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ASSESSMENT OF DIFFERENT LOCAL INTERPOLATIONS FOR A GLOBAL-LOCAL SUPERPOSITION 2D FINITE ELEMENT

André Schwanz de Lima
Alfredo Rocha de Faria
Instituto Tecnológico de Aeronáutica, Praça Marechal Eduardo Gomes, 50 - Vila das Acacias, São José dos Campos - SP, 12228900
andresl@ita.br; arfaria@ita.br

Abstract. To accurately model a composite laminate structural behavior, a laminate theory must represent the zig-zag shape of the in-plane displacements and assure the continuity of both displacements and transverse stresses. These are named the C^0_z requirements. Among the models that are able to fulfil the C^0_z requirements and obtain accurate responses, the global-local superposition theories appear as one of the most promising techniques. In these theories, the displacement field is composed of a global interpolation function that is defined over the entire thickness of the laminate, and a local interpolation function that is defined along the thickness of each layer. The elements built under this hypothesis possess a number of degrees of freedom independent of the number of layers, what makes this technique very attractive in computational aspects. The work presents the assessment of different polynomial expansions for the local interpolations of a 2D finite element to model composite structures based on global-local superposition.

Keywords: beam element, composite material, zig-zag behavior, global-local superposition

1. INTRODUCTION

Composite structures are replacing traditional metallic structures in many engineering applications, especially in high-strength light-weighted structures, as it is the case of the aerospace sector. Nevertheless, the mechanical behavior of composite materials is much more complex than traditional engineering materials given its non-homogeneous nature.

Advanced composite materials generally present a layerwise construction scheme, where mechanical properties may vary greatly from layer to layer (e.g. sandwich structures). Also, to assure structural integrity, the displacement field must be continuous through the thickness. These particularities originate the C^0_z requirements, as named by (Carrera, 1997) that need to be fulfilled in order to correctly describe the displacement, strain and stress fields of a composite structure.

The Finite Elements Method is a powerful tool used to model complex geometries such as used in engineering applications, but when it comes to composite modeling, traditional modeling techniques and elements are not suitable because they fail to fulfil the C^0_z requirements. As a result, many works propose strategies and formulations in order to assure full or partial attainment of these requirements, obtaining accurate and physically consistent results.

Considering the unknown variables used to model the composite problem, three major lines of research can be identified: Displacement-based theories, where only displacements are the unknown variables; Stress-based theories, where stresses are the unknown variables; Mixed theories, where both displacements and stresses are unknowns.

Considering the displacement-based formulations, (Li and Liu, 1995) proposed a model where global in-plane displacement variables used in traditional laminate theories are combined with local layerwise displacement variables which are responsible for assuring the transverse shear stress continuity and zig-zag behavior of the in-plane displacements. This theory was addressed as the superposition theory (ST). Later, this strategy was compared to other displacement-based theories by (Liu and Li, 1996) and it was pointed as one of the most promising strategies to model composite structures, considering computational cost and numerical accuracy.

Nevertheless, in the superposition theory originally proposed, only constant global behavior for the transverse displacements is assumed, neglecting transverse normal strains. Therefore, in order to fully attain C^0_z requirements, a post processing technique via integration of the equilibrium equations was required to assess transverse normal stresses. To solve this problem, the papers (Shariyat, 2010a), (Shariyat, 2010b) and (Shariyat, 2011) assumed superposition for in-plane and transverse displacements, respecting both transverse shear and normal stress continuity, dismissing the use of post-processing techniques and achieving accurate results compared to the elasticity solution.

Recently, (Shariyat et al., 2015) proposed a variation of the global-local theory to study sandwich plates neglecting the local interpolation of the transverse displacement *w* and transverse normal stresses and strains, obtaining more accurate results compared to other shear deformation theories. Notwithstanding, the same author in (Shariyat, 2010b) recognized the transverse normal stresses and strains as significant and therefore not negligible. The same position is adopted by the present work.

Also, (Shariyat, 2010b) assumes three local variables for the displacement field, according to the double superposition hypothesis proposed by (Li and Liu, 1997). This results in not only global displacement variables, which are physically intuitive, but also in one local displacement variable for each displacement direction, which is purely mathematical in nature.

More recently, (Lezgy-Nazargah, 2017) proposed a generalized layered global-local beam theory for elasto-plastic analysis of thin-walled members, including transverse local displacement field and obtaining reasonable agreement with three dimensional 3D commercial finite element analysis. The model presented used much less degrees of freedom, reinforcing the advantage of global-local approach in respect to computational cost.

In order to explore this computational advantage, (Lima and Faria, 2018) proposed a global-local 2D formulation that considered a local linear displacement for both in-plane and transverse displacement fields using only global degrees of freedom with physical significance. This formulation achieved accurate results for moderately thick laminate structures under bending using a linear local interpolation field, but transverse normal stresses were not as accurately predicted as the transverse shear stress, requiring a post-processing technique.

Consequently, the present work assess the global-local formulation originally proposed by (Lima and Faria, 2018) using different polynomial distributions for the local displacement field in order to enhance results accuracy.

2. NUMERICAL MODEL

The proposed global-local model for a 2D element possesses the following general expressions for the displacement fields along the thickness direction (z-direction):

$$u^k(x,z) = u_b(x)H_0 + \phi_b(x)H_1 + u_t(x)H_2 + \phi_t(x)H_3 + u_{k-1}F_0 + u_k F_1 \tag{1}$$

$$w^k(x,z) = w_b(x)H_0 + \psi_b(x)H_1 + w_t(x)H_2 + \psi_t(x)H_3 + w_{k-1}F_0 + w_k F_1 \tag{2}$$

The local displacement variables u_k and w_k of the k-th layer, the functions in x-direction, as well as the functions in z-direction H_0, H_1, H_2 and H_3 are detailed in (Lima and Faria, 2018). The functions F_0 and F_1 defined in Tab. 1 are polynomial functions that satisfy the same assumptions of (Lima and Faria, 2018), with ζ_k being the local through-the-thickness coordinate defined between -1 and 1, bottom and top of the k-th layer respectively.

Table 1. Interpolation functions F_0 and F_1 in thickness direction.

Local Interpolation	F_0	F_1
0-1 ST[1]	$\left(\dfrac{1-\zeta_k}{2}\right)$	$\left(\dfrac{1+\zeta_k}{2}\right)$
2-3 ST	$\left(\dfrac{-\zeta_k+\zeta_k^2}{2}\right)$	$\left(\dfrac{\zeta_k+\zeta_k^2}{2}\right)$
1-2 ST	$\left(\dfrac{\zeta_k^2-\zeta_k^3}{2}\right)$	$\left(\dfrac{\zeta_k^2+\zeta_k^3}{2}\right)$
0-3 ST	$\left(\dfrac{1-\zeta_k^3}{2}\right)$	$\left(\dfrac{1+\zeta_k^3}{2}\right)$

[1] (Lima and Faria, 2018)

The 0-1 ST interpolation represents the linear interpolation scheme already defined in (Lima and Faria, 2018).

Because the interpolation scheme in the x-direction remains unaltered, the element continues to possess only global degrees of freedom (independent of the number of layers) with clear physical significance, what is an advantage in comparison with other advanced models.

The only superposition theory presented in the work by (Liu and Li, 1996) that was not tested was the 1-3 ST. Although this theory is indicated by the authors to have the best results, there is a mathematical impossibility of eliminating all the local degrees of freedom in this theory, therefore it is not suitable for the finite element formulation developed in the present work.

3. NUMERICAL RESULTS AND DISCUSSIONS

The structural responses of the different interpolation schemes were assessed in the case of a simply supported beam under cylindrical bending induced by a constant distributed load of 10^6 N/m as in (Lima and Faria, 2018), in comparison to the analytical solution by (Pagano, 1969). Beams with different thickness and layups were tested.

Material properties for a unidirectional lamina oriented with 0°, directions (1, 2, 3) are aligned with (x, y, z) directions are the same used by (Zhen et al., 2012): $E_1 = 172.5$ GPa, $E_2 = E_3 = 6.9$ GPa, $G_{12} = G_{13} = 3.45$ GPa, $G_{23} = 1.38$ GPa, $v_{12} = v_{13} = v_{23} = 0.25$.

All results are plotted in $x = 3L/4$ using a normalized thickness coordinate $\bar{z} = z/h$, defined from -0.5 (bottom surface) to 0.5 (top surface), where h is the beam thickness and L is the beam length.

The first beam is a three-layered 0°/90°/0° thick beam possessing length-to-thickness ratio $S = L/h$ equal to 4, with results are depicted in Fig. 1. The second beam is a moderately thick beam with length-to-thickness ratio $S = 20$ possessing the same stacking sequence, with results illustrated in Fig. 2. The third beam is also a moderately thick beam with length-to-thickness ratio $S = 20$ but now composed of twelve layers of 0° and 90° plies alternated, characterizing a unsymmetrical laminate behavior with results illustrated in Fig. 3, where the shaded areas indicate the plies oriented in 90°.

There were significant changes to the response of the structure depending on the polynomial terms used in the solution, with the exception of the transverse displacement w and the transverse normal strain ε_z, revealed almost insensitive to the local interpolation.

An important information that can be extracted from a quick observation of Fig. 1 is that the odd-order terms (linear and cubic) of the local displacements are responsible for the shape of the solution. This is evident from the curves for u, ε_x, τ_{xz}, γ_{xz} and σ_z, where the interpolations that possess the cubic term, 2-3 ST and 0-3 ST, present a very similar response, as well as the 0-1 ST and 1-2 ST interpolations, that possess the linear term.

The presence of the cubic term in z-direction brings the shear distribution to a shape more similar to the analytic response, but the magnitude of the shear stress and strain is overestimated. The axial displacement distribution is impaired, but the error in transverse displacement is reduced, specially in the 2-3 ST. It can also be observed that the presence of the quadratic term reduces the errors in the approximations for the thick beam problem while it does not change the shape of the solution.

Reducing the thickness to a moderately thick situation while keeping the layup, the in-plane displacement, strain and stress, u, ε_x, and σ_x respectively, are accurately predicted independent of the local interpolation scheme. Nevertheless, the transverse stresses σ_z and τ_{xz}, and the γ_{xz} strain still present the behavior observed in the thick beam situation, while the linear local interpolation (0-1 ST) presents the more accurate results for the transverse displacement and shear quantities as can be seen in Fig. 2.

Increasing the number of layers and consequently reducing layer thickness in the moderately thick configuration, the linear local interpolation still holds as the best approximation of the exact solution. The cubic order solutions exhibit large errors in the transverse shear stress and strain inside the layers while it presents reasonable agreement with the exact solution in the interfaces between layers. Hence, with the increase of the number of layer it is expected that the prediction of σ_z, τ_{xz} and γ_{xz} is expected to be less accurate.

Apart the behavior of the different local interpolation schemes, an important observation with respect to the bending of the multilayered composite structures can be made. If we compare the results for w and σ_z for both thick beam and moderately thick beam configuration with different layups, the same pattern is observed, a smooth behavior through the thickness. This may tempt the researches to justify dismissing the use of local interpolation for the transverse displacement, but it is important to remember that if this local interpolation is not present, the continuity of the transverse normal stress cannot be enforced, violating the C^0_z requirements.

4. CONCLUSIONS

The superposition hypothesis proved to be an effective method to model composite structures fulfilling the C^0_z requirements.

Among the different local interpolation schemes assessed in the present work the ones possessing the linear term provided more accurate results, revealing the importance of such term in the solution. Also, the linear 0-1 ST proposed by (Lima and Faria, 2018) proved to be the best alternative among the other local interpolations tested.

Nevertheless, the lack of accuracy in the prediction of the transverse normal stress is a point that needs to be solved. Therefore, the future developments for this work consist in investigating more local interpolation schemes to achieve the same level of accuracy of the results by (Shariyat, 2010b) dismissing the use of local displacement variables.

Finally, it would be interesting to perform experimental tests with multilayered structures in order to verify the capabilities of the proposed concept in composite failure, e.g. delamination.
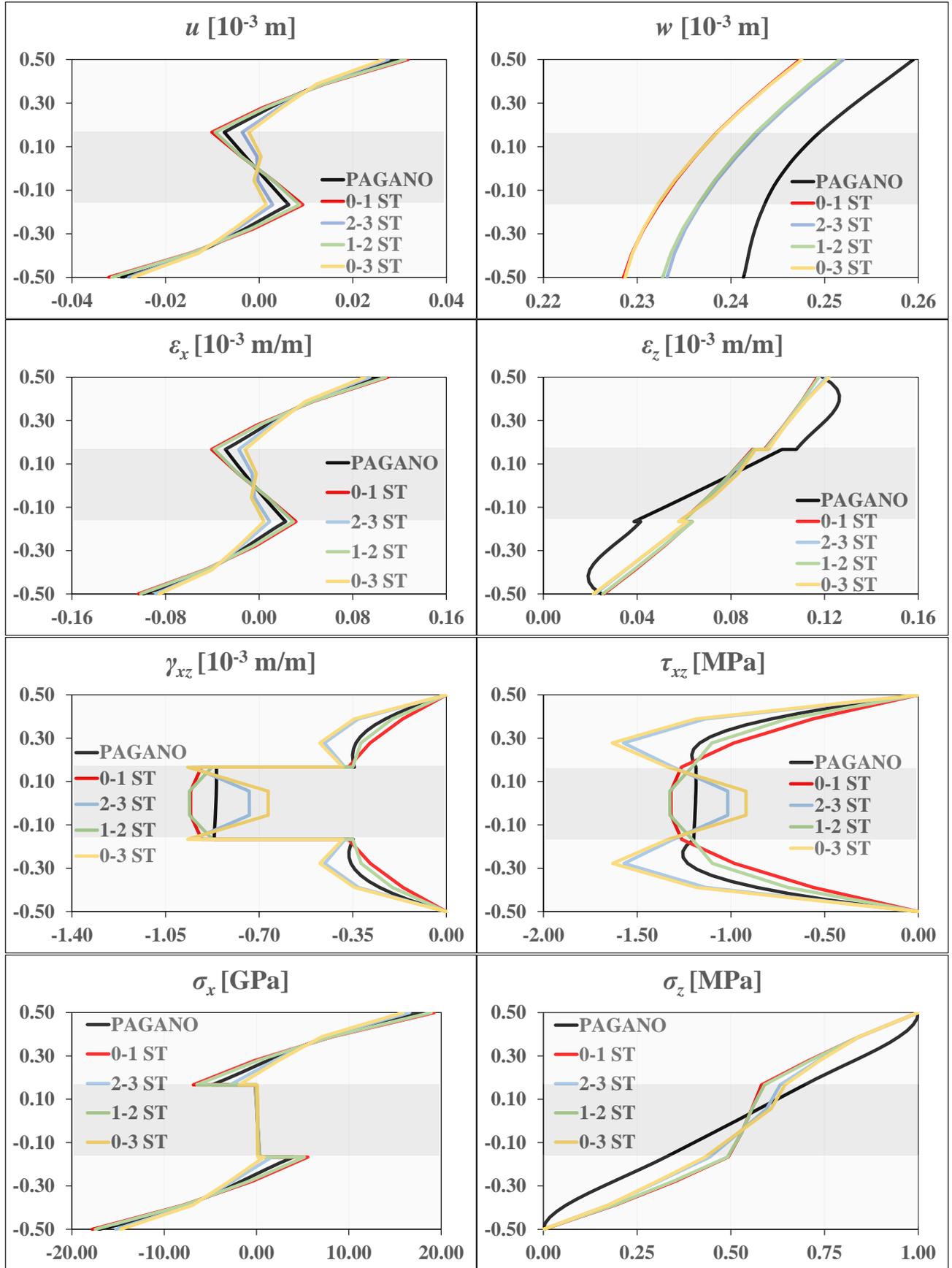
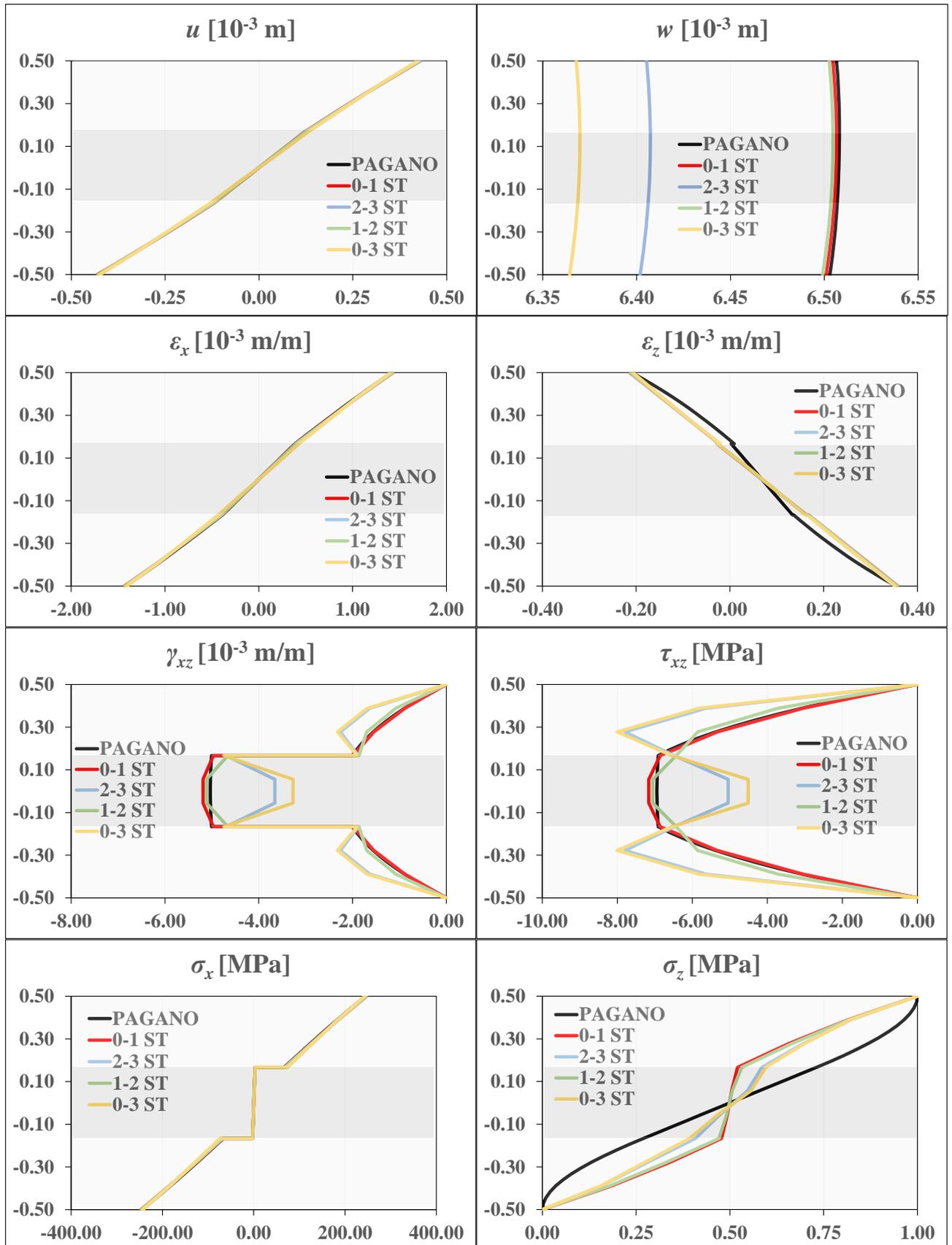Figure 1. Results for the 0º/90º/0º thick beam (S = 4) problem using different superposition schemes.

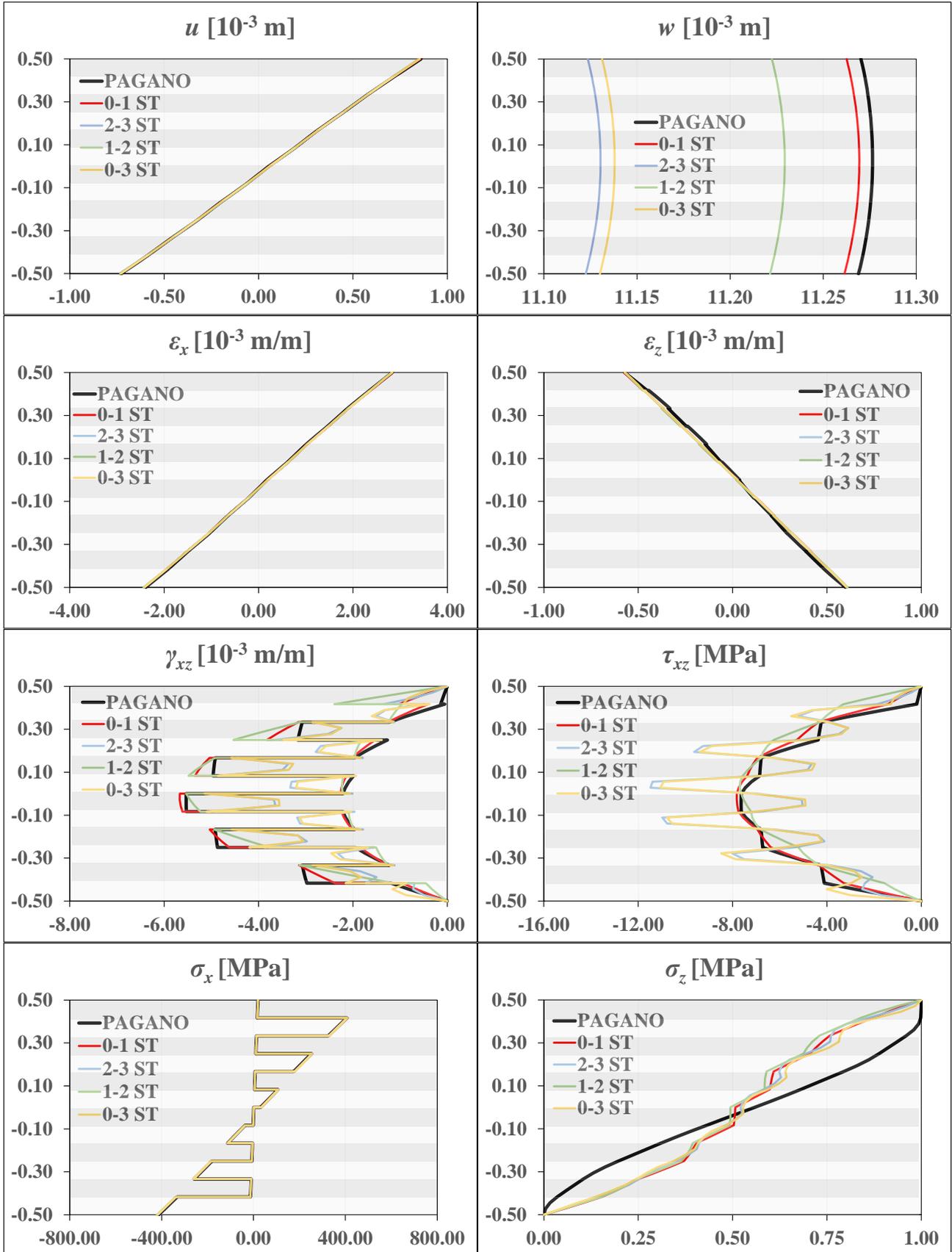Figure 2. Results for the 0º/90º/0º moderately thick beam ($S = 20$) problem using different superposition schemes.

Figure 3. Results for the twelve-layered 0º/90º moderately thick beam ($S = 20$) problem using different superposition schemes.

5. ACKNOWLEDGEMENTS

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