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# DYNAMICS OF EVAPORATION LIQUID DROPLET: THE ROLE OF DRAG COEFFICIENT CALCULATION

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**Abstract.** *Appropriate computation of droplet drag coefficient is essential for determining the dynamics of a spray flow. Depending on the situation, this drag coefficient might have to be corrected in order to consider evaporation and deformation effects. Therefore, the present study aims to evaluate empirical correlation for drag coefficient calculation that have already been proposed in the literature by means of numerical simulations. First, for model validation, droplet motion and evaporation is computed considering the drag coefficient of a rigid sphere. Then, the influence of transfer processes and non-sphericity on the drag coefficient is investigated. The results reveal that while evaporation reduces the drag coefficient the deformation increases it.*

**Keywords:** *drag coefficient, droplet motion, evaporation, deformation.*

## 1. INTRODUCTION

Evaporation of liquid droplets in high temperature gas environment plays an important role in technical applications, such as furnaces, chemical reactors, gas turbines and internal combustion engines (Lefebvre and McDonell, 2017). Modeling of droplet dynamics and evaporation in two-phase flows is a challenging problem, due to the effects of transient liquid heating, gas phase convection, variable thermophysical properties, and deformable entities. One aspect of the problem which is of technological interest is the dynamics of the evaporative liquid droplet, specially the determination of drag coefficient. Therefore, better understanding the effect of drag coefficient on the dynamics of droplet evaporation could improve evaporative spray simulations. Numerical simulations with the Lagrangian approach compute the droplet motion based on Newton's second law, considering the external forces acting on the droplet. Hence, the correct a drag model to predict the dynamic force acting on the droplet is of crucial importance. There are many empirical and theoretical correlations available in the literature for the determination of drag coefficient (Mashayek and Ashgriz, 2011). Most of the evaporative spray simulations consider the evaporating liquid droplet as a solid non-deforming sphere (Jones *et al.*, 2010; De *et al.*, 2011; Azami and Savill, 2016; Abdelsamie and Thévenin, 2017). However, if the heat and mass transfer between the droplet and the surrounding gas is high enough, the evaporation process can become an important factor in the calculation of the drag force (Schwarzkopf *et al.*, 2016).

The advective or bulk motion of the vapor directed away from the evaporating surface, known as the Stefan's flow, causes a blowing effect, which reduces the drag coefficient. In this work, we used a modification of the drag coefficient for the evaporation of droplets proposed by Eisenklam *et al.* (1967), who inferred it by analyzing the theories of the laminar boundary layer, slow viscous flow and stagnant flow.

Moreover, it is known that a spherical drop can undergo significant deformation and, also, that drag coefficients for non-spherical bodies are significantly different from those of spheres. This deformation effect increase the drag coefficient, since the droplet assume a flatten shape. Hence, the drag coefficients based on correlations for spheres, which are still being used in most spray simulations, need to be revised. In this work, we used a modification of the drag coefficient for the deformation of droplets developed by Liu *et al.* (1993) proposed the TAB-based suggestion, recurrently used in breakthrough calculations, where its analogy is given by Taylor (1963) and its incorporation for Sprays is defined by O'Rourke and Amsden (1987).

The purpose of this study is, therefore, to investigate the effect of deformation and evaporation, on the drag coefficient of an isolated droplet. A n-decane droplet with initial diameter  $D_{d0} = 10 \mu\text{m}$  and temperature  $T_{d0} = 300 \text{ K}$  evaporating in air at  $T_g = 1500 \text{ K}$  and  $p_g = 1.0 \text{ MPa}$ , where the drop with initial velocity  $u_{d0} = 15 \text{ m/s}$  is in a quiescent air environment. First, the simulation results considering a drag coefficient correlation for a solid sphere is compared with the results from Abramzon and Sirignano (1989). Once the computational model is validated, the drag coefficient correlation is modified in order to consider evaporation and deformation effects.

## 2. PROBLEM MODELING

### 2.1 Evaporation

Droplet evaporation is modeled by taking two key process into account mass and energy transfer. These processes are described by differential equations, which express the temporal changes of droplet size and temperature:

$$\frac{dm_d}{dt} = -\dot{m}_d, \quad (1)$$

where  $m_d$  is the droplet mass and  $\dot{m}_d$  is the droplet evaporation rate that leads directly to droplet size reduction:

$$\frac{dD_d}{dt} = -\frac{2\dot{m}_d}{\pi\rho_l D_d^2}, \quad (2)$$

and

$$\frac{dT_d}{dt} = \frac{Q_S}{m_d c_{pl}}, \quad (3)$$

where  $c_{pl}$  is the liquid droplet specific heat capacity and recalling that  $Q_S$  is the power transferred to promote the droplet thermal energy variation per unit of time, which is transferred as heat. In the present work  $\dot{m}_d$  and  $Q_S$  are modeled with Abramzon and Sirignano model (1989).

### 2.2 Displacement model

For engineering applications involving spray evaporation, the droplet drag force and the gravitational force, also known as body force, are predominant compared to other forces, as Basset history, added mass, Magnus, Saffman, buoyancy and pressure gradient terms Shirokar *et al.* (1996). Under these conditions and considering the Lagrangian approach, droplet motion and momentum equations are:

$$\frac{d\mathbf{u}_d}{dt} = \frac{\mathbf{u}_g - \mathbf{u}_d}{\tau_d} + \mathbf{g}, \quad (4)$$

where  $\mathbf{x}_d$  and  $\mathbf{u}_d$  are droplet position and velocity, respectively,  $\mathbf{u}_g$  is the carrier gas velocity, and  $\mathbf{g}$  is the gravitational acceleration. The droplet relaxation time,  $\tau_d$ , is determined by:

$$\tau_d = \frac{4}{3} \frac{\rho_l}{\rho_g} \frac{D_d}{C_D |\mathbf{u}_g - \mathbf{u}_d|}, \quad (5)$$

where  $\rho_l$  and  $\rho_g$  respectively refer to liquid droplet and gas phase densities. The drag coefficient,  $C_D$ , is given by semi-empirical correlations. A frequently used one is the Schiller-Naumann correlation for solid non-evaporating spheres, which is given by Clift *et al.* (2005):

$$C_D = \frac{24}{Re_d} \left(1 + \frac{1}{6} Re_d\right)^{\frac{2}{3}} \text{ if } 1 < Re_d \leq 800 \quad (6)$$

in which the droplet Reynolds number,  $Re_d$ , is defined as:

$$Re_d = \frac{\rho_g D_d |\mathbf{u}_g - \mathbf{u}_d|}{\mu_g}, \quad (7)$$

where  $\mu$  is the dynamic viscosity.

Since the Stefan's flow reduces the drag coefficient (Abramzon and Sirignano, 1989), Sazhin *et al.* (2005) have suggested a modification of  $C_D$  for evaporating droplet:

$$C_{Dev} = \frac{C_D}{(1 + B_M)^{\alpha_M}}, \quad (8)$$

where  $B_M$  is the Spalding mass transfer number given by:

$$B_M = \frac{Y_{vs} - Y_{vg}}{1 - Y_{vs}}, \quad (9)$$

recalling that  $Y_{vs}$  and  $Y_{vg}$  are the vapor mass fraction at the droplet surface and far away, respectively, and  $\alpha_M$  is:

$$\alpha_M = \begin{cases} 1, & \text{if } B_M < 0.78 \\ 0.75, & \text{if } B_M \geq 0.78 \end{cases}. \quad (10)$$

Moreover, considering that droplets are deformable entities Liu *et al.* (1993) proposes a submodel that uses the TAB model approach to estimate the distortion of droplets inserted into a high relative velocity flow. The suggested new correlation for the  $C_D$  is given by:

$$C_{Ddef} = C_D(1 + 2.632y) \quad \text{if } 0 \leq y \leq 1 \quad (11)$$

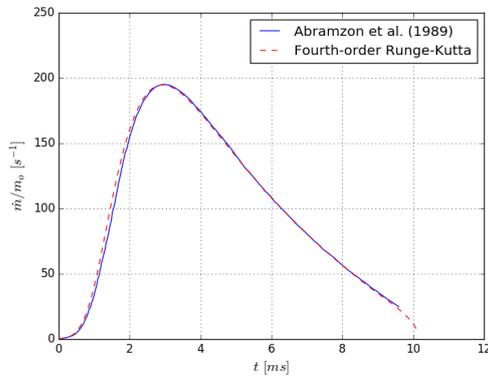
where  $y$  is the degree of drop distortion.

### 3. RESULTS AND DISCUSSION

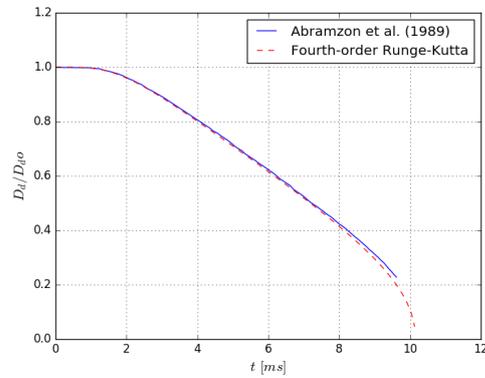
#### 3.1 Model Validation

The time evolution of four droplet parameters, ie evaporation rate, temperature, non-dimensional cycle and drag coefficient are shown in Figures 1. The simulation in figure 1 was similar to that proposed by Abramzon and Sirignano (1989) in which the fourth-order Runge-Kutta method was used using the drag coefficient (6) in which it neglects the evaporation and deformation effects of the drop.

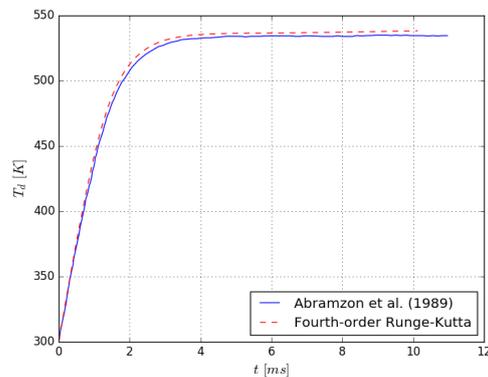
Figure 1. The temporal evolution of parameters



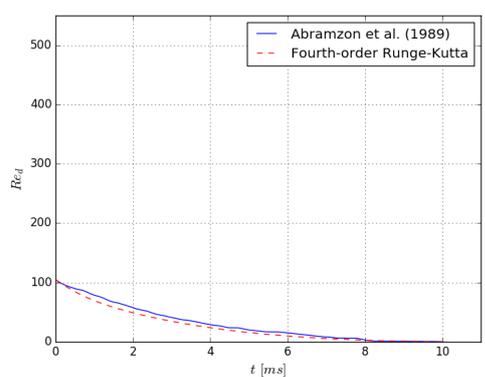
(a) Droplet evaporation rate temporal evolution.



(b) Non-dimensional droplet diameter temporal evolution.



(c) Droplet temperature temporal evolution.

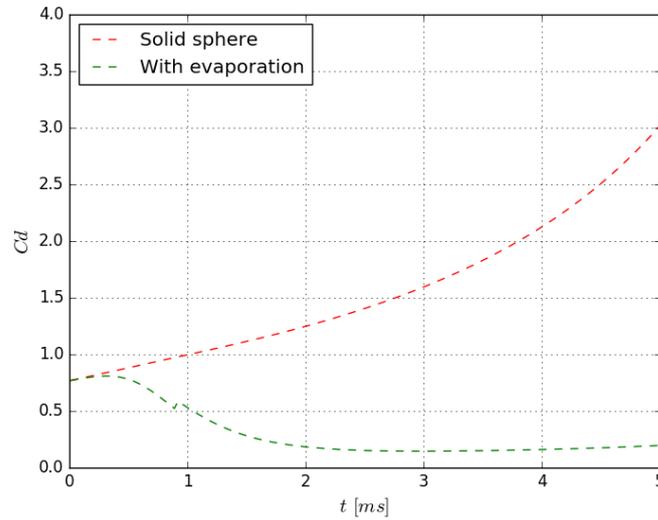


(d) Droplet Reynolds temporal evolution.

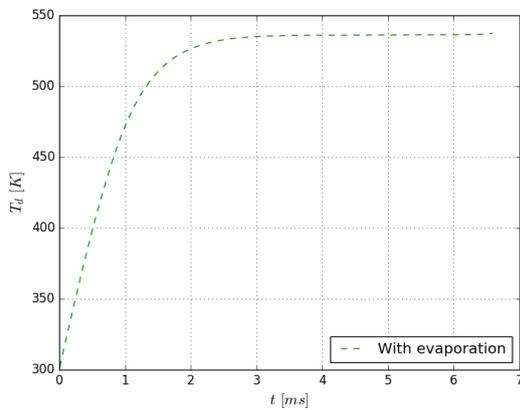
### 3.2 Effects of evaporation on drag coefficient

The time evolution of the drag coefficient, spalding heat transfer number and temperature are shown in Figures 2. For consider the effect of evaporation, the drag coefficient (8) was used.

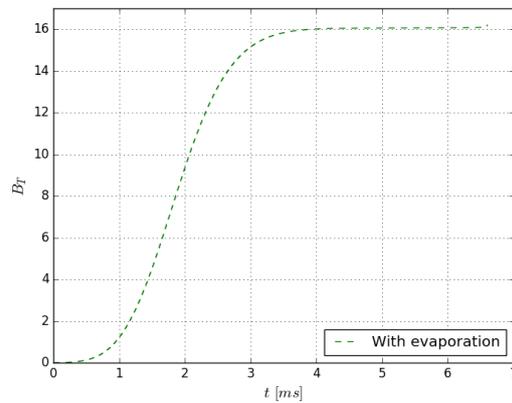
Figure 2. The temporal evolution of parameters



(a) Droplet drag coefficient temporal evolution.



(b) Droplet temperature temporal evolution.



(c) Temporal evolution of droplet spalding heat transfer number.

In Fig. 2b and Fig. 2c the temporal increase in temperature and of the Spalding heat transfer number can be analyzed as the sensitive phase of the drop and therefore without phase change. Subsequently, the constant behavior corresponds to latent phase of the drop, recording the evaporation process. In Fig. 2a, the temporal discrepancy of the drag coefficient in the latent phase for the solid and evaporative sphere can be understood due to the formation of the vapor layer on the drop surface during evaporation, eventually causing the drag to decrease.

### 3.3 Effects of deformation on drag coefficient

For the inclusion of the deformation effect in the drag coefficient calculation the relation (11) was used. In order to analyze the behavior of the three conditions, the figure 3 was generated.

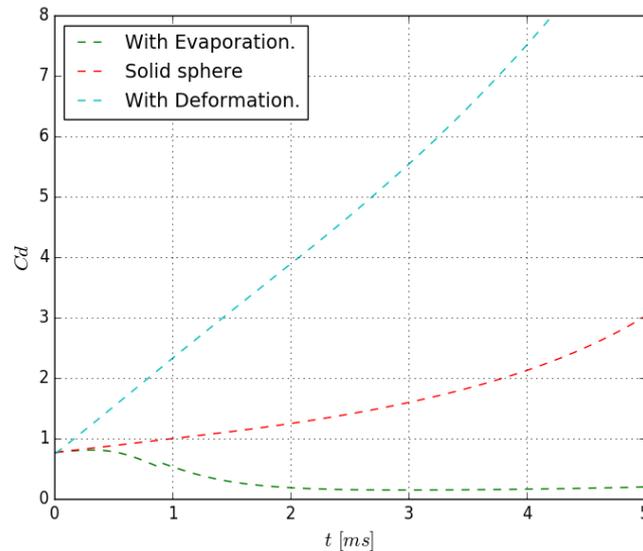


Figure 3. Temporal evolution of the drop drag coefficient for the three considerations.

A high drag coefficient for deformable drop was physically expected due to the increased surface area of the drop as it deforms, the behavior would be to compensate for this increase.

## 4. CONCLUSION

It is possible to notice some discrepancy in the drop drag coefficient when incorporating the effects of evaporation and deformation, thus making the correction of the coefficient necessary for simulation. All the results obtained met the expected physical expectations, however, the code needs validation for two correlations used, as well as the appropriate revision of the intervals in which these relationships can be used.

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