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DEVELOPMENT OF A COUPLED GEOMECHANICAL MODEL FOR A COMPOSITIONAL RESERVOIR SIMULATOR USING THE FINITE VOLUME METHOD

João Henrique Bessa Gomes
Pontifícia Universidade Católica do Rio Grande do Sul, CEP 90619-900, Porto Alegre, RS
joao.hgomes@hotmail.com

Francisco Marcondes
Universidade Federal do Ceará, Campus do Pici, CEP 60455-900, Fortaleza, CE
marcondes@ufc.br

Kamy Sepehrnoori
The University of Texas at Austin
kamys@mail.utexas.edu

Abstract. *Traditional petroleum reservoir simulators usually apply a single parameter for the rock compressibility to account for the effect of addition and removal of fluid in the porous media mechanical structure. The geomechanical effect however has been shown to be a more complex phenomenon that is intrinsically connected to the evolution of reservoir pore pressure distribution and therefore must be taken into careful consideration, especially for stress-sensitive formations. The main objective of this work is to present an iteratively coupled solution between a compositional reservoir simulator and a linear elastic geomechanical model and its application to oil recovery processes. The simulator UTCOMP was used in the implementation of the geomechanical code and its coupling to the reservoir model. The discretization approach for both models is the Element-based Finite Volume Method (EbFVM), allowing the use of unstructured grids for the representation of the physical domain. The implementation is validated through specific problems with analytical solutions available in the literature. The results obtained show good agreement between the analytical solution and the simulation from the coupled code, indicating that the implementation of the geomechanical model and its coupling to UTCOMP was successfully implemented. A case study shows the influence of the geomechanical effect on the pressure distribution and on the oil recovery factor.*

Keywords: *Petroleum Reservoir Simulation, Geomechanical Analysis, Iterative Coupling, Element-based Finite Volume Method (EbFVM), Unstructured Grids.*

1. INTRODUCTION

The evolution of the processing capabilities of computers in the last 40 years enabled the development of more robust simulators capable of handling more rigorous and sophisticated petroleum reservoir modeling. Given the inherent complexity of the fluid flow through porous media phenomenon, the traditional reservoir models were developed based on a set of limiting assumptions that, while simplified the mathematical treatment, also made those models less accurate and restricted in their application. One of these limitations regards the treatment of the geomechanical effects experienced by the reservoir during the injection and production of fluids. Traditional approaches use a single parameter, the rock compressibility, to account for the variation of pore volume with reservoir pore pressure. Usually, this parameter is kept constant throughout the simulation, yielding a linear relation that not always accurately represent the real behavior of the reservoir.

The geomechanical effect in porous media was first studied by Terzaghi (1923), although the study was focused in water-saturated soils. According to this soil consolidation theory, an initial state of mechanical equilibrium exists between the fluid pressure inside the pores and the force exerted by weight of the rock column. The fluid removal and, therefore, the pressure drop will cause the rock column weight to compact the medium. Terzaghi (1943) expanded this theory by defining the effective stress as the difference between the total stress applied to the medium and the pore pressure, effectively linking the fluid flow and geomechanical effects to one another. Effective stress would then represent the fraction of total stress responsible for effects like compaction of the porous media. Therefore, variations in effective stress are responsible for changes in rock properties, such as porosity and permeability, which in turn will affect the fluid flow in the reservoir (Pan, 2009).

The main objective of this work is to implement a two-dimensional geomechanical model and couple it with the UTCOMP simulator; a compositional reservoir simulator developed at The University of Texas at Austin (Chang, 1990). The discretization technique used for both the geomechanical and the reservoir model is the Element-based Finite Volume Method (EbFVM) (Baliga and Patankar, 1980), which allows the use of unstructured grids to represent the domain more accurately. The geomechanical model implemented is a linear-elastic plane strain formulation that is coupled to the reservoir model through an iterative coupling method, as proposed by Chin et al. (2002). Results for the coupled simulation are validated through comparison with analytical solutions. Study cases for different oil recovery processes are then tested with and without the geomechanical coupling to quantify the impact of the geomechanical effect on hydrocarbon recovery.

2. MATHEMATICAL MODEL

2.1 Fluid Flow Model

The equations that describe the multiphase/multicomponent flow through porous media used in UTCOMP were first derived by Chang (1990) and are based on a set of restricting assumptions that can be found in the aforementioned work. To obtain the equations, a material balance between pore volume and total reservoir fluid volume is done for each component in each phase. Neglecting the physical dispersion terms, the following equation is obtained:

$$\frac{1}{V_b}\frac{\partial N_i}{\partial t}+\vec{\nabla}\cdot\left(\sum_{j=1}^{n_p}\xi_j x_{ij}\vec{u}_j\right)+\frac{q_i}{V_b}=0,\ \ i=1,\ldots,n_c+1, \tag{1}$$

in which N_i is the number of moles of component i; ξ_j is the molar density of phase j; x_{ij} is the mole fraction of component i in phase j; $\vec{u}_j$ is the Darcy velocity of component i in phase j.; q_i is the well molar flow rate of component i; V_b is the bulk volume of the control volume, n_c is the total number of components except water and n_p is the number of phases.

Assuming that the fluid volume occupies the entirety of the pore volume, a volume balance is done in the control volume and a pressure equation is obtained:

$$\left[\frac{\partial\phi^*}{\partial P}-\frac{1}{V_b}\left(\frac{\partial V_T}{\partial P}\right)_{N_k}\right]\frac{\partial P}{\partial t}-\sum_{i=1}^{n_c+1}\overline{V}_{Ti}\sum_{j=1}^{n_p}\vec{\nabla}\cdot\left[\xi_j x_{ij}\frac{\overline{\overline{K}}k_{rj}}{\mu_j}\left(\nabla P_j-\gamma_j\nabla D\right)\right]-\sum_{i=1}^{n_c+1}\overline{V}_{Ti}\frac{q_i}{V_b}=0,, \tag{2}$$

where ϕ^* is the true porosity; V_T is the total fluid volume; $\overline{\overline{K}}$ is the absolute permeability tensor; k_{rj} is the relative permeability of phase j; μ_j is the viscosity of phase j; P_j is pressure of phase j; γ_j is the specific gravity of phase j and D is the depth of the reservoir.

Upon the discretization of Eq. (2), a matrix system is formed with pressure as primary variable. Once the system is solved implicitly, *i.e.,* simultaneously for all grid blocks, the molar composition of each component is updated by solving a mass balance equation (discrete version of Eq. (1)) explicitly, *i.e.,* for each grid block individually, characterizing an IMPEC (Implicit Pressure, Explicit Compositions) formulation.

2.2 Geomechanical Model

The equations used to describe the geomechanical effect in the reservoir are based in the effective stress law, as proposed by Terzaghi (1943):

$$\overline{\overline{\sigma}}=\overline{\overline{\sigma}}'-\alpha P\overline{\overline{\delta}}, \tag{3}$$

where $\overline{\overline{\sigma}}$ is the total stress tensor; $\overline{\overline{\sigma}}'$ is the effective stress tensor; α is the Biot-Willis coefficient and $\overline{\overline{\delta}}$ is the Kronecker delta.

Eq. (3) represents the basic relationship between stress and pore pressure that serves as a starting point to the model. In order to obtain the final equation for the model, the linear elastic constitutive equation, given by Eq. (4), the definition of strain, shown in Eq. (5), and Eq. (3) are applied into a force balance, yielding a final equation to solve for displacement.

$$\vec{\vec{\sigma}}' = 2\mu_l \vec{\vec{\varepsilon}} + \lambda_l tr(\vec{\vec{\varepsilon}})\vec{\vec{\delta}} \,, \tag{4}$$

$$\vec{\vec{\varepsilon}} = \frac{\left[\nabla\vec{u} + \left(\nabla\vec{u}\right)^T\right]}{2} \,. \tag{5}$$

In the equations above, $\vec{\vec{\varepsilon}}$ is the strain tensor, λ_l and μ_l are the Lamé's constants, $tr(\cdot)$ is the trace operator and $\vec{u}$ is the displacement vector.

As mentioned previously, compaction effects can cause changes in porosity and permeability of the reservoir. These changes are accounted in the model by the following equations (Pan, 2009):

$$\phi^* = 1 - \left(1 - \phi_0\right) e^{-\varepsilon_v} \,, \tag{6}$$

$$\frac{K_{ij}}{K_{ij,0}} = \left(\frac{\phi^*}{\phi_0^*}\right)^n \,, \tag{7}$$

where ε_v is the volumetric strain and is equal to the trace of the strain tensor, ϕ_0 is the initial reservoir porosity, $K_{ij,0}$ represent the initial value for the components of the permeability tensor and n is an empirical constant dependent on the problem; it typically ranges from 6 to 15 (Haddad et al., 2015).

True porosity can be related to traditional reservoir porosity, ϕ_0, used in standard models by the following relationship:

$$\phi = \left(1 - \varepsilon_v\right) \phi^* \,. \tag{8}$$

This relationship allows the use of the material balance presented in Eq. (1), which is similar to the traditional material balances normally used in compositional petroleum reservoir simulators.

2.3 Discretization

The Element-based Finite Volume Method (EbFVM) was used to discretize the physical domain for both models. According to Maliska (2004), the main advantage of the EbFVM approach is the combination of concepts of elements and shape functions from the Finite Element Method and the conservative aspect of the standard Finite Volume Method, as the balances are performed in control volumes, conserving the properties inside these volumes. In two dimensional grids, the domain is elements that can be triangles or quadrilaterals. Each element is then subdivided into sub-control volumes (SCVs). The control volumes in which the equations are going to be solved are formed by the combination of all SCVs that share a same vertex. Therefore, the main variables for the coupled model, pressure and displacement, are calculated in the vertices of the grid, characterizing a cell-vertex formulation. For example, in Fig. 1, to assemble the control volume for vertex 3 (colored in black), the equations must be integrated in the SCVs colored in blue, namely SCV 3 of element 1, SCV2 of element 4, SCV 1 of element 3, SCV 1 of element 5 and SCV 2 of element 2.

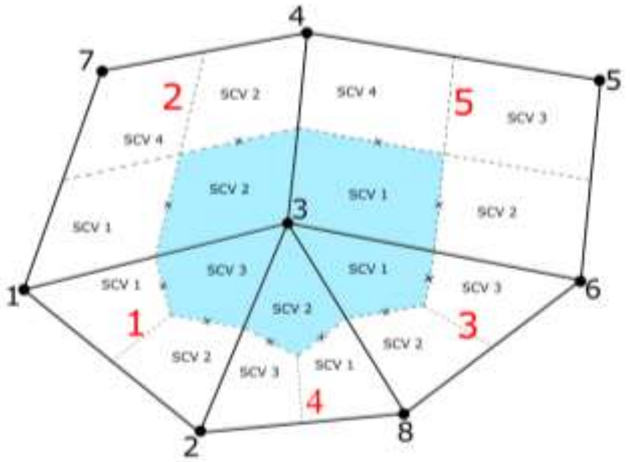

Figure 1: Sample unstructured grid using the EbFVM discretization method.

2.4 Iterative Coupling

As seen previously, pore pressure is the result of the reservoir model and is a parameter for the geomechanical model. In this model, porosity and permeability are calculated and are also parameters for the reservoir model. Therefore, there must be a sequencing for solving the equations in order to obtain solutions as accurate and realistic as possible. In this work, an iterative coupling technique presented by Chin et al. (2002) is used.

This approach initially solves the geomechanical model based on the initial reservoir pressure distribution to determine an initial stress state in equilibrium. Porosity and permeability are provided to the reservoir model as the algorithm enters its recurrent section. For each time step, the reservoir model is solved for pressure, that is passed to the geomechanical model as a parameter. This model is then solved, calculating new values for porosity and permeability that are sent back to the reservoir model for pressure recalculation. This iterative process is repeated until the pressure reaches convergence and the code advances to the next time level.

3. RESULTS

3.1. Validation

In order to validate the implementation, a case with existing analytical solution was simulated. The problem selected models a one-dimensional consolidation case with a constant load applied that it best known as Terzaghi's problem. The case consists of a rectangular column of porous rock saturated with water. The boundaries of the column are rigid and impermeable, except for the top boundary, which is open to the atmosphere. The column is initially at atmospheric pressure until a load is applied and the pressure increases to reach mechanical equilibrium, as predicted by the effective stress law. The top boundary of the column remains at atmospheric pressure. Table 1 presents the input data used for the validation case.

Table 1. Input data for Terzaghi's problem

Column height (m)	6.0
Column length (m)	1.0
Applied stress (GPa)	1.0
Fluid compressibility (Pa^{-1})	3.030×10^{-10}
Fluid viscosity (Pa·s)	0.001
Solid compressibility (Pa^{-1})	2.778×10^{-11}
Biot-Willis coefficient	0.778
Poisson coefficient	0.2
Young's modulus (GPa)	14.4
Initial porosity	0.19
Initial absolute permeability (m^2)	1.90×10^{-15}
Water saturation	1.0

Figure 1 shows the variation of the pressure on the base of the column with time, as well as the comparison of the numerical results obtained for the present implementation with the analytical solution provided by Schiffmann (1960). The results show a good agreement between the analytical and numerical results for pressure. Given the coupled nature of the phenomenon studied, the good match indicates not only that the implementation was satisfactory, but also that the coupling procedure adopted is effective in its purpose.

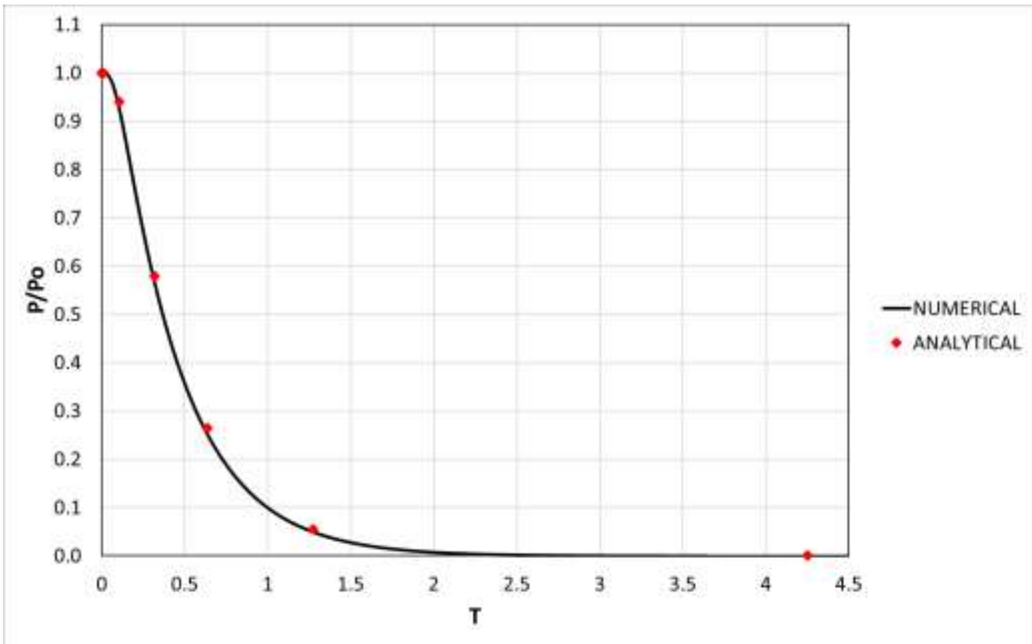

Figure 1: Numerical and analytical results for dimensionless pressure at the base of the column vs. dimensionless time.

2.5.2. Case Study: Gas Injection with 3-pseudocomponent fluid mixture

This case presents a secondary recovery process, in which a gas mixture, predominantly CO_2, is injected into a reservoir to provide the energy necessary to drive the reservoir fluids towards the production well. Wells are placed on the borders of the domain. Given the considerable depth of the reservoir, it is a more realistic approach to place the wells in smaller segments, which emulates the real-life well completion strategies for injection and production. A completion interval of 58.8 ft was chosen for both wells. The injection well was placed near the bottom of the border and the production well placed near the top. This arrangement takes advantage of the lower density of the injected gas to to provide a better sweeping of the reservoir fluids. Reservoir hydrocarbon fluids are lumped in 3 pseudocomponents and up to the phases (oil, gas and water) can be found simultaneously in the pore space. This case should be able to showcase the capability of the coupled simulator to handle multiphase fluid using the compositional model. A single unstructured grid was used for the case. It is composed of 612 vertices and 561 quadrilateral elements. Figure 2 shows a schematic of the case. Tables 2 and 3 presents, respectively, the input properties and the fluid components properties.

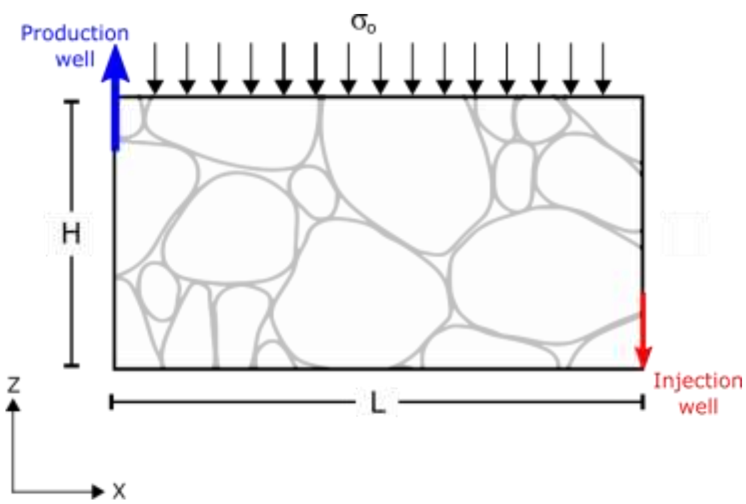

Figure 2: Schematics for the case study reservoir.

Table 2. Input data for the case study

Simulation time (days)	150.0
Reservoir height (m)	76.2
Reservoir length (m)	152.4
Reservoir thickness (m)	76.2
Applied load (Pa)	4.137×10^{7}
Initial porosity	0.30
Initial absolute permeability (m²)	9.87×10^{-13}
Initial water saturation	1.0
Initial pressure (Pa)	1.93×10^{7}
Production well bottom-hole pressure (Pa)	1.93×10^{7}
Injection well gas flow rate (m³/day)	141.58
Biot-Willis coefficient	0.79
Poisson's ratio	0.01
Young's modulus (Pa)	9.0×10^{9}

Table 3. Fluid components properties for the case study

Component	Initial reservoir composition	Injection fluid composition
CO_2	0.01	0.95
C_1	0.19	0.05
n-C_{16}	0.80	-

Figures 3 and 4 show the production curves of oil and gas for simulation with and without the geomechanical coupling ("UTCOMP+GEOM" and "UTCOMP", respectively). The coupled simulation presented a similar behavior to the uncoupled one, with slight value differences, as expected. By considering the geomechanical effect, the results predict, in comparison to the uncoupled solution, an earlier beginning of oil production, as well as a slightly earlier gas breakthrough.

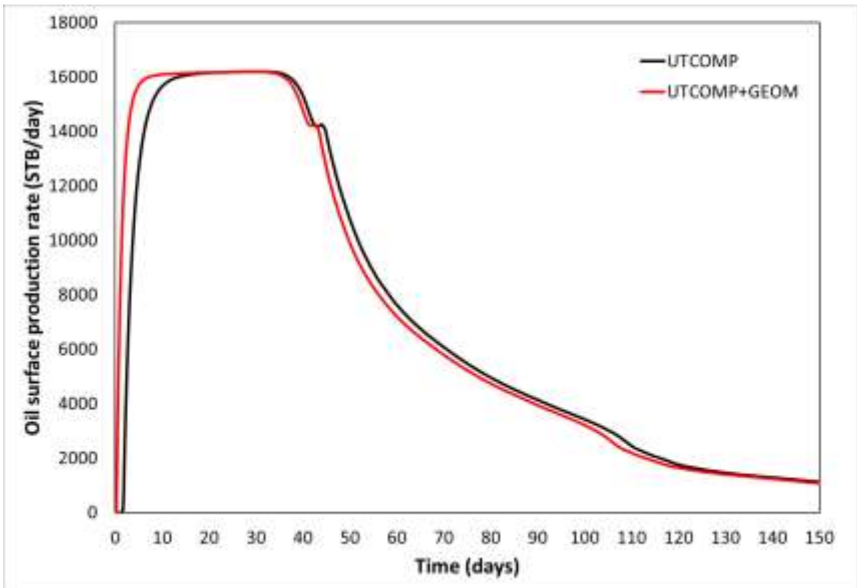

Figure 3: Oil production results for the case study.

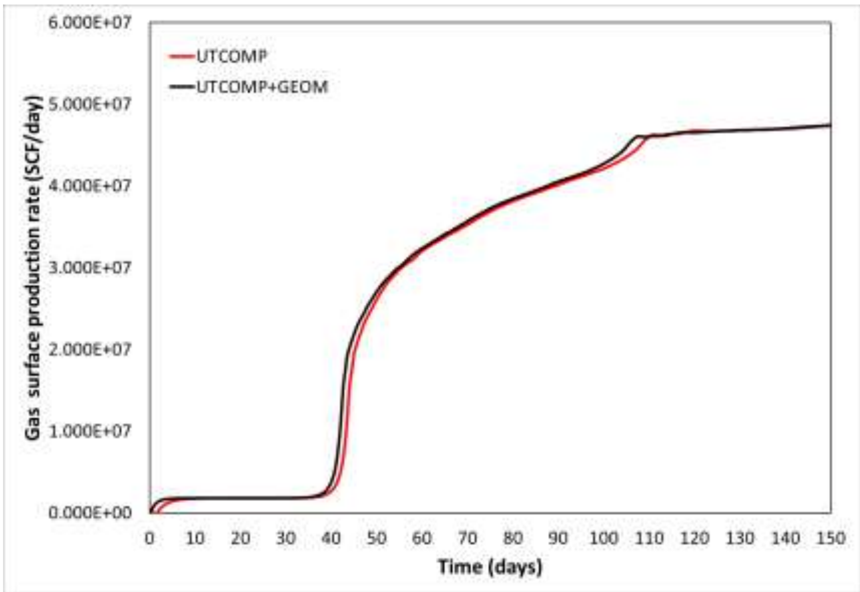

Figure 4: Gas production results for the case study.

Other variables showed a similar behavior. The oil recovery factor, in Fig. 5, was higher for the coupled solution, although both curves converged to close values by the end of the simulation.

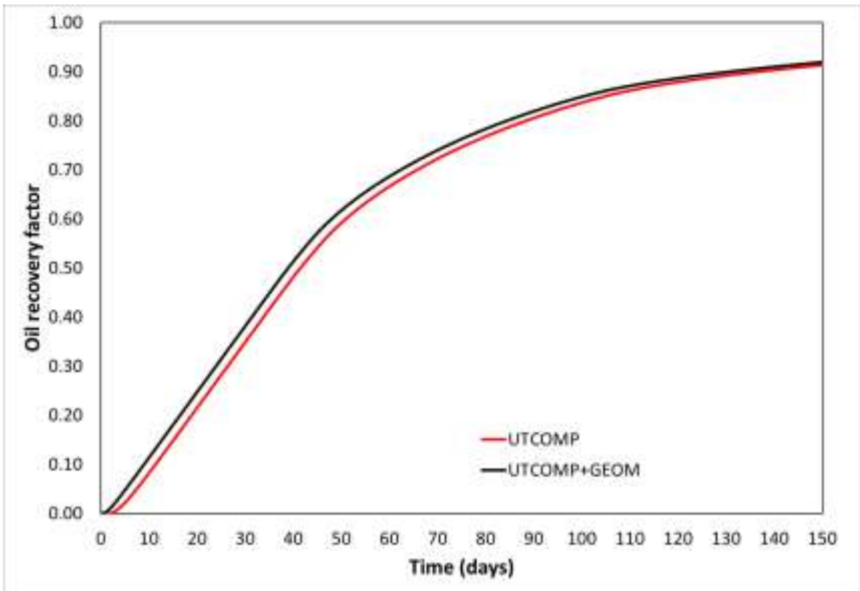

Figure 5: Oil recovery factor results for the case study.

In the case study, only the porosity is being used as a coupling parameter between the geomechanical and fluid flow models. Attempts were made to include the absolute permeability variation as well, but that lead to instabilities in the simulation. Therefore, by allowing only the porosity to variate, the compaction drive effect is enhanced, benefiting fluid production and translating into a slightly higher oil recovery for the coupled solution. If the permeability was allowed to vary as well, given its power-law nature shown in Eq. (7), decreases in its value were likely to be seen for the coupled solution, which would impact negatively on the oil recovery.

4. CONCLUSIONS

This work presented the development of a geomechanical model coupled to the UTCOMP compositional reservoir simulator. For both the fluid flow and mechanical models, the Element-based Finite Volume Method (EbFVM) was used as the discretization approach, allowing the use of a single unstructured grid for both discretized models. The results for the case study show that, by taking into account the geomechanical effect, the prediction obtained for oil production was slightly higher, since the compaction of the reservoir acts to drive the hydrocarbon fluids towards the production well. However, since the effect of diminishing permeability was not considered, the results obtained tend to be optimistic.

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6. REFERENCES

Baliga, B. and Patankar S. 1980. "A new finite-element formulation for convection-diffusion problems". *Numerical Heat Transfer.* Vol. 3.

Chang, Y-.B., 1990. *Development and application of an equation of state compositional simulator*. Ph.D. thesis, University of Texas at Austin, Texas, United States.

Chin, L. Y., Thomas, L.K., Sylte J. E., and Pierson, R G. 2002. Iterative coupled analysis of geomechanics and fluid flow for rock compaction in reservoir simulation. *Oil & Gas Science and Technology - Rev IFP,* v57, no.5, pp. 485-497

Haddad, M., Al-Shalabi, E. W.,and Sepehrnoori, K., 2015. "Development and Validation of an Iteratively Coupled Geomechanics Module for a Compositional Reservoir Simulator, UTCOMP". *In SPE Reservoir Characterisation and Simulation Conference and Exhibition*. Abu Dhabi, UAE.

Maliska, C. R., 2004. *Transferência de Calor e Mecânica dos Fluidos Computacional.* LTC, 2nd edition.

Pan, F., 2009. *Development and application of na coupled geomechanics model for a parallel composition reservoir simulator*. Ph.D. thesis, University of Texas at Austin, Texas, United States.

Schiffmann, R. L. 1960. Field applications of soil consolidation under time-dependent loading and varying permeability. *Highway Research Board Bulletin,* no. 248, pp. 1-25.

Terzaghi, K. 1923. "Die berechnung der durchlassigkeltsziffer des tones aus dem verlauf der hydrodinamischen spannungserchelnungen". *Sitzungsbere. Akad. Wissen. Wien Math. Naturwis.*, vol. 132, pp. 105-124.

Terzaghi, K., 1943. *Theoretical Soil Mechanics.* Wiley & Sons, 1st edition.

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