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WAVE PROPAGATION IN CRACKED 1D PHONONIC CRYSTAL BY THE WAVE SPECTRAL ELEMENT METHOD

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Abstract. *In this paper, we investigate the wave propagation and the band gap creation in 1D phononic crystal (PC) rods with periodic cracks using the Wave Spectral Element method (WSE). WSE consists of the combination the Spectral Element Method (SEM) with the Floquet-Bloch's theorem. PCs are artificial periodic structures with high impedance distinction of material property and/or geometry which can generate band gaps, where waves are forbidden to propagate. Cracks are modeled as mass-less translational springs using Castigliano's theorem and fracture mechanic laws. They are embedded in different locations along the PC. In the paper, it is analyzed how the crack flexibility and position influences the responses obtained and in the formation of band gaps. The results are compared with that obtained with PC without crack and validated with Spectral Transfer Matrix (STM) method.*

Keywords: *phononic crystals, band gaps, wave propagation, crack*

1. INTRODUCTION

The analysis of wave propagation in periodic structures had its beginning with (Brillouin, 1946). He focused on problems of solid state physics, electrical engineering and electronics. Then, elastic wave propagation in periodic structures, e.g., rods and beams, was investigated (Mead, 1970; Orris and Petyt, 1974; Mead, 1975). In the past few years, many studies have been devoted to artificial materials. When the wave phenomena they exhibit depend on the periodic arrangement of scatterers inside the medium, they are usually called phononic crystals (PCs) (Kushwaha *et al.*, 1993).

When the composite materials and structures consist of two or more different materials periodically, there will be stop band or band gaps characteristic, in which the elastic/acoustic wave cannot propagate. This kind of phenomenon enables the periodic structures to be applied in the acoustic filters, vibration suppression and noise isolation technology, which has drawn enormous attention (Sigalas and Economou, 1993). Conform to the generation mechanisms, the band gaps can be divided into Bragg band gap and local resonance (LR) band gap, the Bragg band gap results from the structural periodicity while LR band gap, originating from the locally resonant behavior of microstructure in each unit cell (Zhang *et al.*, 2015).

The addition of imperfections or cracks into PC provide an extra reflector that may lead to additional band-gaps or other unique physical phenomena (Biwa and Ishii, 2017; Golub and Zhang, 2015). The determination of the dispersion relations and the band gaps in structures with the periodic cracking model has already been studied. We can find various engineering structures that can simulate a periodic crack, such as; long bridges, ducts and conveyors, as well as in fine systems such as skyscrapers. Some bridges and viaducts are designed from series of spans supported by pillars. In the correspondence of each pier, the spans are connected only by the upper deck. Therefore, the joint at the pier behaves as a cracked section where the ligament is represented by the depth of the upper deck. The dynamic response of elongated solids with a distribution of periodic cracks is used in the practical evaluation of composite armor properties as well as protection sheets and windscreens of armored vehicles (Carta *et al.*, 2014).

Several numerical methods have been used for the study of band gaps in phononic crystals and metamaterials, we can highlight some as, wave finite element (WFE) (Mencik, 2014), boundary element method (BEM) (Li *et al.*, 2013) and plane wave expansion (PWE) (Cao *et al.*, 2004). In this paper we will use the WSE and STM method for the study of band gaps in cracked phononic crystals. The paper is organized as follows. In Section 2, we present briefly the dynamic stiffness matrix formulation for a PC cracked rod unit-cell used in the periodic system and some important articles are cited for the understanding of this work. In Section 3, the validation of the WSE with the STM is done through the analysis of the dispersion curves for the phononic crystal, the influence of cracks depth on the appearance and width of the bang gaps is analyzed.

2. PHONONIC CRYSTAL CRACKED ROD MODELING

The more detailed formulation for a 1D rod using the WSE can be found in (Goldstein *et al.*, 2010). Here we present briefly the dynamic stiffness matrix formulation for a PC cracked rod unit-cell used in the periodic system. Applying Floquet-Bloch periodicity condition to the unit-cell PC cracked rod results in an eigenvalue/vector problem whose formulation produces the force-displacement relationships.

Figure 1 shows a 1D PC cracked rod model consisting of a periodic array of unit cells with two different materials ; steel and nylon. Figure 2 shows the cells analyzed.

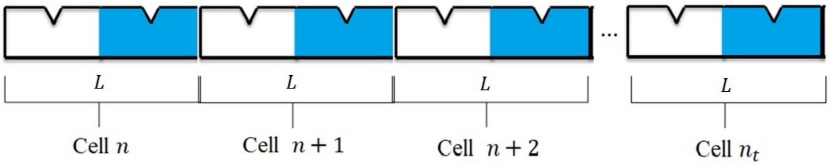
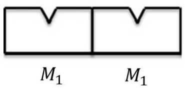
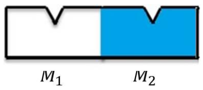


Figure 1. Schematic representation of a PC rod with crack with N unit cells.



(a) unit cell with same material



(b) unit cell with different materials

Figure 2. Analyzed models of the unit cells where M_1 refers steel and M_2 refers nylon.

WSE method uses the concept of transfer matrix methods to compute the wave modes along periodic structures. The periodic structures which are considered here are assumed to be 1D periodic in the sense that they are composed of identical unit-cells along a certain straight direction. By considering the wave modes, the frequency forced response of periodic structures can be computed in an efficient way. The computational tasks involved in the WSE method can be summarized as follows:

1) SE model of a unit-cell: Consider the unit-cell $(material_1 + material_2)$ of the periodic rod shown in Figure 1, made with two material properties. It consists in obtaining the dynamic stiffness matrix of the unit-cell, which can be modeled using only one rod spectral element, one for each elastic property changes. Same considering that this spectral element uses an approximated solution by Fourier series, it is still more precise than others approximated models like finite element (FE) method. It has no restrictions to accurately capture the wavelengths of the wave motion in a infinite structure.

2) Transform the dynamic stiffness matrix in the transfer matrix and applying Floquet-Bloch theorem: Partitioning the dynamic stiffness matrix in left and right boundary state vectors, the transfer matrix of the unit-cell can be easily derived. Then, applying Floquet-Bloch theorem it has the wave eigenproblem given by,

$$Tq = e^{\mu}q, \tag{1}$$

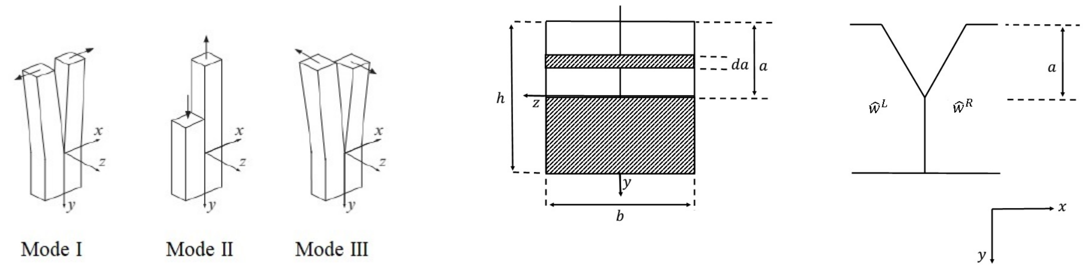
where T is the transfer matrix, q is the state vector, e is the exponential function and $\mu = ikd$ is the attenuation constant with k as the wavenumber, d the unit-cell length and i the imaginary unit.

2.1 Cracking modeling

From theorem of the Castigliano, the flexibility at the crack location for the one-dimensional rod spectral element can be obtained by (Palacz and Krawczuk, 2002):

$$c_{ij} = \frac{\partial^2 U}{\partial S_i \partial S_j} \tag{2}$$

where U is the elastic strain energy due the crack and Q is the nodal force on the element. A crack can be subjected to three basic modes of loading as shown in Figure 3(a), designated by modes I, II and III. Mode I is called normal traction crack, mode II is called flat shear crack and mode III anti-plane shear crack, in this paper we will consider only crack mode I is present in the rod element and in the Figure 3(b) is showed the Cracked rod cross-section at crack position where a is the depth of dimensionless crack given by (a/h) where h is the height of the cross-section.



(a) Basic modes (b) cross-section at crack position

Figure 3. Basic modes of displacement on crack surfaces and cracked rod cross-section at crack position.

3. NUMERICAL RESULTS

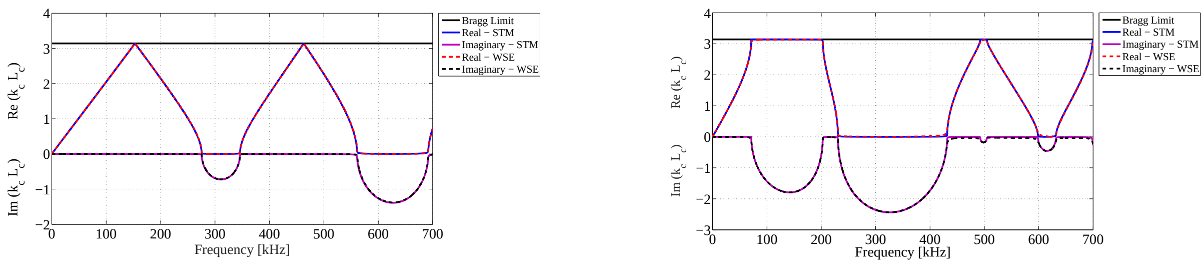
For the numerical examples, it was considered the two models of unidimensional cracked structure, shown in the Figure 2. The material properties and geometry dimensions are listed in the Table 1.

Table 1. PC rod geometric parameters and material properties.

Property and Geometry	Value
Modulus of elasticity	$E_A = 21 \times 10^{10} Pa$, $E_B = 30 \times 10^9 Pa$
density	$\rho_A = 7800 kg/m^3$, $\rho_B = 1150 kg/m^3$
Loss factor	$\eta_A = 0.0013$, $\eta_B = 0.001$
Cross section area	$A = 10^{-4} m^2$
Number of unit cells	$N = 5$
Rod length	$L = 0.075m$
Unit cell length	$L_c = 0.015m$
Percentage of material	$M_1 = 67\%$ and $M_2 = 33\%$
Relative depth of the crack	$a = 0.2$

3.1 Validation

In order to validate the WSE method, the Spectral Transfer Matrix (STM) method was implemented to obtain the dispersion curves of the cracked PC unit cell for the two models showed in the Figure 2. The first model was considered a unit cell of steel with two cracks while the second model was considered a cracked cell of steel and nylon. The Figure 4 shows the comparison between the WSE and STM method. As seen, the results exhibit good agreement. The dispersion diagram relates the wavenumber of the unit cell with the analyzed frequency range. The region of bandgap starts when the real part of $k_c\,L_c$ is equal to 0 or π, where the imaginary part of $k_c\,L_c$ exhibit a non-zeros value indicating the predominant behavior of evanescence waves.



(a) Dispersion Curves model 1 (b) Dispersion Curves model 2

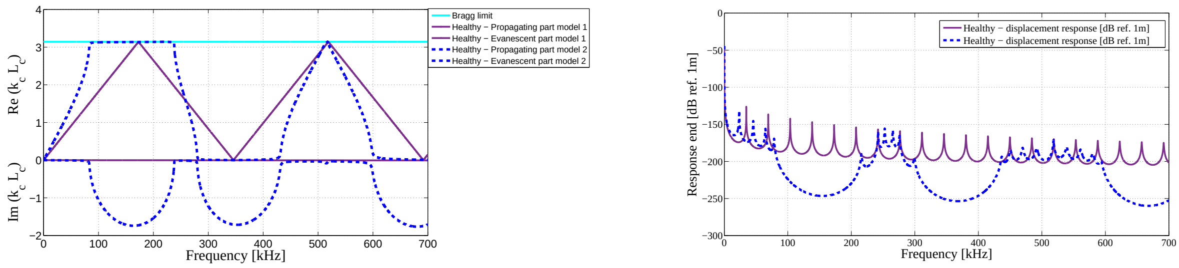
Figure 4. Comparison of the dispersion curves obtained by STM and WSE methods.

With the validation of the WSE method, it is possible to apply the method for the study of cracked phononic crystals and to analyze the behavior of band gaps.

3.2 Analyze of damaged phononic crystal

The PC rod parameters and material properties are listed in Table 1, considering model 1 in Figure 1. The structural damping, η_A, η_B, also known as loss factors, are included as a complex Young's modulus, $E_A = E_A(1 + i\eta_A)$, $E_B = E_B(1+i\eta_B)$. The analyzed cells are shown in Figure 2 where in the Figure 2(a) the cell is composed of the same material with two cracks in each part already in Figure 2(b) is shown the cell with two different materials both with crack.

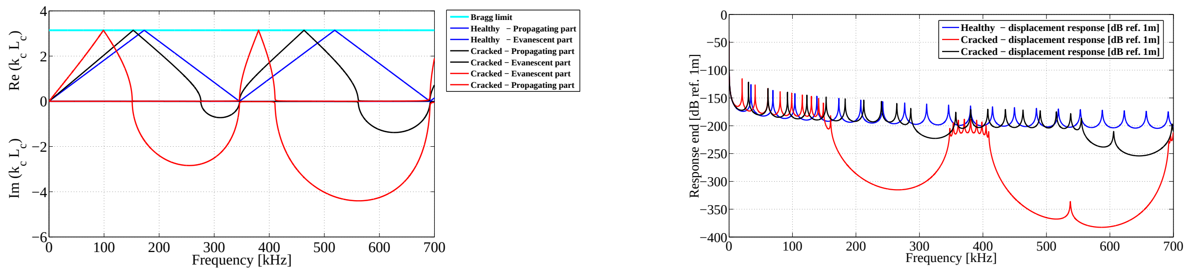
Figure 5(a) shows the dispersion curves for the healthy unit cell used in model 1 and model 2 the model 1 represents the unit cell with the same material and model 2 represents the unit cell with different materials, in Figure 5(b) shows the displacement response for the complete structure using five unit cells $N_c = 5$, the formation of band gaps through the dispersion curves and displacement response for the unit cell composed of different materials is observed in frequency ranges from $f_1 = 90 - 220Hz$, $f_2 = 270 - 420Hz$ and also at $f_3 = 600Hz$. For forced response the structure was excited with a unitary force at the beginning of the structure and displacement at the end of the rod was obtained.



(a) Dispersion Curves (b) Displacement

Figure 5. Dispersion Curves and displacement using model 1.

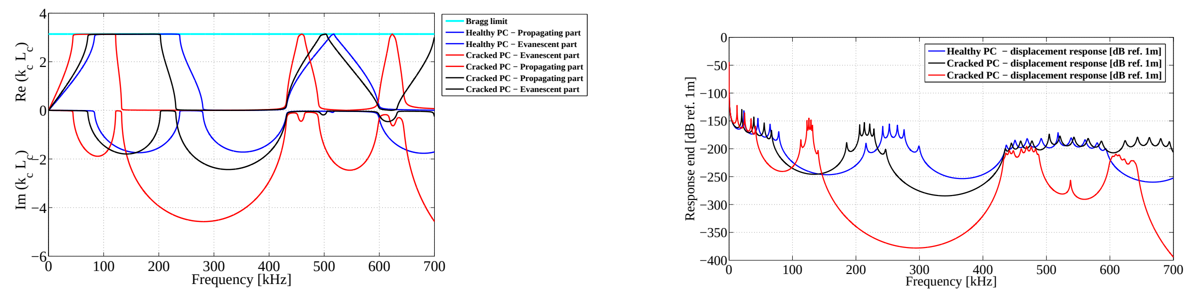
In the Figure 6 is shown as the variation of the depth of the crack influences in the dispersion curves and the displacement response for case the unit cell with same material is noticed that the bigger the depth of the crack the greater the band gaps found for the same analyzed frequency range, where the cyan line represents the Bragg limit, blue lines represent the healthy structure, black lines represent the cracked structure with a dimensionless depth $a = 0.2$ and the red lines represent the cracked structure with a dimensionless depth $a = 0.4$



(a) Dispersion Curves (b) Displacement

Figure 6. Dispersion Curves and displacement using model 1.

Figure 7 is shown as the variation of the depth of the crack influences in the dispersion curves and the displacement response for case the unit cell with different materials.



(a) Dispersion Curves (b) Displacement

Figure 7. Dispersion Curves and displacement using model 2

Where the cyan line represents the Bragg limit, blue lines represent the healthy structure, black lines represent the cracked structure with a dimensionless depth $a = 0.2$ and the red lines represent the cracked structure with a dimensionless depth $a = 0.4$. The formation of band gaps is observed in frequency ranges from $f_1 = 90 - 220Hz$, $f_2 = 270 - 420Hz$ and also at $f_3 = 600Hz$ for the healthy case. With the presence of the crack the band gaps were in $f_1 = 40 - 130Hz$, $f_2 = 140 - 420Hz$, $f_3 = 430Hz$ and $f_4 = 490 - 600Hz$ for the cracked structure with a dimensionless depth $a = 0.4$. Note that the presence of crack causes the presence of band gaps as observed in the analyzed examples.

4. CONCLUSION

Band gaps in 1D phononic crystal (PC) rods with periodic cracks was presented. A numerical methodology called wave spectral element (WSE), based on a combination of SEM and Floquet-Bloch's theorem was proposed as an engineering tool to calculate 1D phononic crystal (PC). Numerical simulations were made for PC rod made of a unitary cell. Two unit cell models were analyzed, one with the same material (model 1) and the other with different materials (model 2). One can observe through the dispersion curves and displacement the formation of band gaps due to crack and it was noticed that the bigger the depth of the crack the bigger the band gaps and the band gaps formation was also observed for the case of the unit cell with different materials. The structure was analyzed with free-free boundary conditions. Through the obtained results an analysis can be performed for the detection of cracks in phononic crystals through the dispersion graphs and the results of this work can be used to design systems with filtering properties for example.

5. ACKNOWLEDGEMENTS

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