COBEM UBERLÂNDIA 2019

ABCM Associação Brasileira de Engenharia e Ciências Mecânicas

25th ABCM International Congress of Mechanical Engineering
October 20-25, 2019, Uberlândia, MG, Brazil

REVISTING THE BEHAVIOR OF BI-DIRECTIONAL COUPLED CHUA CIRCUIT

José Manoel Balthazar
Universidade Estadual de São Paulo Julio de Mesquita Filho- Faculdades de Engenharia Bauru- Campus Bauru and Universidade Federal Tecnológica do Paraná - Campus Ponta Grossa
Mauricio a. Ribeiro
Universidade Federal Tecnológica do Paraná - Campus Ponta Grossa
Wagner Barth Lenz
Universidade Federal Tecnológica do Paraná - Campus Ponta Grossa
Remei Haura Junior
Universidade Estadual de São Paulo Julio de Mesquita Filho- Faculdades de Engenharia Bauru- Campus Bauru and Centro Universitário Vale do Iguaçu (UNIGUAÇU)
Angelo Marcelo Tusset
Universidade Federal Tecnológica do Paraná - Campus Ponta Grossa

Abstract. *In this paper, a nonlinear coupling of two Chua circuits is presented. This circuit has been studied extensively because it has the advantage of being very simple, presenting a complex dynamics that presents chaos. That is, this behavior is presented when the Chua circuit is coupled or uncoupled. In addition to being a circuit with potential in various areas such as telecommunications. The coupled system behavior analysis allows phenomena such as information exchange and synchronization using such circuits. For dynamic analysis and stability we use classical tools of dynamic systems theory, such as Lyapunov exponent and the bifurcation diagram. We also use statistical methods of analysis of fractional systems in the integer derivative system of Chua. This analysis was performed with a chaotic system and a periodic state system, so the intensity of linear and nonlinear coupling was studied by this tooling. As a result it can also be noted that one circuit of Chua does not necessarily have the same behavior of the other, even though there is a strong coupling between them. It is important to note that because it is a chaos circuit its behavior will be totally different depending on the initial conditions.*
Keywords: *oscillating circuit, coupled chua circuit, behavior of the Chua circuit*

1. INTRODUCTION

Synchronization of periodic signals is a well-known phenomenon in physics. However, chaotic systems which do not have a stable behavior, are systems that defy synchronization. Two identical chaotic systems have distinct trajectories, even though both of them maps out the same attractor in phase space. With the study about the synchronization of chaos, many potential areas of application have emerged, among them communication systems.An information signal is transmitted using a chaotic signal as a broadband carrier and synchronization is used to recover the signal (Pecora and Carroll, 1990) and (Wu and Chua, 1993). One circuit that can be used in communication systems is the Chua circuit that presents chaos. Suggested by Leon Chua, this circuit has been studied extensively because presents the advantage of being very simple and presents a complex dynamics regards bifurcation and chaos (Chua et al.*, 1993). This simplified circuit has an inductor, a resistor, two capacitors and a non-linear element called Chua diode, as can be seen in Figure 1 (Matsumoto, 1984).*

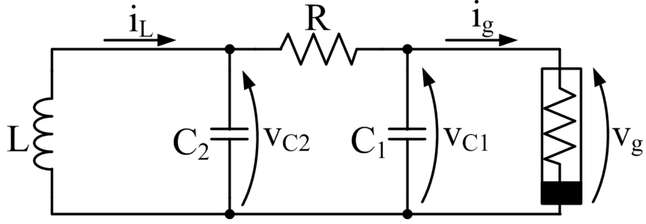

Figure 1. Chua circuit. Adapted from (Chua *et al.*, 1993)

Where R is a resistor, C_1 and C_2 are capacitors, L is an inductor and the nonlinear component is called Chua diode, i_L and i_g are the current through inductor and Chua diode respectively, v_g is the voltage across the Chua diode,v_{C1} and v_{C2}

are the voltages across capacitor C_1 and C_2 respectively. Many works were proposed analyzing the behavior of the Chua oscillator, with the purpose of analyzing its chaotic behavior and finding attractors varying its parameters (Matsumoto, 1984) and (Chua, 1995). Other works propose a coupling between two circuits of Chua. This coupling can be establishing a master and slave relationship between the Chua circuits (Itoh and Chua, 2013) and (Bilotta et al.*, 2014).But in other cases there is not necessarily a synchronism between both circuits and may have hyper-chaotic behavior (Cannas and Cincotti, 2002). The following is the proposed circuit with its mathematical modeling.*

This paper is organized as follows: section 2 presents the studied topology. Section 3 presents the mathematical model of the circuit. In section 4 the methodologies used to verify chaos in the system are presented. Section 5 presents the results obtained from the structure simulation and finally in section 6 presents the conclusions.

2. CHUA OSCILLATOR COUPLED MODELING

The Chua oscillator coupled modeling is proposes as bidirectional Chua's circuit and its structure (Cannas and Cincotti, 2002). The circuit is composed by two identical Chua's circuits, this structure can be seen in Figure 2.

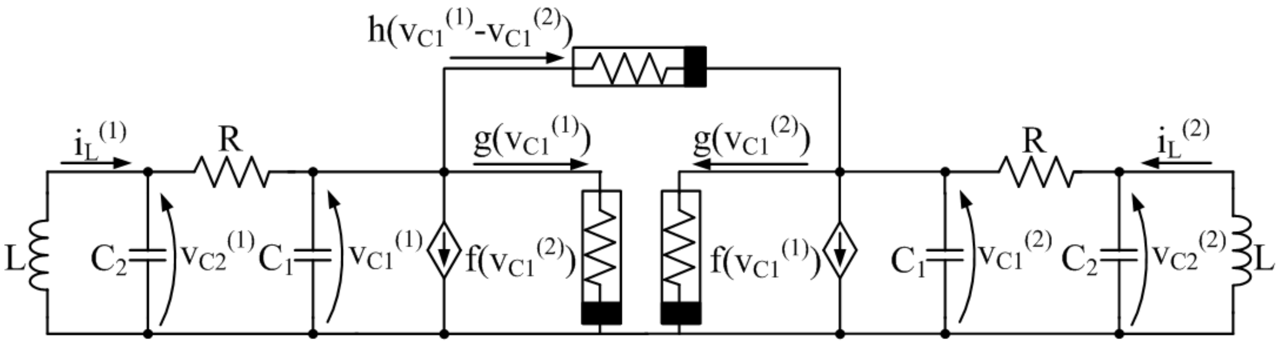

Figure 2. Coupled Chua circuit. Adapted from (Cannas and Cincotti, 2002)

Where R is a resistor, C_1 and C_2 are capacitors, L is an inductor. This circuit has three Chua diode, being $h\left(v_{C1}^{(1))}-v_{C1}^{(2))}\right)$ the characteristics of the coupled Chua diodes,$i_L^{(1)}$ and $i_L^{(2)}$ are the currents through inductors of the respective Chua circuits, $v_{C1}^{(1)}$, $v_{C2}^{(1)}$, $v_{C1}^{(2)}$ and $v_{C2}^{(2)}$ are the voltages across the capacitors, $g\left(v_{C1}^{(1)}\right)$ and $g\left(v_{C1}^{(2)}\right)$ are the voltage defined in each non-linear Chua resistor, $f\left(v_{C1}^{(1)}\right)$ and $f\left(v_{C1}^{(2)}\right)$ are a non-linear voltage controlled current source, driven by voltage v_{C1} of the other circuit.

3. MATHEMATICAL MODEL

A function was chosen for the nonlinear resistor that which causes chaos (Huang et al.*, 1996). Thus, the voltage across the Chua resistor is given by Eq. 1.*

$$g\left(v_{C1}\right)=\gamma_1 v_{C1}+\gamma_3 v_{C1}^3 \tag{1}$$

The resistor model which makes the coupling between the two circuits is given by Eq. 2.

$$h\left(v_{C1}^{(1)}-v_{C1}^{(2)}\right)=\frac{K_c}{2}\gamma_3\left(v_{C1}^{(1)}-v_{C1}^{(2)}\right)^3 \tag{2}$$

A non-linear voltage-controlled current source, driven by the voltage v_{C1} of the other circuit, is introduced into each Chua circuit and is defined by Eq. 3.

$$f\left(v_{C1}\right)=d\gamma_3 v_{C1}^3 \tag{3}$$

Considering the main equations and conducting the analysis of the circuit, it is possible to construct the equations of the system dynamics, composed of a set of six differential equations given by Eq. 4.

$$\begin{cases} C_1 \dfrac{dv_{C1}^{(1)}}{dt} = -\dfrac{v_{C1}^{(1)} - v_{C2}^{(1)}}{R} - g\left(v_{C1}^{(1)}\right) - h\left(v_{C1}^{(1)} - v_{C1}^{(2)}\right) - f\left(v_{C1}^{(2)}\right) \\ C_2 \dfrac{dv_{C2}^{(1)}}{dt} = \dfrac{v_{C1}^{(1)} - v_{C2}^{(1)}}{R} + i_L^{(1)} \\ L \dfrac{di_L^{(1)}}{dt} = -v_{C2}^{(1)} \\ C_1 \dfrac{dv_{C1}^{(2)}}{dt} = -\dfrac{v_{C1}^{(2)} - v_{C2}^{(2)}}{R} - g\left(v_{C1}^{(2)}\right) + h\left(v_{C1}^{(1)} - v_{C1}^{(2)}\right) - f\left(v_{C1}^{(1)}\right) \\ C_2 \dfrac{dv_{C2}^{(2)}}{dt} = \dfrac{v_{C1}^{(2)} - v_{C2}^{(2)}}{R} + i_L^{(2)} \\ L \dfrac{di_L^{(2)}}{dt} = -v_{C2}^{(2)} \end{cases} \tag{4}$$

The state equations can be re-written dimensionless as follows:

$$\begin{cases} \dfrac{dx_1}{d\tau} = -\alpha c x_1 + \alpha y_1 - \alpha x_1^3 - d\alpha x_2^3 - K_c \dfrac{\alpha}{2}(x_1 - x_2)^3 \\ \dfrac{dy_1}{d\tau} = x_1 - y_1 + z_1 \\ \dfrac{dz_1}{d\tau} = -\beta y_1 \\ \dfrac{dx_2}{d\tau} = -\alpha c x_2 + \alpha y_2 - \alpha x_2^3 - d\alpha x_1^3 + K_c \dfrac{\alpha}{2}(x_1 - x_2)^3 \\ \dfrac{dy_2}{d\tau} = x_2 - y_2 + z_2 \\ \dfrac{dz_2}{d\tau} = -\beta y_2 \end{cases} \tag{5}$$

Where the following considerations have been made for system dimensionless:

$$\begin{cases} x_1 = v_{C1}^{(1)} \sqrt{R\gamma_3} \\ y_1 = v_{C2}^{(1)} \sqrt{R\gamma_3} \\ z_1 = i_L^{(1)} R \sqrt{R\gamma_3} \\ x_2 = v_{C1}^{(2)} \sqrt{R\gamma_3} \\ y_2 = v_{C2}^{(2)} \sqrt{R\gamma_3} \\ z_2 = i_L^{(2)} R \sqrt{R\gamma_3} \\ \alpha = \dfrac{C_2}{C_1} \\ \beta = \dfrac{R^2 C_2}{L} \\ c = 1 + \gamma_1 R \\ \tau = \dfrac{t}{R C_2} \end{cases} \tag{6}$$

With the system modeling and system dimensionless, we started to study the methodologies used to verify the chaos in the circuit.

4. METHODOLOGY

One of the tests used to verify chaos in the system is the 0-1 Test for Chaos. This test is used for determining whether a given deterministic dynamical system is chaotic or non-chaotic, the input is the time-series data and the output is 0 *or 1, depending on whether the dynamics is chaotic or periodic. The test is universally applicable to any deterministic dynamical system (Gottwald and Melbourne, 2004) , (Gottwald and Melbourne, 2005), (Gottwald and Melbourne, 2008),(Gottwald and Melbourne, 2009a) (Gottwald and Melbourne, 2009b).*

4.1 0-1 Test for chaos

The 0-1 Test for Chaos is directly applied to a time series data, based on the statistical properties of a single coordinate. Basically, the 0-1 Test for Chaos consists of estimating a single parameter K_c (Bassinello et al.*, 2018) and (Bernardini and Litak, 2016). The test considers a system variable $x(j)$, where two new coordinates (p,q) are defined as follows :*

$$\begin{cases} p(n,\bar{c}) = \sum_{j=0}^{n} x(j)cos(j\bar{c}) \\ q(n,\bar{c}) = \sum_{j=0}^{n} x(j)sin(j\bar{c}) \end{cases} \tag{7}$$

Where $\bar{c}\epsilon(0,\pi)$ is a constant. The mean square displacement of the new variables $p(n\bar{c})$ and $q(n\bar{c})$ is given by:

$$M(n,\bar{c}) = \lim_{n\to\infty} \frac{1}{N} \sum_{j=1}^{n} [(p(j+n,\bar{c}) - p(j,\bar{c}))^2 + (q(j+n,\bar{c}) - q(j,\bar{c}))^2] \tag{8}$$

Where n=1,2,...,N and, therefore, the parameter K_c is obtained in the limit of a very long time by:

$$K = \frac{cov(Y, M(\bar{c}))}{\sqrt{var(Y)var(M(\bar{c}))}} \tag{9}$$

Where vectors $M(\bar{c}) = [M(1,\bar{c})], M(2,\bar{c}), ..., M(n_{max}\bar{c})]$ and $Y = 1, 2, ..., n_{max}$.

$$\begin{cases} cov(x,y) = \frac{1}{n_{max}} \sum_{n=1}^{n=max} (x(n) - \bar{x})(y(n) - \bar{y}) \\ \qquad var(x) = cov(x,y) \end{cases} \tag{10}$$

Where $\bar{x}$ and $\bar{y}$ are the average of $x(n)$ and $y(n)$, respectively. As a final result, the value of the searched parameter K_c is obtained taking the median of 100 different values of the parameter $\bar{c}\epsilon(0,\pi)$ (Bernardini and Litak, 2016). If k_c value is close to 0, the system is periodic. On the other hand, if K_c value is close to 1, the system is chaotic.

4.2 Lyapunov exponent

Considering a system with n ordinary differential equations with a hypersphere of initial conditions centered in $\vec{x}(t_0)$ (Eckmann et al.*, 2018) . In this way, as time passes, this volume deforms. Assuming that, over the j-th size, the initial radius $d_j(t_0)$ has varied exponentially in time, so that the relation between $d_j(x_0)$ and the corresponding value at time t, given by $d_j(t)$, has a value:*

$$d_j(t) = d_j(t_0)e^{\lambda_j(t-t_0)} \tag{11}$$

while $j = 1, 2, \dot{;} n$.
Rewriting Eq. 11 and considering $t \to \infty$

$$\lambda = \lim_{t\to\infty} \lim_{d_j(t_0)\to 0} \frac{1}{t} \ln \left[\frac{d_j(t)}{d_j(t_0)}\right] \tag{12}$$

Therefore, the values of λ_j obtained by Eq. 11 are named as exponents of Lyapunov.

5. RESULTS AND DISCUSSION

To make Chua's circuit simulations the following constants were determined: α=10, β=16 and c=-0.143 and initial conditions for chaotic regime $(x_1, y_1, z_1) = (0.1, -0.1, 0.1)$ and $(x_2, y_2, z_2) = (0, 0, 0)$ for periodic regime. In this way we analyze the behavior of the system and we apply 0-1 Test for Chaos to analyze the coupling parameters K_c and d. So, we select two cases. The first case, the variable $d = 0.07$ and $K_c = 0.05$ made the oscillators are chaotic and for case 2 with the variable $d = 0.07$ and $K_c = 0.8$ made the oscillators periodic. The Figure 5 and the Figure 6 presents the system phase portraits for the periodic and chaotic case respectively.

With the initial conditions determined, it was possible to construct 0-1 Test for Chaos in the Figure 3 and the bifurcation graphs in the Figure 4 and to verify where the coupled system presents chaos. The graphs can be seen in the Figure 3 and were constructed by varying the coupling factor K_c in the interval delimited by the x-axis of the graphs and setting a value for the other coupling factor d. The figures are presented below.

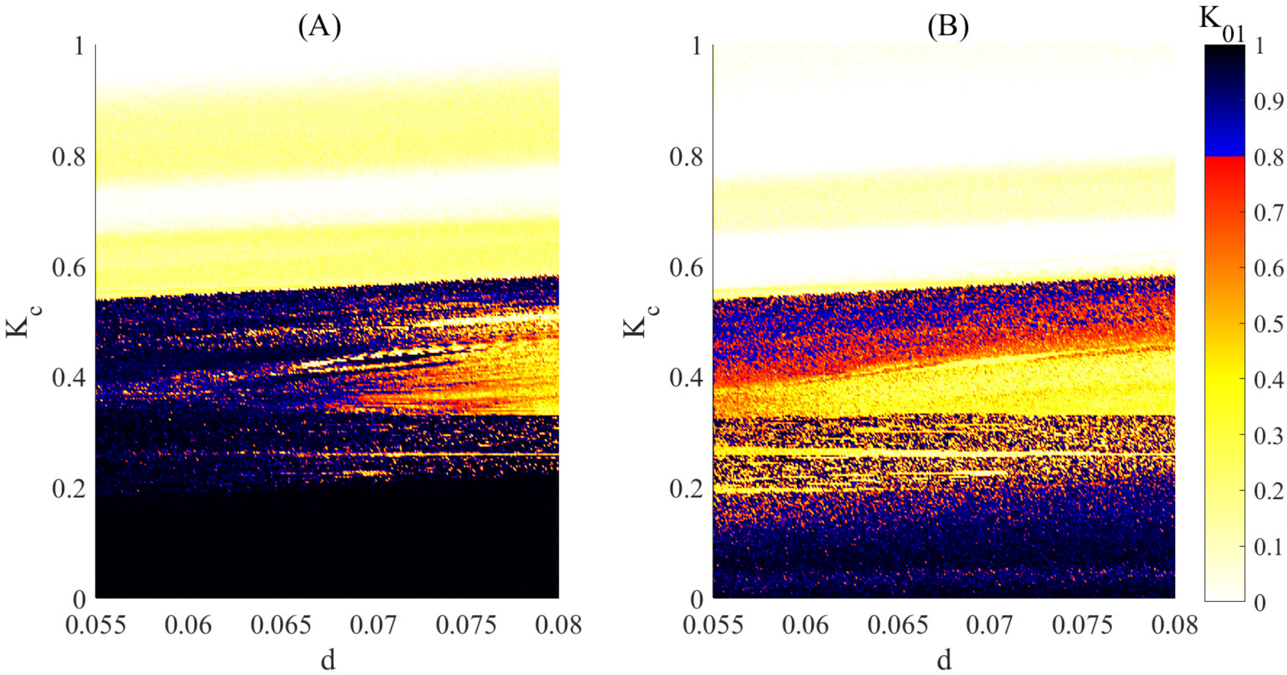

Figure 3. Chua circuit 0-1 Test for Chaos. (A) x_1 and (B) x_2

Then we calculate the bifurcation diagrams for the value of $K_c = 0.8$ and $K_c = 0.05$, since the oscillators have an intermediate value of coupling and we analyze two cases. The Figure 4 represents the bifurcation diagram for this system varying d. It is possible to notice that the two circuits of Chua are not necessarily synchronized.

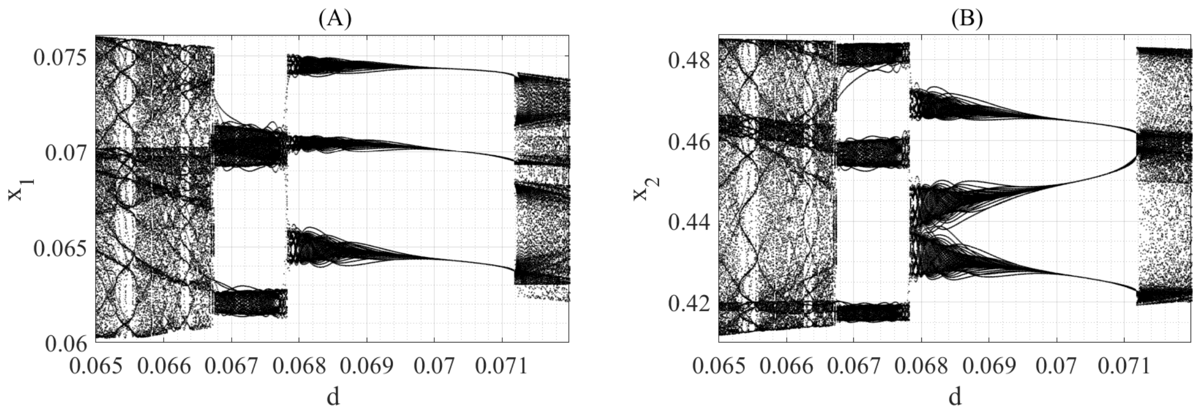

Figure 4. Bifurcation Diagram. (A) x_1 and (B) x_2

The Figures 5 represent the system for $K_c = 0.8$ and $d = 0.07$. With the following Lyapunov exponent $\lambda = -0.00124$ indicating that the system is periodic.

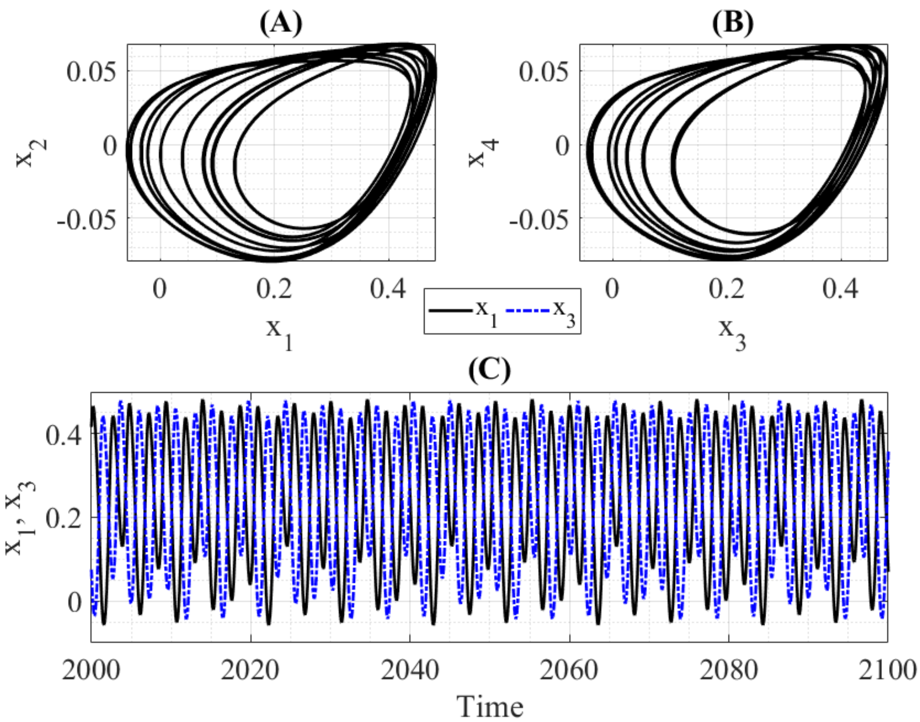

Figure 5. Chua circuit considering $K_c = 0.8$ and $d = 0.07$. (A) Phase portrait of the 1^{th} Chua circuit. (B) Phase portrait of the 2^{nd} Chua circuit and (C) Time series x_1 and x_3.

It can be seen that in Figure 5 the phase portrait shows periodic behavior, it can be seen that the K_c value chosen for the construction of this figure represents in 0-1 Test for Chaos a value with a low coupling factor which demonstrates this periodic behavior.

The Figure 6 represents the chaotic behavior of the coupled system, where $d = 0.07$ and $K_c = 0.05$, with the following Lyapunov exponent $\lambda = 0.346489$ indicating that the system is periodic.

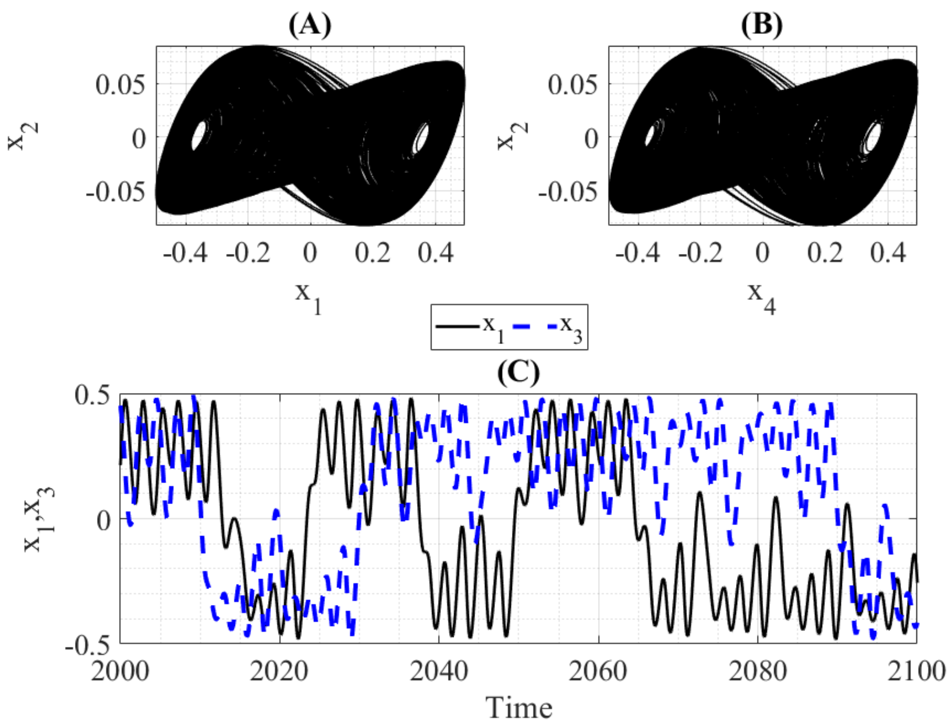

Figure 6. Chaotic behavior of Chua circuit considering $K_c = 0.05$ and $d = 0.07$. (A) Phase portrait of the 1^{th} Chua circuit. (B) Phase portrait of the 2^{nd} Chua circuit and (C) Time series x_1 and x_3.

In Figure 6 the phase portrait on the other hand shows a chaotic behavior, it can be seen still that the value of K_c chosen for the construction of this figure represents in 0-1 Test for Chaos a value with a high coupling factor which demonstrates a chaotic behavior. It can also be noted from Figure 5 and Figure 6 through the phase portrait that one circuit of Chua does not necessarily have the same behavior of the other, even though there is a strong coupling between them. This behavior can be noted by looking at the bifurcation diagram, the behavior is not the same for the circuits, even though there is a strong coupling. It is important to note that because it is a chaos circuit its behavior will be totally different depending on the initial conditions or different values of d and Kc. Being in practice difficult to control all variables to have exactly the desired behavior of the circuit

6. CONCLUSION

The Chua circuit is well studied because of its simplicity. In this work the behavior of a coupling between two of these circuits is studied using a Chua diode for this. With the determined initial conditions it is possible to construct the bifurcation diagram and to analyze the behavior of the circuit in a partial way.However, it is necessary to use another method to be sure of the behavior of the circuit, the other method used is the 0-1 Test for Chaos indicating the intensity of the coupling and developing a better characterization of the system.

Constructing graphs of the bifurcation and the 0-1 Test for Chaos, it is possible to verify the behavior of the coupled Chua circuit and the regions where it has chaos or is periodic. For that, is made the construction of the graph of the 0-1 Test for Chaos, besides being made the graph of the bifurcation to verify the regions where there is chaos. Determined the equations and the initial conditions two cases are analyzed, one in which the system is periodic and the other that presents chaos in order to analyze the different behaviors of this circuit. With this, it is possible to construct graphs that determine if the system is chaotic in a certain range of variation of the coupling factor K_c. It is verified that even with the coupling the behavior of circuits are different from each other and with maximum coupling the system becomes chaotic throughout the period of variation of K_c.

7. ACKNOWLEDGEMENTS

The authors acknowledge financial support by CNPq, CAPES, FAPESP and FA, all Brazilian research funding agencies.

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