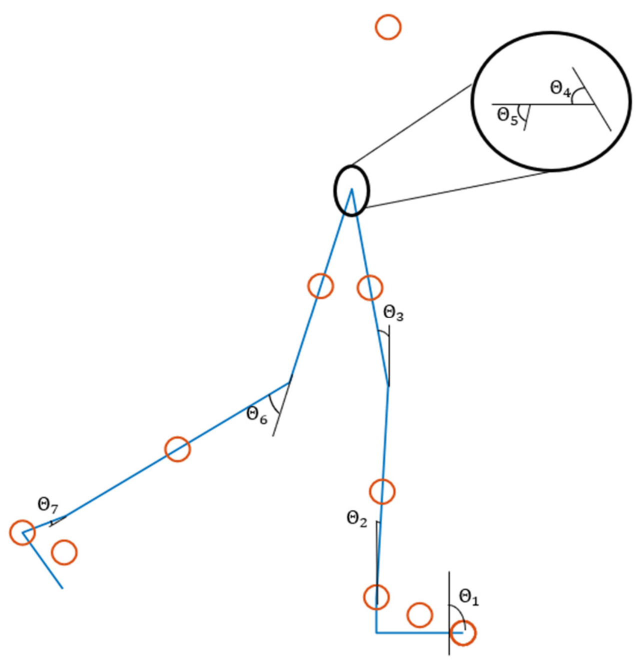

Figure 1. Proposed seven degree of freedom gait model

2.1 Constraints

Two types of constraints were imposed to the system regarding its time dependency.

The time-dependent constraints which include the joint limits, torque limits, ground penetration limit, hip progression velocity, stability. Those constraints are presented in the following inequalities:

$$\mathbf{q}^i \le \mathbf{q}(t) \le \mathbf{q}^s, \qquad 0 \le t \le T \tag{2}$$

$\mathbf{q}^i$ and $\mathbf{q}^s$ are the lower and upper values for each joint rotation.

$$\boldsymbol{\tau}^i \le \boldsymbol{\tau}(t) \le \boldsymbol{\tau}^s, \qquad 0 \le t \le T \tag{3}$$

$\boldsymbol{\tau}^i$ and $\boldsymbol{\tau}^s$ are the lower and upper values for the toque limits.

The single support phase of human gait is characterized by a unilateral contact between the ground and the stand foot. It is represented by the following constraints:

$$\begin{aligned} & y_i(t) = 0, \quad \dot{x}_i(t) = 0, \quad \dot{y}_i(t) = 0 \quad i \in \Omega \\ & y_i(t) \geq \varepsilon, \quad i \notin \Omega \quad 0 \leq t \leq T \end{aligned} \tag{4}$$

Ω is a set of points that are in contact to the ground. During the first 70% of the SSP, both heel and toe are in contact to the ground, after that period only the toe is in contact to the ground. ε is defined as any number greater than zero, it means that other points that aren't in contact with the ground should be above it.

The velocity of progression of the body should be constant, but it is very difficult to achieve and not very realistic. Then it was defined that the hip anteroposterior velocity ($v_{hip,x}$) should be inside an interval defined by a maximum and minimum velocity, v_{max} and v_{min}, respectively.

$$v_{\min}(t) \leq v_{hip,x}(t) \leq v_{\max}(t), \qquad 0 \leq t \leq T \tag{5}$$

The last time-dependent constraint is the dynamic stability, it was used the Zero Moment Point (ZMP) concept brought by Vukobratic & Borovac (2004). The ZMP must be inside the polygon of support, during the SSP the polygon of support is defined by the region under the stand foot. This is a 2D model, therefore only the x coordinate is evaluated. This conditions is presented below:

$$x_{heel}(t) \leq x_{ZMP}(t) \leq x_{toe}(t), \qquad 0 \leq t \leq T \tag{6}$$

x_{heel}, x_{ZMP}, x_{toe} are respectively the horizontal coordinate of the heel, ZMP and toe.

The time-independent constraints were applied in order to avoid the swing leg touching the ground and to define the step length. Restraining the knee rotation of the swing leg (q_j) during the midstance phase (t_{ms}) is a sufficient condition to avoid the swing leg dragging on the ground.

$$55° \leq q_j \leq 65°, \qquad t = t_{ms} \tag{7}$$

The initial and final coordinates of the toe and heel of swing leg are prescript to define the step length:

$$x_{ts}(0) = -L/2, \quad y_{ts}(0) = 0, \quad x_{hs}(T) = L/2, \quad y_{hs}(T) = 0 \tag{8}$$

The subscript *ts* and *hs* means respectively the toe and heel of swing leg. The cycle duration of the single support phase is T.

2.2 Objective Function Routine Calculation

The first step of the objective routine is to define the initial control points (PC). The experimental value of angular displacement (Winter, 2009) for each degree of freedom is defined as the first and last control point. Afterwards, the intermediate values of the points are found by a linear interpolation of the extreme points. The 5th degree B-Splines are used to find the variation of the angular displacement of each degree of freedom. The angular velocity and acceleration are obtained by deriving the base functions with respect to time. The Cartesian position, velocity and acceleration in the global reference system are obtained using the following 4×4 transformation matrices:

$$\mathbf{A}_j = \mathbf{T}_1\mathbf{T}_2\mathbf{T}_3 \cdots \mathbf{T}_j = \mathbf{A}_{j-1}\mathbf{T}_j \tag{9}$$

$$\mathbf{B}_j = \dot{\mathbf{A}}_j = \mathbf{B}_{j-1}\mathbf{T}_j + \mathbf{A}_{j-1}\frac{\partial \mathbf{T}_j}{\partial q_j}\dot{q}_j \tag{10}$$

$$\mathbf{C}_j = \dot{\mathbf{B}}_j = \ddot{\mathbf{A}}_j = \mathbf{C}_{j-1}\mathbf{T}_j + 2\mathbf{B}_{j-1}\frac{\partial \mathbf{T}_j}{\partial q_j}\dot{q}_j + \mathbf{A}_{j-1}\frac{\partial^2 \mathbf{T}_j}{\partial q_j^2}\dot{q}_j^2 + \mathbf{A}_{j-1}\frac{\partial \mathbf{T}_j}{\partial q_j}\ddot{q}_j \tag{11}$$

where $\boldsymbol{T_j}$ is the Denavit-Hartenberg transformation matrix between $\boldsymbol{j}$ and $\boldsymbol{j-1}$ degrees of freedom, with j = 1 to *n*, where *n* is the number of degrees of freedom. In addition, $\boldsymbol{q_j}$, $\dot{\boldsymbol{q}}_j$ and $\ddot{\boldsymbol{q}}_j$are respectively the angular displacement, velocity and acceleration of node j. The initial values of the transformation matrices are: $\boldsymbol{A_0} = [\boldsymbol{I}]$ $\boldsymbol{A}_0 = [\boldsymbol{I}]$and $\boldsymbol{B}_0 = \boldsymbol{C}_0 = [\boldsymbol{0}]$.

Thus, the Cartesian position, velocity and acceleration of any point belonging to the system are obtained as

$$^0\mathbf{r}_j = \mathbf{A}_j\mathbf{r}_j; \quad ^0\dot{\mathbf{r}}_j = \mathbf{B}_j\mathbf{r}_j; \quad ^0\ddot{\mathbf{r}}_j = \mathbf{C}_j\mathbf{r}_j \tag{12}$$

where $\boldsymbol{r}_j$, $\dot{\boldsymbol{r}}_j$, $\ddot{\boldsymbol{r}}_j$ are the position, velocity and acceleration augmented vector in local coordinate of the *j*-th Cartesian system.

To calculate the applied torque at each degree of freedom, a backward recursive dynamic methodology is employed (Hollerbach, 1980)

$$\tau_i = tr\left[\frac{\partial \mathbf{A}_i}{\partial q_i}\mathbf{D}_i\right] - \mathbf{g}^T\frac{\partial \mathbf{A}_i}{\partial q_i}\mathbf{E}_i - \mathbf{f}_k^T\frac{\partial \mathbf{A}_i}{\partial q_i}\mathbf{F}_i - \mathbf{G}_i^T\mathbf{A}_{i-1}\mathbf{z}_0 \tag{13}$$

where $\mathbf{D}_i$ is a 4×4 transformation matrix and $\mathbf{E}_i$, $\mathbf{F}_i$ and $\mathbf{G}_i$ are 4×1 transformation vectors, given by

$$\begin{aligned} \mathbf{D}_i &= \mathbf{I}_i \mathbf{C}_i^T + \mathbf{T}_{i+1} \mathbf{D}_{i+1} \\ \mathbf{E}_i &= m_i \, {}^i\mathbf{r}_i + \mathbf{T}_{i+1} \mathbf{E}_{i+1} \\ \mathbf{F}_i &= {}^k\mathbf{r}_f \delta_{ik} + \mathbf{T}_{i+1} \mathbf{F}_{i+1} \\ \mathbf{G}_i &= \mathbf{h}_k \delta_{ik} + \mathbf{G}_{i+1} \end{aligned} \tag{14}$$

Here, m_i are the masses, $\boldsymbol{I}_i$ is defined as the inertial matrix of link i, $\boldsymbol{f}^T{}_k$ are the external forces and $\boldsymbol{M}^T{}_k$ the external moments for the link k defined in global coordinates, $\boldsymbol{g}^T$ is the gravity vector, ${}^i\boldsymbol{r}_i$ is the position of the center of mass of link i in its own local coordinate system, ${}^k\boldsymbol{r}_f$ is the position of the external force in the local coordinate system k The vector $\boldsymbol{z}_0 = [0; 0; 1; 0]$ is defined for revolution joint and $\boldsymbol{z}_0 = [0; 0; 0; 0]$ for prismatic joint. The initial value of $\boldsymbol{D}_{n+1} = \boldsymbol{E}_{n+1} = \boldsymbol{F}_{n+1} = \boldsymbol{G}_{n+1} = [\mathbf{0}]$.

The *GlobalSearch* function from *Matlab*® (MathWorks, 2015) was used to run the optimization routine. The constrains are added to the objective function by summing a penalizing factor of 9999 every time that the constraints were violated.

The Figure 2 shows the steps to evaluate the objective function and the proposed constraints. Initially, 9 control points are entered for each degree of freedom, then the angular displacement ($\mathbf{q}$), velocity ($\dot{\mathbf{q}}$) and acceleration ($\ddot{\mathbf{q}}$) are interpolated using the 5th-degree B-splines. The cartesian kinematic values are calculated using the equation (12), so the matrix $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$ can be used on the recursive dynamic formulation to retrieve the joint torques. The last step is to calculate the objective function using the joint torques on equation (1) and adding a 9999-penalty factor for each time that a constraint is broken. The *Globalseach* function will run until find the objective function converge to the minimum value.

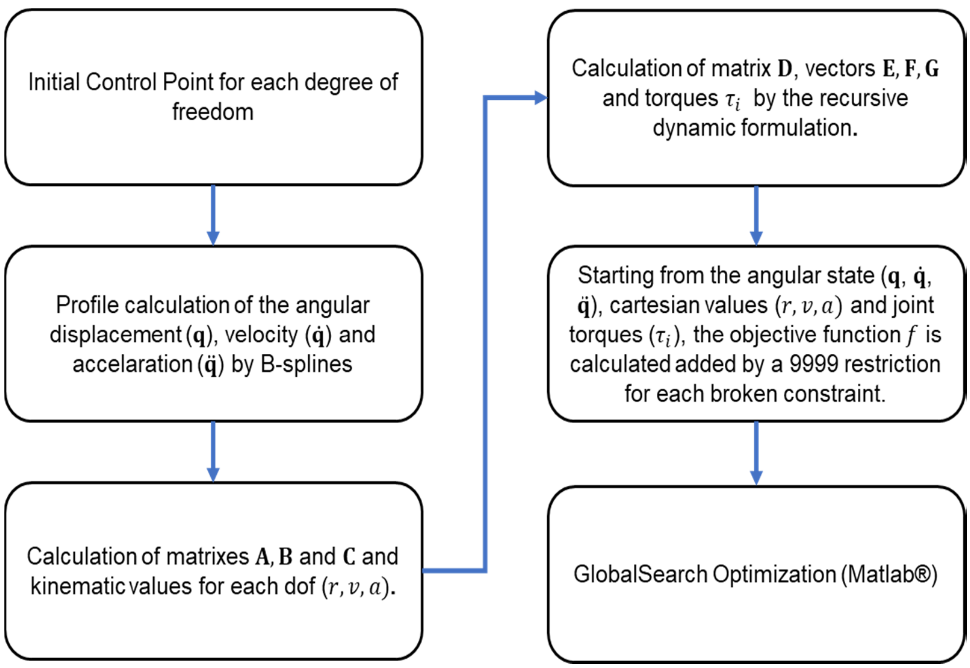

Figure 2. Routine to evaluate the objective function and optimization calculation

2.3 Foot Adaptation

One important aspect in the human walking modelling is the feet description. Here the flat foot adaptation is adopted, where the foot is modeled with no joint separating the midfoot from the forefoot. In such case, the entire ground-foot contact happens at a single point for any applied rotation. The initial rotation angle is null until a defined instant is reached, afterwards the rotation increases up to the end of the single support phase. The Figure 3 depicts the rotation progress, when the foot is flat to the ground (Figure 3a) and after any α negative rotation (Figure 3b).

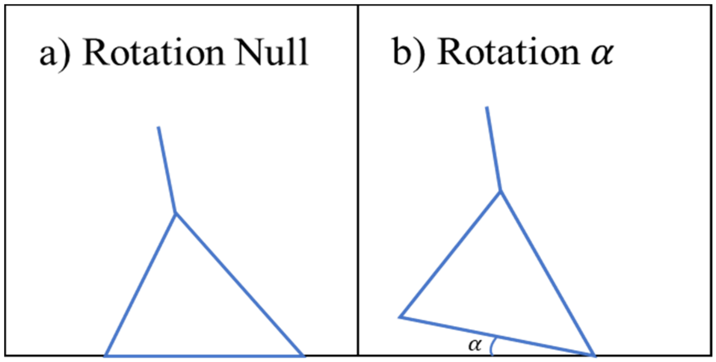

Figure 3. Foot Rotation Adaptation

The 5th degree B-splines did not interpolate properly the displacement for the foot rotation. This happens because for the flat-foot simplification the displacement is null until 70% of the single support phase and the B-splines curve oscillates around the zero-horizontal line, but never stays constant and equal to zero. Any negative rotation means that the foot penetrates the ground, which is not realistic. An adjust had to be made regarding the control points (CP). It was defined then that the CPs for the foot rotation would be set up to the value of zero until 70% of SSP, after the CP would be defined by the optimization routine.

3. RESULTS

After running the routine a few times, the optimization process converged into favorable results. The main results are the joint displacement, the joint torques and the ground reaction force. They are compared with the experimental results from Winter (2009), during the single support phase.

At the beginning of the simple support phase, the predicted value for the ankle rotation is around 4° lower than the laboratory value (Figure 4a), but still within expected range. After 35% of the march, the modelling result tends to grow more sharply, reaching a maximum value of 15 °, while the values of Winter (2009) reach only 7 °. The results of Abdel-Malek and Arora (2013) show a maximum limit of up to approximately 18 ° for ankle dorsiflexion. However, the increasing of the rotation in the final stretch of the SSP should not happen so abruptly. One possible explanation for this unconformity is that the foot rotated about 10 ° (less than expected from laboratory) and in order to compensate this low rotation, the ankle had an excessive dorsiflexion. One of the restrictions on movement is the step length. The body has a shorter sagittal progress because of the reduced foot rotation, requiring a bigger effort from the ankle to reach the final position.

The Figure 4b shows the result as well for the knee rotation. It has a very close approximation of the experimental value.

Figure 5b compares the result of the knee torque of the support leg with the experimental values. The results are shown to be favorable, especially the trend of the curve. It should be noted that the graph shows a discontinuity due to the adaptation used to simulate the foot. The ankle torque (Figure 5a) failed to represent the experimental values, this is explained because of the excessive dorsiflexion illustrated at the Figure 4a at the end of the SSP. A positive torque applied at the ankle would reduce the excessive dorsiflexion, hence it would diminish the difference between the model response and the experimental data, not only the ankle rotation but also the ankle torque.

The last parameters to compare with are the ground reaction forces. The horizontal GRF (Figure 6a) has the shape of the results found by Winter (2009). The vertical GRF presented the characteristic "M" shape (Figure 6b), however the part of the graph where the concave is up is not so accentuated as should be. One possible explanation is the heel of the stand leg should have lost contact with the ground by about 30% of the gait, this contact is lost by 40% of the gait because of the simplification though. Therefore, the foot is flat to the ground for a longer period, resulting on a higher ground reaction force, mostly between 20% to 40% of the gait.

a) b)

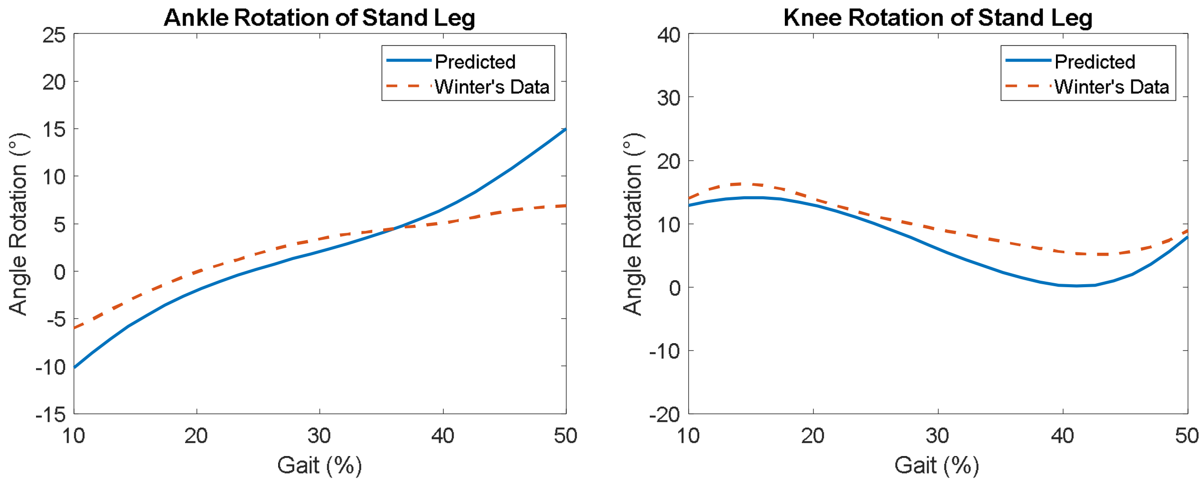

Figure 4. Ankle and Knee Rotation. The solid line is the result acquired from the model and the dashed line is the Winter's Experimental Data.

a) b)

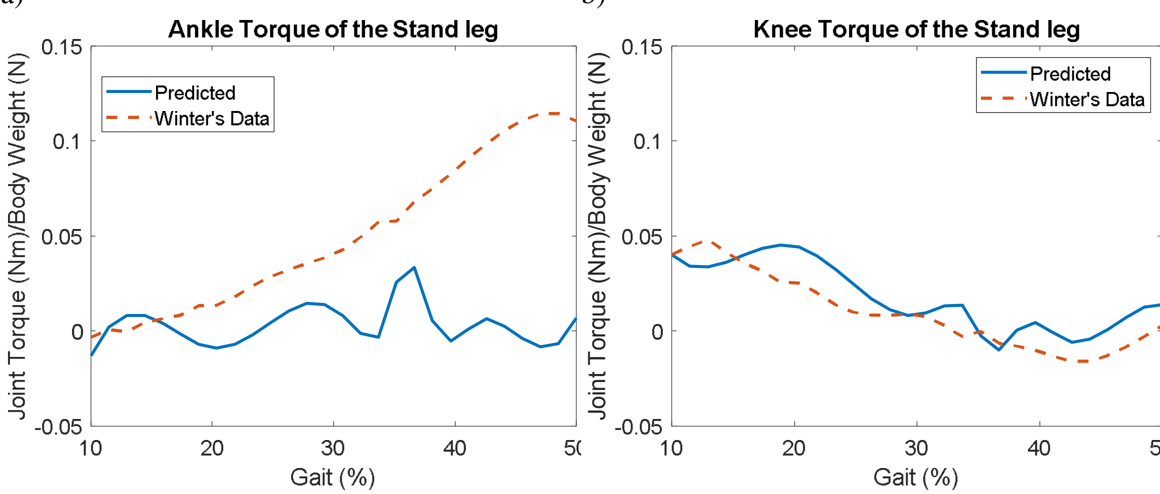

Figure 5. Ankle and Knee Rotation. The solid line is the result acquired from the model and the dashed line is the Winter's Experimental Data.

a) b)

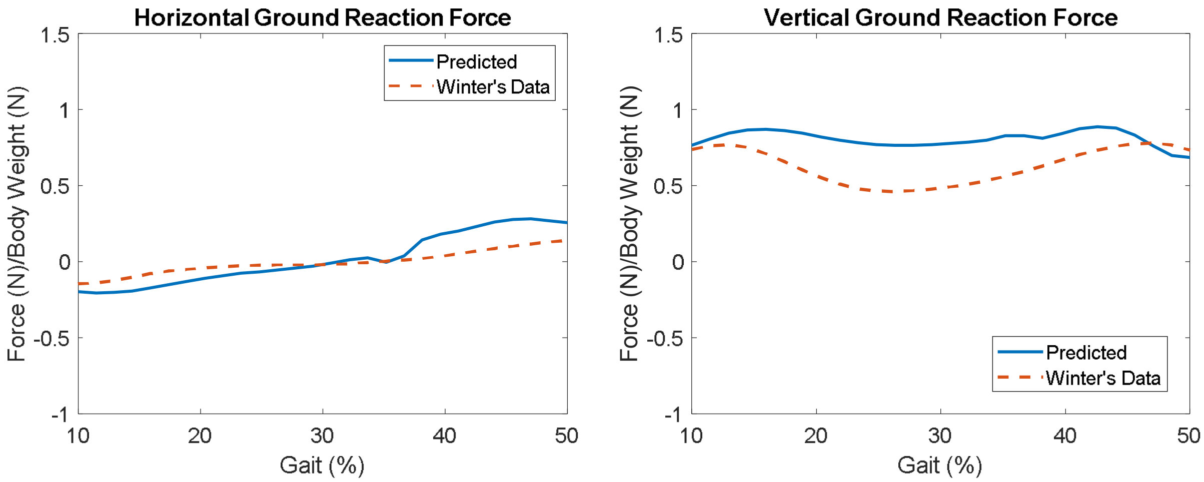

Figure 6. Ground Reaction Forces. The solid line is the result acquired from the model and the dashed line is the Winter's Experimental Data.

Figure 7 shows the model set at 10%, 23%, 37% and 50% of the single support phase. It is noticed that there is no rotation outside the human limit, the physical limits of the ground are not broken, and the stand leg works as an inverted pendulum (Kuo et al., 2005).

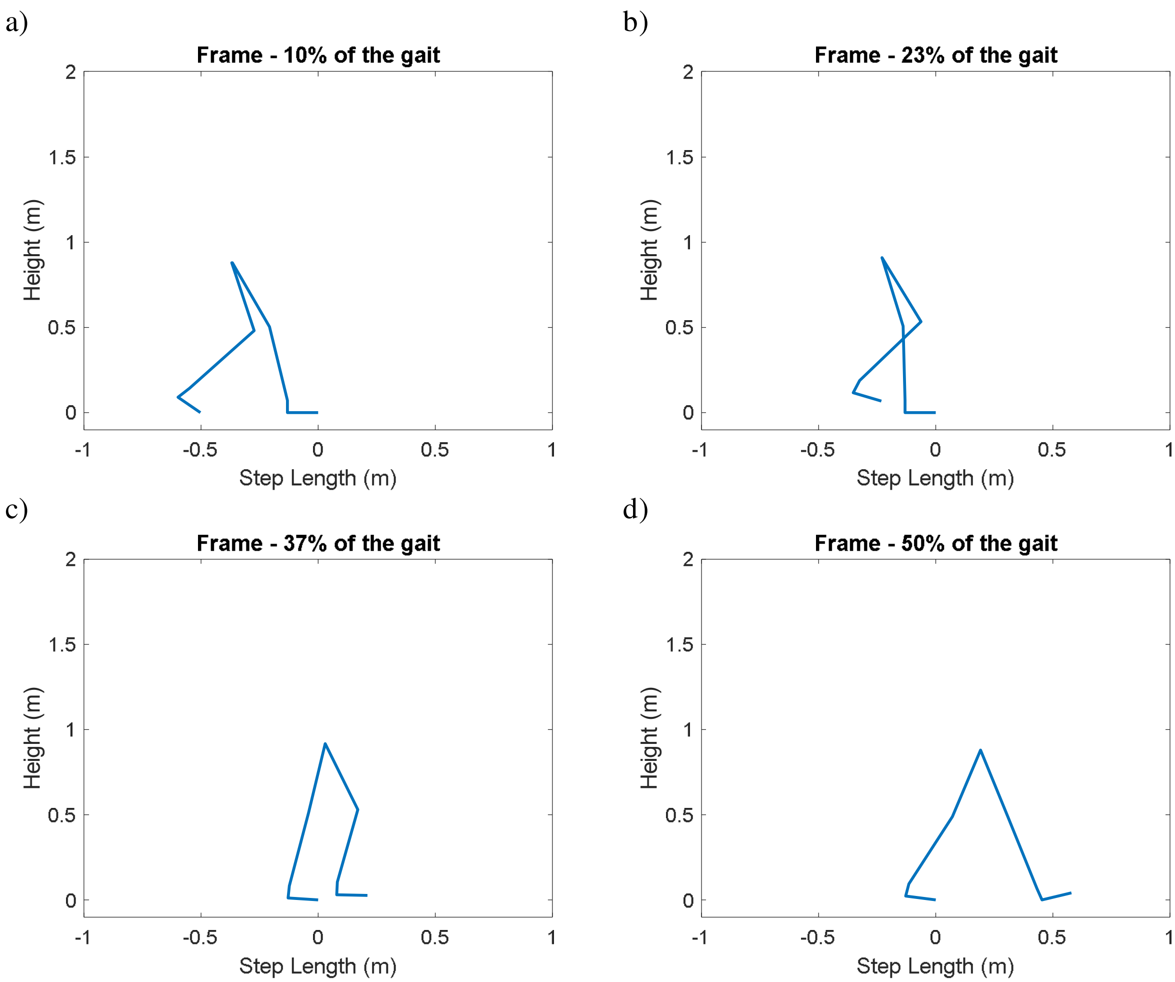

Figure 7. Development of the SSP of the gait. Here is shown 4 instants at 10%, 23%, 37% and 50%.

4. CONCLUSION

The 2D seven dof model used the concept of predictive dynamics, based on the work of Abdel-Malek and Arora (2013). The main feature of this methodology is not needing previous known experimental data, it is only required a performance measure and physical constraints. Another advantage that it should be highlighted is the capability of analyzing the cause and effect of any hypothesis, such as different height, weight or any limitation. It also should be noted that simulating different speeds might lead to incorrected results, because depending of the walking speed, the human body starts to run, and the proposed model should not be applied. In general, use another methodology is used to simulate running.

It was observed that the main issue of this model is to simulate the flat foot, which means the foot is plane and there is no division between forefoot and midfoot. During the first 70% of the support phase, the foot model does not rotate and the algorithm using B-splines is not very efficient to interpolate the foot rotation. To overcome this issue, it was presented an alternative to split the control points for the foot rotation. Those control points were manually set to zero for the first 70% of the single support phase, meanwhile for the rest of the SSP the optimization algorithm defined the most appropriated control points.

The kinematic results of the model showed favorably compared with the Winter's experimental data, frequently used as reference in the literature. Nevertheless, the dorsiflexion was greater than expected possibly because of an insufficient foot rotation. Consequently, greater angulation is required to complete the step length and ensure that the swinging leg reaches the ground at the end of the SSP.

Torques and ground reaction forces were calculated from the equation of motion using the inverse-dynamics process. The calculated torque on the knee showed to be very similar to Winter's data. On the other hand, ankle torque was not accurate. This is a direct consequence of the over dorsiflexion.

Finally, the vertical ground reaction force presented its characteristic "M" shape, only with a less pronounced positive concavity. The horizontal ground reaction force was also very similar to the expected values. The GRF are the a very important indicator of the validity of the model.

5. ACKNOWLEDGEMENTS

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