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COMPARISON OF STRUCTURED CORNER POINT AND UNSTRUCTURED GRIDS FOR COMPOSITIONAL RESERVOIR SIMULATION

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Abstract. *Approaches based on corner point grids used in commercial simulators neglect the cross terms that arises from the transformation of the physical domains to the computational domains. This approximation may lead to errors in mass flux calculation due to the cross terms influence in non-orthogonal grids. The numerical solution may also lose its accuracy depending on the level of refinement of the grid. One approach to overcome these issues is the Element-based Finite Volume Method (EbFVM), which uses the finite volume method based on elements. This method is expected to be more accurate since it's locally conservative. The main goal of the present work is to compare corner point grids and unstructured grids based on EbFVM to solve compositional multicomponent/multiphase flow in petroleum reservoirs. In order to compare both methods, the corner point grid must be preprocessed and converted into an unstructured grid to eliminate any hanging nodes that may exist. Both approaches are implemented in an in-house compositional simulator UTCOMP that has been developed at the University of Texas at Austin. The comparison of both methods will be performed in terms of oil and gas production curves, saturation fields, and CPU time. Using equivalent grids, it was observed that the EbFVM is more accurate than formulations that use structured corner points. To increase the precision of corner point approaches a more refined grid and large computational time may be required.*

Keywords: *Petroleum Reservoir Simulation, Element-based Finite Volume Method (EbFVM), Corner Point grids*

1. INTRODUCTION

Reservoir simulation is a powerful tool to understand the reservoir behavior and to predict the oil and gas production. Cartesian grids have been widely used and studied in the oil industry because of their simplicity. However, geological effects may cause deformations and displacements of the geological layers of the reservoir. According to Fung *et al*. (1992) the use of the Finite Volume Methods with Cartesian grids has a lot of issues when trying to represent the complex geometry of non-orthogonal structures. A fully rectangular orthogonal grid is unable to represent properly reservoir properties. An alternative to the use of Cartesian grids is the corner point grids, which are the standard grids in most commercial simulators to represent complex reservoirs. Each gridblock of this type of grid is defined individually by the coordinates of its eight vertices, allowing it to model non-orthogonal reservoir with faults and variable dips (Sammon, 2000). Unlike the reservoir production, the numerical solution is dependent of the grid and numerical approach used to discretize the material balance equations. Commercial simulators, which use corner point grids, usually neglect the cross derivatives that arises from the transformation of equations from the physical domains to the computational domains (Marcondes *et al*., 2008). This is done mainly to maintain the model simple and its solution similar to the Cartesian grids; this approach results in a seven-point scheme discretization for 3D grids. However, this approximation may lead to substantial errors in mass flux calculation due to the cross terms influence in highly non-orthogonal grids (Fernandes *et al*., 2014).

Due to the aforementioned reasons, there is a high motivation to develop methods using unstructured grids. This type of grid may represent any type of geometry and it is expected to achieve more accurate results when compared to other approaches. An important approach to discretize the reservoir is unstructured grids in conjunction with the Finite

Volume Method (FVM). This mathematical approach was first adopted by Baliga and Patankar (1980) using triangular grids; they named this technique as Control Volume Finite Element Method (CVFEM). However, according to Maliska (2004), this nomenclature gives the wrong impression of using a Finite Element Method (FEM) based on control volumes. Actually, the CVFEM is a FVM, which uses the concept of elements. For that reason, Maliska (2004) proposed the nomenclature Element-based Finite Volume Method (EbFVM), which is going to be used in the present work. The EbFVM is expected to be more flexible to handle complex geometries and also more accurate. Marcondes and Sepehrnoori (2010) compared the EbFVM with Cartesian grids for compositional reservoir simulation. Using equivalent grids, it was shown that Cartesian grids require more grid blocks when compared to the EbFVM in order to achieve the same accuracy. However, the EbFVM is computationally more expensive than corner point grids due the large stencil of the Jacobian matrix.

In the present work, both grids will be implemented in UTCOMP, an in-house compositional simulator developed at The University of Texas at Austin. The corner point grids are going to be generated using a commercial software. In order to use the corner point grid to create the grid for the EbFVM, the corner point must be preprocessed. This is necessary because corner point grids may have faulted blocks. To apply the EbFVM, the hanging nodes should be eliminated with a grid converter. The comparison of the results of the two types of grid will be presented in terms of oil and gas production, saturation fields and CPU time.

2. MATHEMATICAL MODEL

The assumptions made and physical model used in the compositional reservoir simulator are fully described by Chang (1990). The flow is represented by up to four phases: an aqueous phase and three hydrocarbon phases, which consists of oil, gas, and a nonaqueous liquid. The additional liquid phase is used due the three-phase behavior observed in mixtures of carbon dioxide and hydrocarbons.

In reservoir simulation, the flow consists of several hydrocarbon components present in these four phases. The material balance for each component, neglecting the physical dispersion term, is given by

$$\frac{\partial}{\partial t}\left(\phi\sum_{j=1}^{n_p}\xi_j S_j x_{ij}\right)+\vec{\nabla}\cdot\left(\sum_{j=1}^{n_p}\xi_j x_{ij}\vec{u}_j\right)-\frac{q_i}{V_b}=0 \tag{1}$$

where ϕ is the medium porosity, ξ is the molar density of phase j, S is the fraction of pore space occupied by phase j, x_{ij} is the mole fraction of component i in phase j, and $\vec{u}_j$ represents the velocity of phase j. The parameter q_i / V_b is the source/sink term where q_i is the molar flow rate of component i and V_b is the bulk volume of a grid block that holds a well. Considering that the pore volume is filled completely by the total fluid volume, we obtain an addition equation for pressure that is given by

$$\left[\phi^0 C_f-\frac{1}{V_b}\left(\frac{\partial V_t}{\partial P}\right)_{N_i}\right]\frac{\partial P}{\partial t}+\bar{V}_{tw}\vec{\nabla}_{tw}\cdot\left(\xi_w\vec{u}_w\right)+\sum_{i=1}^{nc}\bar{V}_{ti}\sum_{j=2}^{np}\vec{\nabla}\cdot\left(\xi_j x_{ij}\vec{u}_j\right)+\sum_{i=1}^{nc+1}\bar{V}_{ti}\frac{q_i}{V_b}=0 \tag{2}$$

where V_t is the total fluid volume of components, C_f is the rock compressibility, and P is the pressure. The physical properties of hydrocarbon phases are evaluated the Peng and Robinson (1976) equation of state.

2.1 Corner Point Grids

One of the parameters used in the material balance is the transmissibility, which is defined as the inverse of the flux resistivity. The transmissibilities for corner point grids are calculated using two approaches presented by Fernandes *et al*. (2014). The first one uses the standard method used in commercial simulators, which neglects cross derivatives. The second approach takes into account the cross terms for flux calculation. They are respectively referred in the present work as woct (without cross terms) and wct (with cross terms). Both methods use a transformation of coordinates from a physical to a parametric computational plane. Since corner point elements are defined by its eight vertices, this type of grid may have faulted and non-orthogonal blocks with hanging nodes. To deal with this displacement, transmissibility in the interface of neighbor blocks cannot be addressed using the aforementioned method. Peaceman (1996) proposed a harmonic average to calculate transmissibility between faulted blocks in any direction, which is given by

$$\frac{1}{T_{ab}}=\frac{A_c}{\dfrac{A_a}{T_a}+\dfrac{A_b}{T_b}} \tag{3}$$

where T_{ab} is the transmissibility in the interface, A_c is the contact area in the interface, A is the total area of the block in the specified direction, and T is the half-block transmissibility, which is defined by the transmissibility from the centroid to the contact surface.

The half block transmissibility is calculated using a method of tubes considering a transformation from a physical to a computational plane using Eq. (4). The integration is done using a three-point Gaussian quadrature.

$$T_a = \int_{\varepsilon=\varepsilon_c}^{\varepsilon=1} \frac{1}{\displaystyle\int_{\eta=-1}^{\eta=1}\int_{\gamma=-1}^{\gamma=1} \frac{K_a D(\varepsilon,\eta,\gamma) cos\psi(\varepsilon,\eta,\gamma)}{\frac{ds(\varepsilon,\eta,\gamma)}{d\varepsilon}} d\eta d\gamma} d\varepsilon \qquad (4)$$

The transmissibility is integrated in a parameterized cube that ranges from -1 to 1 on the coordinates ε, η, and γ of the computational plane. D is the term of differential flow area, $cos\psi$ takes into account the deformation of the block, $ds / d\varepsilon$ and K_a are respectively the differential length and permeability in the given direction.

The flux between blocks is calculated with the approach used by commercial simulators, which modifies the seven-point discretization. Neighbor blocks are defined for each cell and the flux is calculated in the principal directions increasing the stencil of the Jacobian matrix.

2.2 Element-based Finite Volume Method

The EbFVM defines the elements of the grids by its nodes and connectivities. Although the EbFVM 3D grids can be composed of hexahedrons, tetrahedrons, prisms, and pyramids, only hexahedrons are going to be used in order to match the hexahedrons of the Corner Point grids. The equations are discretized in a control volume around the nodes of the elements. Each sub-control volume is formed by joining the center of the element to their medians. The EbFVM uses shape functions for performing the geometrical and physical interpolations.

2.3 Grid Converter

To compare both methods, the grids should be equivalent. A grid converter was developed to convert the corner point grid generated by a commercial simulator into an unstructured grid. In order to do that, the corner points of each element are converted into nodes and the connectivities between them are defined to generate the unstructured grid. Any hanging nodes generated in the commercial simulator must be eliminated. New nodes should be defined to generate an unstructured grid without collapsed blocks.

3. RESULTS

Three case studies are presented here. The first case consists of an orthogonal Cartesian grid converted into both corner point and unstructured grids. This is done in order to validate the UTCOMP implementation for corner point grids. The other two cases are used to compare the numerical solution obtained by the EbFVM and the structured corner point grids. The second case study aims to evaluate the effects of hanging nodes over the numerical solution. The last case consists in a distorted corner point grid. As a benchmark, the simulations performed in the UTCOMP are compared with the results obtained from a compositional commercial simulator. Herein, the three case studies presented consists in a quarter of five spot with the same reservoir fluid injection. Tab. 1 presents the reservoir data used for all case studies.

Table 1. Reservoir data.

Property	Value
Porosity	0.35
Initial Water Saturation	0.3
Initial Pressure	1500 Psi
Permeability (X,Y,Z)	10 mD, 10 mD, 10 mD
Formation Temperature	160 F
Injection Rate	1000 MSCF/Day
Producer Bottom Hole Pressure	1300 Psi

The reservoir fluid is composed of six components and only oil and water phase are initially presented in the reservoir. Injection wells are operating at constant surface rate and production wells are operating at constant bottom hole pressure. Initial overall compositions and injection fluid compositions are presented in Tab. 2.

Table 2. Fluid composition data.

Component	Initial Reservoir Composition	Injection Fluid Composition
C_1	0.5	0.77
C_3	0.03	0.2
C_6	0.07	0.01
C_{10}	0.2	0.01
n C_{15}	0.15	0.005
C_{20}	0.05	0.005

The reservoir used in case study 1 is a regular quarter of five-spot. This case study is designed to validate the numerical solutions using unstructured and corner point grids with the Cartesian grid. Figure 1 shows the 56x56x10 cartesian grid along with the reservoir dimensions. The equivalent corner point grid was generated with a commercial simulator and converted into an unstructured grid with 31,360 nodes.

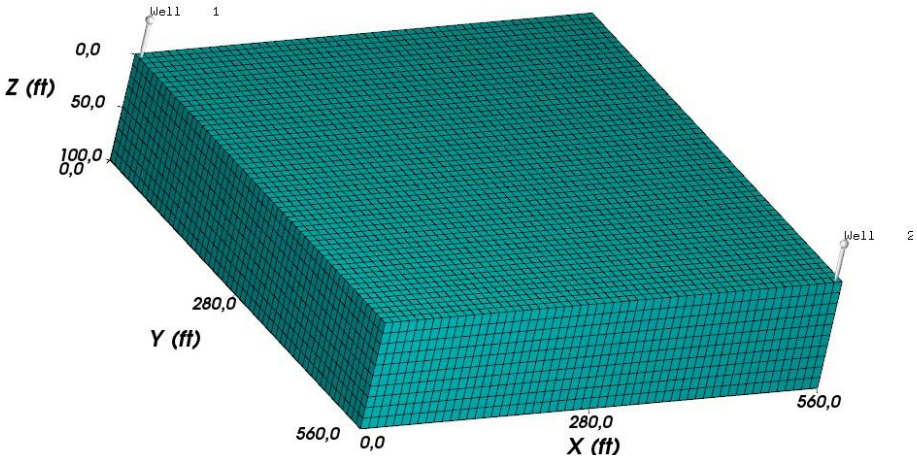

Figure 1. Cartesian grid used in Case study 1.

Figure 2 shows the oil and gas production curves obtained with the grids investigated in this work and also with a largely used by oil industry commercial simulator. Despite neglecting the cross derivatives, the results for Cartesian, corner point, and commercial simulator presented a good match. The grid is orthogonal and the cross derivatives are equal to zero, therefore neglecting them do not affect the numerical solution.

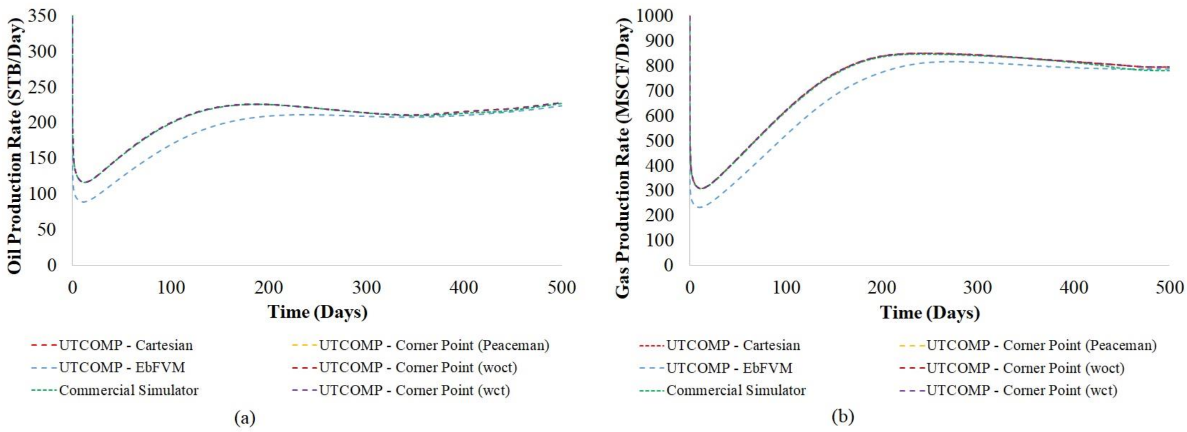

Figure 2. Production curves for Case Study 1.
(a) Oil production curve (b) Gas production curve

However, the curves for the structured grids did not match the production curves obtained with EbFVM approach. To investigate this issue, a simulation was performed increasing the number of grid blocks. Figure 3 shows the production profiles with the previously coarse grid and with a refined grid with 100,000 grid blocks. From this figure, we can clearly verify that when the grid is refined the solution obtained with structure grid converge to the one obtained with the EbFVM.

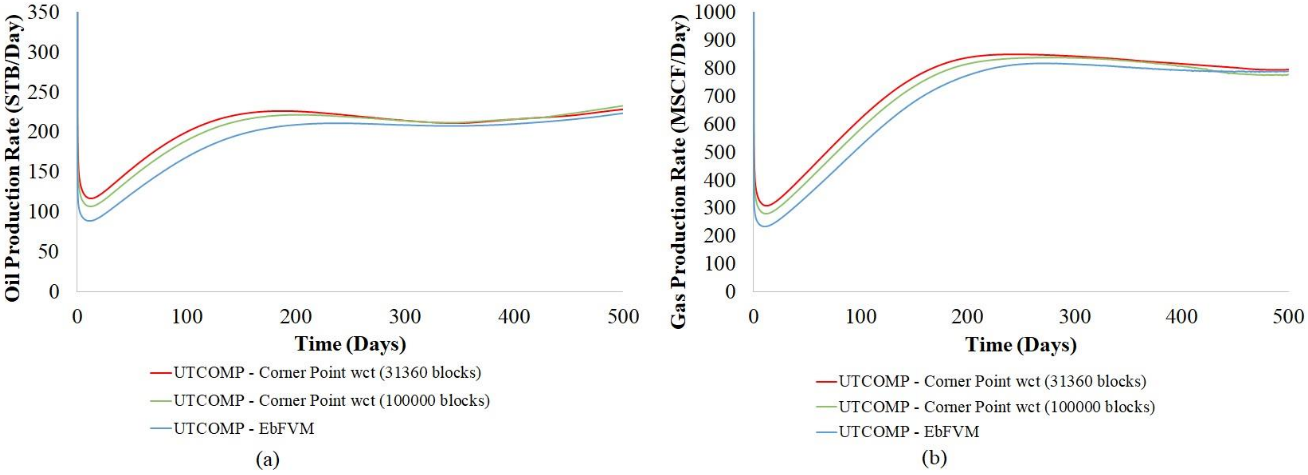

Figure 3. Comparison of production curves for Case Study 1 and refined structured grid.
(a) Oil production curve (b) Gas production curve

Figure 4 presents the oil saturation fields on a cut plane of the reservoir. Despite the production curves do not match, the saturation fields presented the same behavior for unstructured and structured grids.

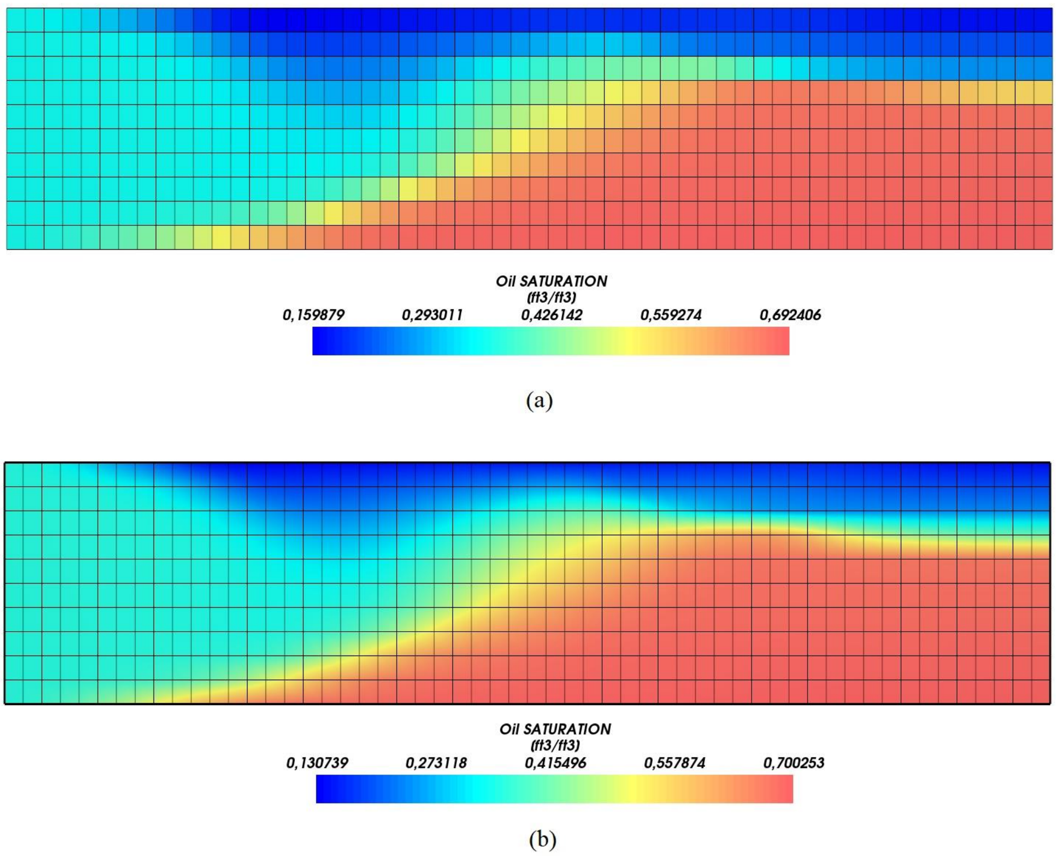

Figure 4.Oil saturation fields for Case Study 1 at 500 days.
(a) Structured grid (b) Unstructured grid with EbFVM

Case study 2 aims to evaluate the numerical solution in structured grids with hanging nodes. To make the comparison, the structured grid is converted into an unstructured grid eliminating the collapsed blocks. Figure 5 presents a plane cut in x-z plane of the original 30x30x10 corner point grid and the preprocessed unstructured grid. In order to eliminate the hanging nodes, new elements must be defined by new nodes and new connectivities. Despite both reservoirs having the same dimensions, the unstructured grid is more refined with 16,200 grid blocks and 18,352 vertices.

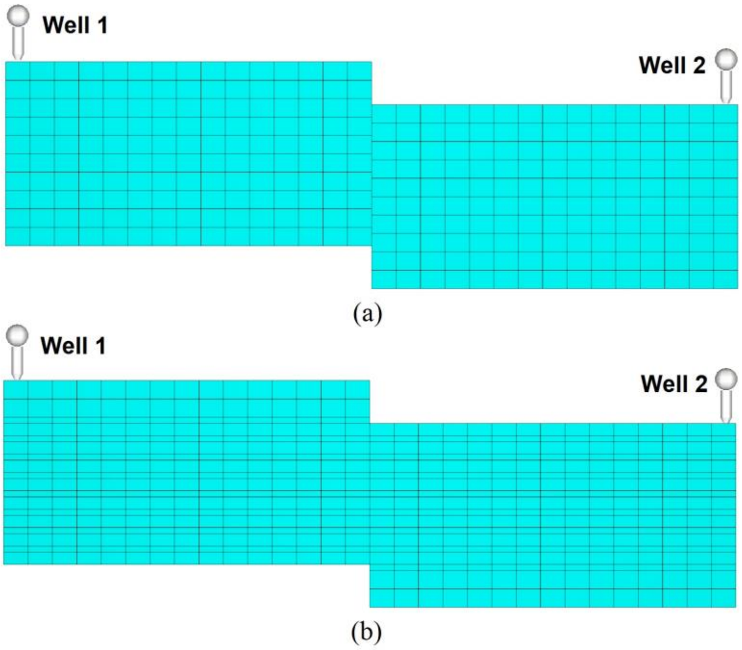

Figure 5. Plane cut in x-z plane of the faulted grid used in Case Study 2.
(a) Original corner point grid generated with commercial simulator (b) Preprocessed unstructured.

The oil and gas production curves are presented in Fig. 6. The formulations that neglects the cross derivatives are in good agreement when compared to the results obtained with the EbFVM. Since there are small differences between the production curves, it can be assumed that cross derivatives can be neglected in faulted grids with fully orthogonal blocks.

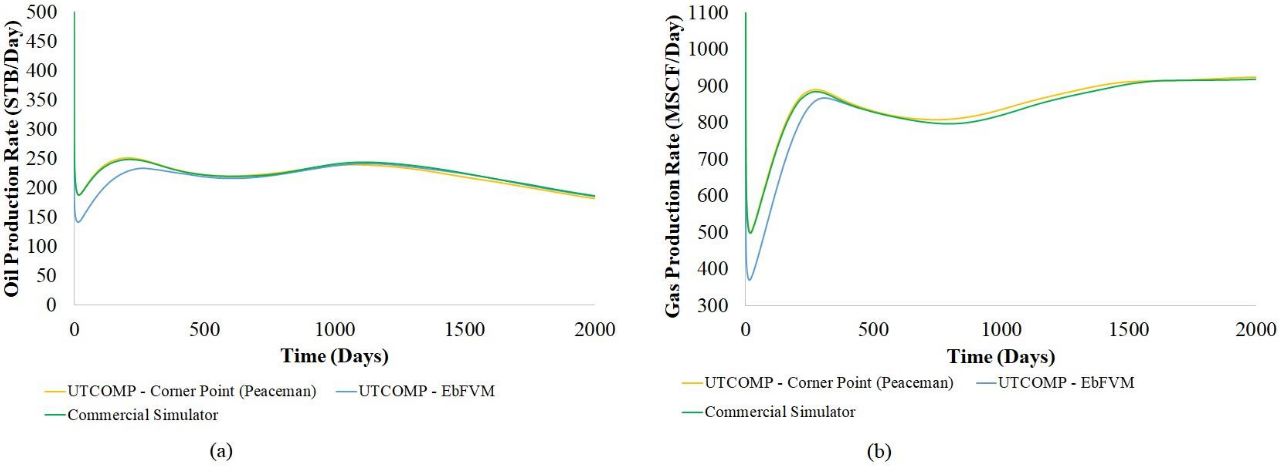

Figure 6. Oil production curve for Case Study 2.
(a) Oil production curve (b) Gas production curve

Figure 7 presents the saturation fields for this case. Although both fields presented similar behaviors, the EbFVM shows a smoother transition in the gas-oil contact zone in regions close to the hanging nodes and production well.

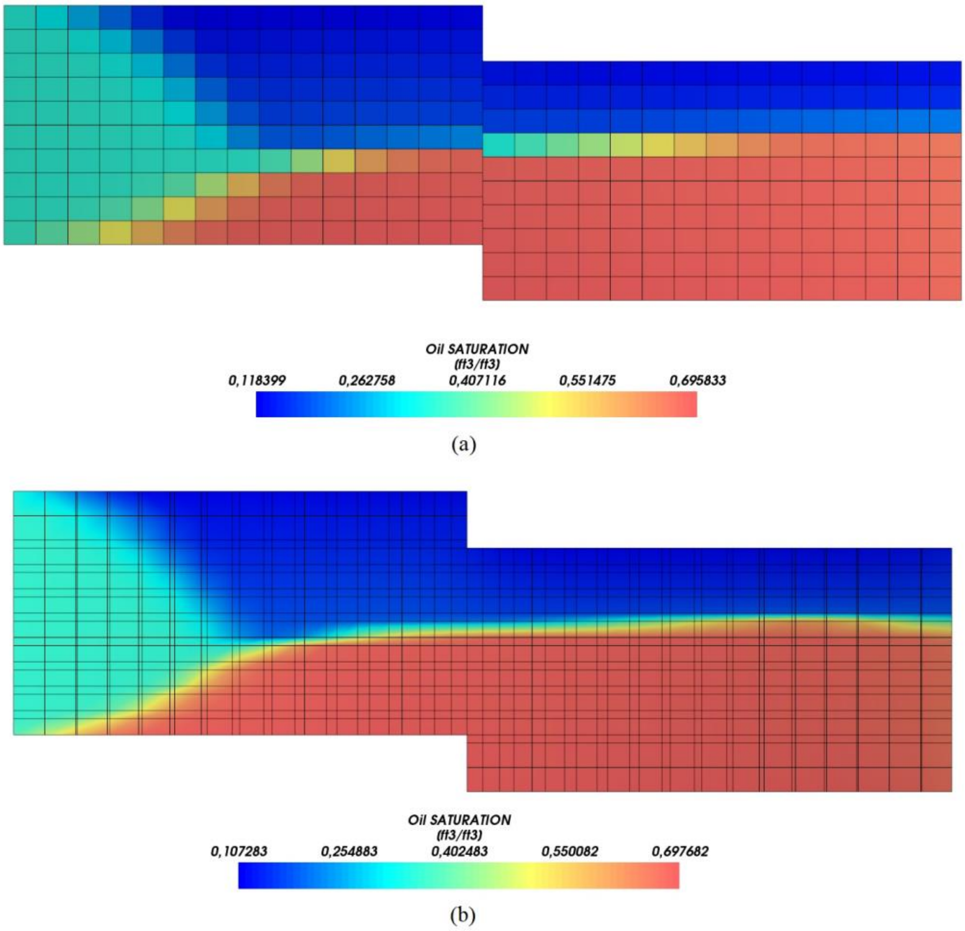

Figure 7.Oil saturation fields for Case Study 2 at 290 days.
(a) Structured grid (b) Unstructured grid with EbFVM

Case study 3 increases the distortion level of the grid to evaluate its effect over the numerical solution. Figure 8 presents the 40x40x10 corner point grid. The preprocessed unstructured grid has 1,8491 nodes.

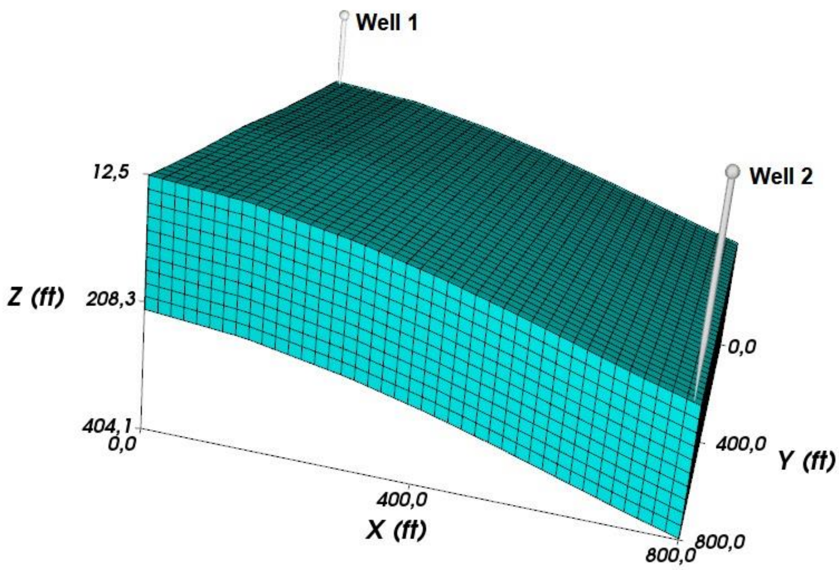

Figure 8. Corner point grid used in Case Study 3.

Production rates for this case study are presented in Fig. 9. Observing only the results for corner point grids, it can be seen that the curves achieve a good match. The way the transmissibility is calculated slightly alters the numerical solution. The wct curve approximated to the EbFVM solution, indicating that distorting the grid causes the cross derivatives to affect the numerical solution. However, the structured grids are not enough refined to match the production curves obtained with the EbFVM.

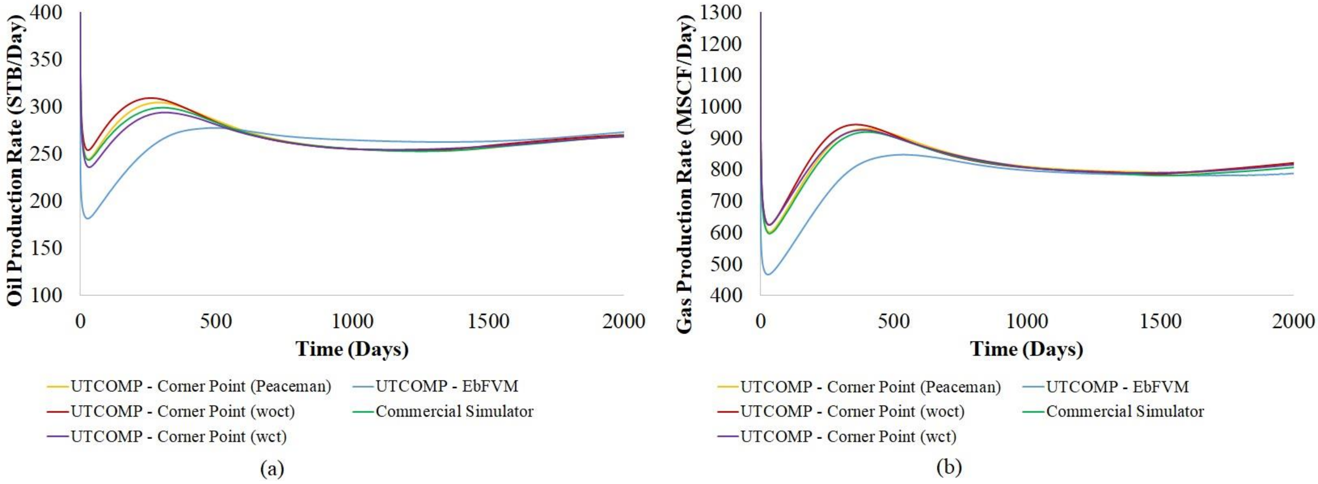

Figure 9. Oil production curve for Case Study 3.
(a) Oil production curve (b) Gas production curve

Saturation fields are presented in Fig. 10. As in previous figures, saturation fields for structured corner point grids and the EbFVM presented similar behavior.

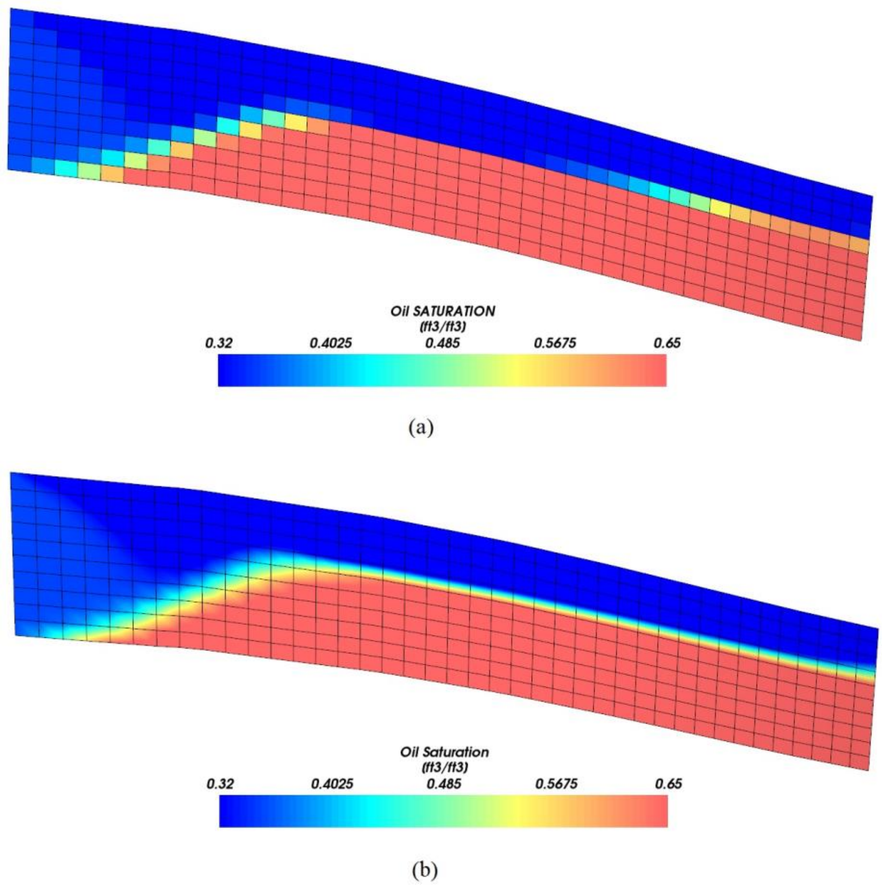

Figure 10.Oil saturation fields for Case Study 3 at 800 days.
(a) Structured grid (b) Unstructured grid with EbFVM

The CPU time for the three cases investigated in this work are presented in Tab. 3. When comparing the formulations used in this work, it is already expected that the EbFVM to be more computationally expensive. However, real reservoirs are usually distorted and faulted. In these situations, it may be required a more refined grid with highly distorted blocks. If this is not considered the accuracy of the solution may decrease when dealing with corner point grids. The EbFVM presents itself an important alternative to the standard techniques used in the oil industry. Despite the large CPU time required, the EbFVM has a more accurate solution over the traditional numerical approaches. It is important to notice that if higher accuracy from the numerical solution is desired, more grid blocks are necessary when using structured grids. If the grid is refined, the computational time would also increase.

Table 3. CPU time (s).

Method	Case 1	Case 2	Case 3
Corner Point (Peaceman)	5,161.8	6,102.0	8,821.1
Corner Point (woct)	3,829.5	-	6,537.0
Corner Point (wct)	3,876.2	-	6,533.6
EbFVM	10,997.0	15,196.3	15,116.1

The CPU time for Case Study 1 including the refined grid is presented in Tab. 4. As it can be observed, the CPU time for the refined grid significantly increased when compared to the original grid and it was larger than the one used by the EbFVM. It is important to stress that structured grid needs to be more refine in order for the production curves match with the ones obtained with EbFVM, which will result in even a larger CPU time for the structured grid.

Table 4. CPU time (s) for Case Study 1.

Method	Time
Corner Point wct (31,360 blocks)	3,876.2
Corner Point wct (100,000 blocks)	15,233.6
EbFVM	10,997.0

4. CONCLUSIONS

This work presented a comparison between unstructured grids using EbFVM and structured grids with the corner point approach. In order to make this comparison, a grid converter was successfully developed to convert the corner point grid generated with a commercial simulator into an unstructured grid. Corner point grids are the standard grids used in the oil industry to represent the real shape of a reservoir. This approach neglects the cross derivatives that arise from the transformation of coordinates from a physical to computational plane. It also observed that corner point grids formulations may require a highly refined grid to maintain accuracy. Despite its simplicity, the corner point approach may lose its accuracy when the grids are distorted. The EbFVM is an alternative to the corner point approach. The way the equations of the EbFVM are discretized takes in account the flux in all directions for each control volume. This indicates a more accuracy of the numerical solution. Although the EbFVM may achieve a higher precision, the method also requires a larger CPU time.

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