25th ABCM International Congress of Mechanical Engineering
October 20-25, 2019, Uberlândia, MG, Brazil

COB-2019-0185
AN EFFICIENT METHOD FOR ESTIMATING AERODYNAMIC ANGLES FOR SMALL UNMANNED AIRCRAFT

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Abstract. *With technological advances, unmanned aerial vehicles (UAV) have become more popular for commercial, military and academic applications. Controlling this type of aircraft, airplanes in particular, requires real time information about the aerodynamic angles α and β (angles of attack and sideslip, respectively). For small unmanned aircraft, measuring these angles is usually impracticable, since measurement vanes are heavy and cumbersome. An alternative to direct aerodynamic angle measurement is estimation. This paper presents a computationally efficient method, based on sensor data and stability and control derivatives, for estimating α and β. Instead of using a Kalman filter to integrate the dynamic equations, the angles are calculated directly from the external forces – obtained from accelerometer data – by solving a linear system. This analytical solution eliminates the need for integration, filter tuning and knowledge of the initial conditions, allowing faster calculations, which is well suited for small embedded computers. An uncertainty analysis is also conducted, and expressions for the variances for the estimators are derived. In addition to the theoretical work, simulations are presented in order to provide a quantitative insight to the method. Although the method is focused on small UAVs, it can be employed for all types of airplanes.*

Keywords: *Aerodynamic angles, angle of attack, estimation, flight dynamics, unmanned aerial vehicles*

1. INTRODUCTION

Although not a new technology, unmanned aerial vehicles (UAVs) have become increasingly popular in the last decades. With improvements in control technology and decreases in components and basic systems costs, UAVs are being used in a wide range of applications, from military operations to academic research. Despite some reliability and regulations issues, the UAV market share is expected to drastically increase in the coming years. Improving guidance and control accuracy and safety is then of fundamental importance.

One of the main issues in aircraft control is to know, with an accepted level of accuracy, the aerodynamic angles α and β (angles of attack and sideslip, respectively). Since aerodynamic forces and moments are intimately connected to α and β values, these angles are critical to control the aircraft movements. Aerodynamic angles are also necessary to estimate wind velocity (Heller *et al.*, 2003), which can be useful to design load alleviation systems, or an auto pilot for atmospheric energy extraction, for example. Larger commercial aircraft usually have probes to measure α and β, but these sensors are usually heavy and cumbersome, and have particular operational issues, such as motion-induced errors and mechanical lags, for example (Zeis *et al.*, 1988).

There are several different approaches to estimating the angles of attack and sideslip without direct measurement. Bach and Parks (1987) propose a regression method based on least squares, which uses data from flight recorders. In this work, the method is employed in post-flight analyses. Zeis *et al.* (1988) present a more complete, non linear approach, also based on the aircraft dynamic response. However, the method employs more degrees of freedom, requiring a more complex inertial measurement unit (IMU), with gyroscopes capable of providing the Euler angles. More computational power is also required. Colgren *et al.* (1999) use a similar method, but it does not require a dynamic pressure probe, employing the inertial velocity instead. Such approach also requires a more complex IMU, and the stability and control derivatives.

Morelli (2012) proposes the estimation of α and β through integration, in the frequency domain, of approximate time derivatives. This approach is used for system identification, and the integration method employed mitigates the cumulative error effect, eliminating the need for a Kalman filter. However, it requires a relatively high airspeed so that the errors in the time derivatives remain small. It also requires knowledge of the initial conditions.

2. ESTIMATION METHOD

In this paper, an adaptation of the method presented in (Heller *et al.*, 2003) is proposed. This approach was developed by Barufaldi (2015) and is based on the direct calculation of the aerodynamic angles from the net force, which is obtained from accelerometer data. The adaptation consists on reformulating the equations and analytically solving a coupled linear system. Since integration is not employed, a Kalman filter is not required, nor is knowledge of the initial conditions.

However, the presented method does require the plant to be properly identified, i.e., the system parameters must be known *a priori*. The aircraft is assumed to be equipped with an inertial measurement unit (IMU) with accelerometers and gyros which, for the sake of simplicity, are positioned at the airplane center of gravity (CG). The maneuvers are assumed to be smooth enough so that the non stationary effects can be neglected (Drela and Youngren, 2010).

2.1 Estimation equations

The net force $\mathbf{F}_R$ acting on the aircraft is composed by the aerodynamic resultant $\mathbf{R}_A$, the weight $\mathbf{W}$ and the thrust $\mathbf{T}$, as shown in Eq. (1):

$$\mathbf{F}_R = \mathbf{R}_A + \mathbf{W} + \mathbf{T} \tag{1}$$

By writing all vectors on the body reference frame, we can rewrite Eq. (1) with its components, as shown in Eq. (2):

$$\begin{bmatrix} F_{R_X} \\ F_{R_Y} \\ F_{R_Z} \end{bmatrix} = m \begin{bmatrix} a_{r_x} \\ a_{r_y} \\ a_{r_z} \end{bmatrix} = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} + mg \begin{bmatrix} -\sin\theta \\ \sin\phi\cos\theta \\ \cos\phi\cos\theta \end{bmatrix} + \begin{bmatrix} T \\ 0 \\ 0 \end{bmatrix} \tag{2}$$

where X, Y and Z are the components of $\mathbf{R}_A$; m is the aircraft mass; g is the acceleration of gravity; a_{r_x}, a_{r_y} and a_{r_z} are the components of the net acceleration with respect to the inertial frame; ϕ and θ are the Euler angles between the body and the inertial frames; and T is the thrust value. Rearranging the terms yields:

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = m \begin{bmatrix} a_{r_x} \\ a_{r_y} \\ a_{r_z} \end{bmatrix} + mg \begin{bmatrix} \sin\theta - {}^{T}\!/_{mg} \\ -\sin\phi\cos\theta \\ -\cos\phi\cos\theta \end{bmatrix} \tag{3}$$

According to Duke *et al.* (1988), the accelerometers measurements a_x, a_y and a_z are related to the components of the net acceleration by Eq. (4):

$$\begin{bmatrix} a_{r_x} \\ a_{r_y} \\ a_{r_z} \end{bmatrix} = \begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix} + g \begin{bmatrix} -\sin\theta \\ \sin\phi\cos\theta \\ \cos\phi\cos\theta \end{bmatrix} \tag{4}$$

Substituting the results of Eq. (4) into Eq. (3) yields:

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = m \begin{bmatrix} a_x - {}^{T}\!/_{m} \\ a_y \\ a_z \end{bmatrix} \tag{5}$$

The aerodynamic components can be expressed in terms of the dynamic pressure $\bar{q}$, the wing reference area S and their respective dimensionless coefficients C_X, C_Y and C_Z, as shown in Eq. (6):

$$[X \;\; Y \;\; Z]^T = \bar{q}S\,[C_X \;\; C_Y \;\; C_Z]^T \tag{6}$$

There are different models for the dimensionless coefficients C_X, C_Y and C_Z, such as in (Mulder, 1986) or (Etkin and Reid, 1995). In this work, we neglect the non stationary effects, because of the smooth manoeuvres assumption. A linear model is employed, as shown in Eq. (7):

$$\begin{bmatrix} C_X \\ C_Y \\ C_Z \end{bmatrix} = \begin{bmatrix} C_{X_\alpha} & C_{X_\beta} & C_{X_p} & C_{X_q} & C_{X_r} & C_{X_{\delta e}} & C_{X_{\delta r}} \\ C_{Y_\alpha} & C_{Y_\beta} & C_{Y_p} & C_{Y_q} & C_{Y_r} & C_{Y_{\delta e}} & C_{Y_{\delta r}} \\ C_{Z_\alpha} & C_{Z_\beta} & C_{Z_p} & C_{Z_q} & C_{Z_r} & C_{Z_{\delta e}} & C_{Z_{\delta r}} \end{bmatrix} \begin{bmatrix} \alpha & \beta & \tilde{p} & \tilde{q} & \tilde{r} & \delta e & \delta r \end{bmatrix}^T + \begin{bmatrix} C_{X_0} \\ C_{Y_0} \\ C_{Z_0} \end{bmatrix} \tag{7}$$

where $\tilde{p}$, $\tilde{q}$ and $\tilde{r}$ are the dimensionless angular rates written on the body reference frame; δe and δr are the elevator and rudder angular deflections, respectively; and C_{X_i}, C_{Y_i} e C_{Z_i} are the stability and control derivatives expressed in the body reference frame.

Substituting the results of Eq. (7) into their respective components in Eq. (6), and then into Eq. (5) yields:

$$C_{Z_0} + C_{Z_\alpha}\alpha + C_{Z_\beta}\beta + C_{Z_p}\tilde{p} + C_{Z_q}\tilde{q} + C_{Z_r}\tilde{r} + C_{Z_{\delta e}}\delta e + C_{Z_{\delta r}}\delta r = \frac{ma_z}{\bar{q}S} \tag{8}$$

$$C_{Y_0} + C_{Y_\alpha}\alpha + C_{Y_\beta}\beta + C_{Y_p}\tilde{p} + C_{Y_q}\tilde{q} + C_{Y_r}\tilde{r} + C_{Y_{\delta e}}\delta e + C_{Y_{\delta r}}\delta r = \frac{ma_y}{\bar{q}S} \tag{9}$$

Equations (8) and (9) form a linear system with two unknowns: α and β. In the system, $\tilde{p}$, $\tilde{q}$, $\tilde{r}$, a_y and a_z can be obtained from IMU data (gyros and accelerometers); $\bar{q}$ is measured with a pitot-static probe; δe and δr can be directly measured with proper sensors, e.g. potentiometers. The stability and control derivatives are known, since the plant is assumed to be properly identified. Hence, rearranging Eqs. (8) and (9) in matrix form yields:

$$\begin{bmatrix} C_{Z_\alpha} & C_{Z_\beta} \\ C_{Y_\alpha} & C_{Y_\beta} \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \begin{bmatrix} C_{11} + C_{12} \\ C_{21} + C_{22} \end{bmatrix} \tag{10}$$

where coefficients C_{ij} are described by Eqs. (11) to (14).

$$C_{11} = \frac{ma_z}{\bar{q}S} \tag{11}$$

$$C_{21} = \frac{ma_y}{\bar{q}S} \tag{12}$$

$$C_{12} = -\left(C_{Z_0} + C_{Z_p}\tilde{p} + C_{Z_q}\tilde{q} + C_{Z_r}\tilde{r} + C_{Z_{\delta e}}\delta e + C_{Z_{\delta r}}\delta r\right) \tag{13}$$

$$C_{22} = -\left(C_{Y_0} + C_{Y_p}\tilde{p} + C_{Y_q}\tilde{q} + C_{Y_r}\tilde{r} + C_{Y_{\delta e}}\delta e + C_{Y_{\delta r}}\delta r\right) \tag{14}$$

Solving the system in Eq. (10) yields the analytical expressions for the estimators $\hat{\alpha}$ and $\hat{\beta}$:

$$\hat{\alpha} = \frac{C_{Y_\beta}\left(C_{11} + C_{12}\right) - C_{Z_\beta}\left(C_{21} + C_{22}\right)}{C_{Z_\alpha}C_{Y_\beta} - C_{Z_\beta}C_{Y_\alpha}} \tag{15}$$

$$\hat{\beta} = \frac{C_{Z_\alpha}\left(C_{21} + C_{22}\right) - C_{Y_\alpha}\left(C_{11} + C_{12}\right)}{C_{Z_\alpha}C_{Y_\beta} - C_{Z_\beta}C_{Y_\alpha}} \tag{16}$$

2.2 Propagation of errors

The variances of the estimators can be calculated in an approximate way, at each instant, as a function of the state variables (Ku, 1966), as shown in Eq. (17) in which f is a non linear function of the variables x_1,...,x_n, each of which having a variance σ_i^2.

$$\sigma_f^2 \approx \sum_{i=1}^{n} \left(\frac{\partial f}{\partial x_i}\right)^2 \sigma_i^2 \tag{17}$$

Thus, it is possible to write the variance of $\hat{\alpha}$ such as in Eq. (18):

$$\sigma_{\hat{\alpha}}^2 = \left(\frac{\partial\hat{\alpha}}{\partial a_y}\right)^2 \sigma_{a_y}^2 + \left(\frac{\partial\hat{\alpha}}{\partial a_z}\right)^2 \sigma_{a_z}^2 + \left(\frac{\partial\hat{\alpha}}{\partial p}\right)^2 \sigma_p^2 + \left(\frac{\partial\hat{\alpha}}{\partial q}\right)^2 \sigma_q^2 + \left(\frac{\partial\hat{\alpha}}{\partial r}\right)^2 \sigma_r^2 + \left(\frac{\partial\hat{\alpha}}{\partial \bar{q}}\right)^2 \sigma_{\bar{q}}^2 + ...$$
$$... + \left(\frac{\partial\hat{\alpha}}{\partial \delta e}\right)^2 \sigma_{\delta e}^2 + \left(\frac{\partial\hat{\alpha}}{\partial \delta r}\right)^2 \sigma_{\delta r}^2 \tag{18}$$

The partial derivatives of $\hat{\alpha}$ are shown in Eqs. (19) to (26):

$$\frac{\partial \hat{\alpha}}{\partial a_y} = -\frac{m C_{Z_\beta}}{\bar{q} S \left(C_{Z_\alpha} C_{Y_\beta} - C_{Z_\beta} C_{Y_\alpha} \right)} \tag{19}$$

$$\frac{\partial \hat{\alpha}}{\partial a_z} = \frac{m C_{Y_\beta}}{\bar{q} S \left(C_{Z_\alpha} C_{Y_\beta} - C_{Z_\beta} C_{Y_\alpha} \right)} \tag{20}$$

$$\frac{\partial \hat{\alpha}}{\partial p} = \frac{b \left(C_{Z_\beta} C_{Y_p} - C_{Y_\beta} C_{Z_p} \right)}{2V_r \left(C_{Z_\alpha} C_{Y_\beta} - C_{Z_\beta} C_{Y_\alpha} \right)} \tag{21}$$

$$\frac{\partial \hat{\alpha}}{\partial q} = \frac{\bar{c} \left(C_{Z_\beta} C_{Y_q} - C_{Y_\beta} C_{Z_q} \right)}{2V_r \left(C_{Z_\alpha} C_{Y_\beta} - C_{Z_\beta} C_{Y_\alpha} \right)} \tag{22}$$

$$\frac{\partial \hat{\alpha}}{\partial r} = \frac{b \left(C_{Z_\beta} C_{Y_r} - C_{Y_\beta} C_{Z_r} \right)}{2V_r \left(C_{Z_\alpha} C_{Y_\beta} - C_{Z_\beta} C_{Y_\alpha} \right)} \tag{23}$$

$$\frac{\partial \hat{\alpha}}{\partial \bar{q}} = \frac{m \left(C_{Z_\beta} a_y - C_{Y_\beta} a_z \right)}{\bar{q}^2 S \left(C_{Z_\alpha} C_{Y_\beta} - C_{Z_\beta} C_{Y_\alpha} \right)} \tag{24}$$

$$\frac{\partial \hat{\alpha}}{\partial \delta e} = \frac{C_{Z_\beta} C_{Y_{\delta e}} - C_{Y_\beta} C_{Z_{\delta e}}}{C_{Z_\alpha} C_{Y_\beta} - C_{Z_\beta} C_{Y_\alpha}} \tag{25}$$

$$\frac{\partial \hat{\alpha}}{\partial \delta r} = \frac{C_{Z_\beta} C_{Y_{\delta r}} - C_{Y_\beta} C_{Z_{\delta r}}}{C_{Z_\alpha} C_{Y_\beta} - C_{Z_\beta} C_{Y_\alpha}} \tag{26}$$

In Eqs. (21) to (23), p, q and r are the dimensional angular rates, such that $\tilde{p} = \frac{b}{2V_r} p$, $\tilde{q} = \frac{\bar{c}}{2V_r} q$ and $\tilde{r} = \frac{b}{2V_r} r$. V_r is the reference airspeed for which the stability derivatives were identified or calculated, $\bar{c}$ is the wing mean aerodynamic chord and b is the wingspan. In a similar way, the variance of $\hat{\beta}$ can be written such as in Eq. (27):

$$\sigma_{\hat{\beta}}^2 = \left(\frac{\partial \hat{\beta}}{\partial a_y} \right)^2 \sigma_{a_y}^2 + \left(\frac{\partial \hat{\beta}}{\partial a_z} \right)^2 \sigma_{a_z}^2 + \left(\frac{\partial \hat{\beta}}{\partial p} \right)^2 \sigma_p^2 + \left(\frac{\partial \hat{\beta}}{\partial q} \right)^2 \sigma_q^2 + \left(\frac{\partial \hat{\beta}}{\partial r} \right)^2 \sigma_r^2 + \left(\frac{\partial \hat{\beta}}{\partial \bar{q}} \right)^2 \sigma_{\bar{q}}^2 + ... \\ ... + \left(\frac{\partial \hat{\beta}}{\partial \delta e} \right)^2 \sigma_{\delta e}^2 + \left(\frac{\partial \hat{\beta}}{\partial \delta r} \right)^2 \sigma_{\delta r}^2 \tag{27}$$

The partial derivatives of $\hat{\beta}$ are shown in Eqs. (28) to (35):

$$\frac{\partial \hat{\beta}}{\partial a_y} = \frac{m C_{Z_\alpha}}{\bar{q} S \left(C_{Z_\alpha} C_{Y_\beta} - C_{Z_\beta} C_{Y_\alpha} \right)} \tag{28}$$

$$\frac{\partial \hat{\beta}}{\partial a_z} = -\frac{m C_{Y_\alpha}}{\bar{q} S \left(C_{Z_\alpha} C_{Y_\beta} - C_{Z_\beta} C_{Y_\alpha} \right)} \tag{29}$$

$$\frac{\partial \hat{\beta}}{\partial p} = \frac{b \left(C_{Z_p} C_{Y_\alpha} - C_{Y_p} C_{Z_\alpha} \right)}{2V_r \left(C_{Z_\alpha} C_{Y_\beta} - C_{Z_\beta} C_{Y_\alpha} \right)} \tag{30}$$

$$\frac{\partial \hat{\beta}}{\partial q} = \frac{\bar{c}\left(C_{Z_q} C_{Y_\alpha} - C_{Y_q} C_{Z_\alpha}\right)}{2V_r\left(C_{Z_\alpha} C_{Y_\beta} - C_{Z_\beta} C_{Y_\alpha}\right)} \tag{31}$$

$$\frac{\partial \hat{\beta}}{\partial r} = \frac{b\left(C_{Z_r} C_{Y_\alpha} - C_{Y_r} C_{Z_\alpha}\right)}{2V_r\left(C_{Z_\alpha} C_{Y_\beta} - C_{Z_\beta} C_{Y_\alpha}\right)} \tag{32}$$

$$\frac{\partial \hat{\beta}}{\partial \bar{q}} = \frac{m\left(C_{Y_\alpha} a_z - C_{Z_\alpha} a_y\right)}{\bar{q}^2 S\left(C_{Z_\alpha} C_{Y_\beta} - C_{Z_\beta} C_{Y_\alpha}\right)} \tag{33}$$

$$\frac{\partial \hat{\beta}}{\partial \delta e} = \frac{C_{Z_{\delta e}} C_{Y_\alpha} - C_{Y_{\delta e}} C_{Z_\alpha}}{C_{Z_\alpha} C_{Y_\beta} - C_{Z_\beta} C_{Y_\alpha}} \tag{34}$$

$$\frac{\partial \hat{\beta}}{\partial \delta r} = \frac{C_{Z_{\delta r}} C_{Y_\alpha} - C_{Y_{\delta r}} C_{Z_\alpha}}{C_{Z_\alpha} C_{Y_\beta} - C_{Z_\beta} C_{Y_\alpha}} \tag{35}$$

3. RESULTS AND DISCUSSION

To test the method, a small fixed-wing UAV was modeled in MATLAB™ , and the non linear equations of movement were integrated with Simulink™ . Sensors were simulated by adding white gaussian noise to system outputs, without bias – in order to clearly illustrate the effects of noise, the sensors were simulated as being quite noisy. The response to a doublet input on the elevator is shown in Fig. 1, where the actual and the estimated angles are presented. The estimation error and the variance are also exhibited.

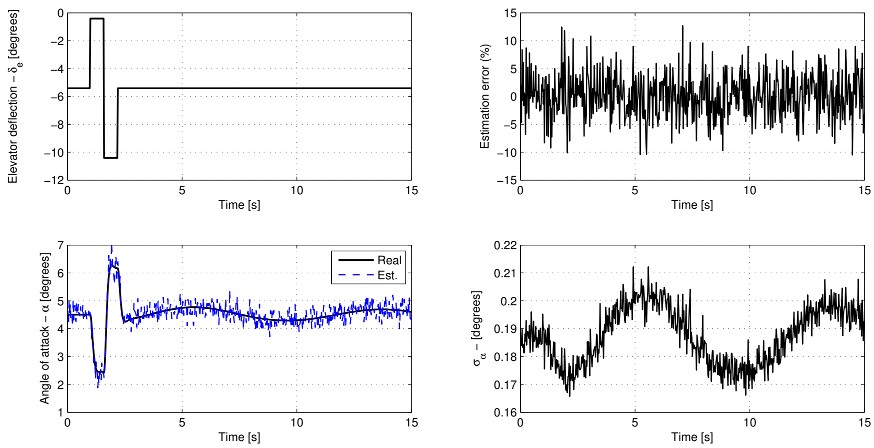

Figure 1. Dynamic response of α to a doublet input on the elevator. Estimated values with noise.

Details of the actual and estimated angles, with noise effect are shown in Fig. 2. The actual value of α is shown by the black solid line. The dashed lines are the result of adding and subtracting the standard deviation to the actual value, for each time t. An "envelope" around the actual value is formed, as can be seen when the noisy estimated value is superposed. This "envelope" gives an idea of the dispersion of estimated data around the actual values.

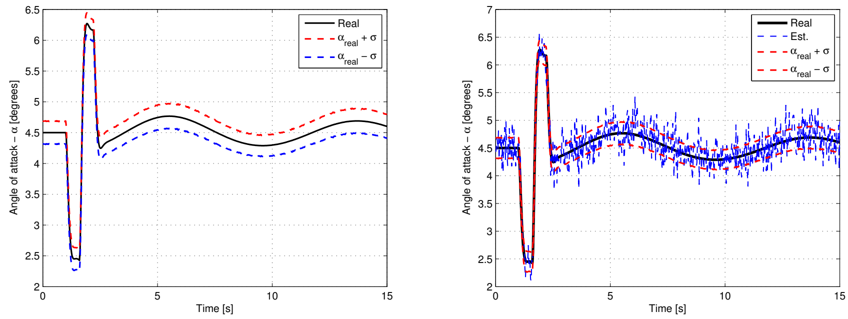

Figure 2. Estimation of α: detail of the "envelope" around the actual value, obtained by superposing the estimation of the standard deviation σ_α. The "envelope" gives an idea of the dispersion of estimated data around the actual values.

The response to a doublet input on the rudder is shown in Fig. 3, where the actual and the estimated angles are presented. The estimation error and the variance are also exhibited.

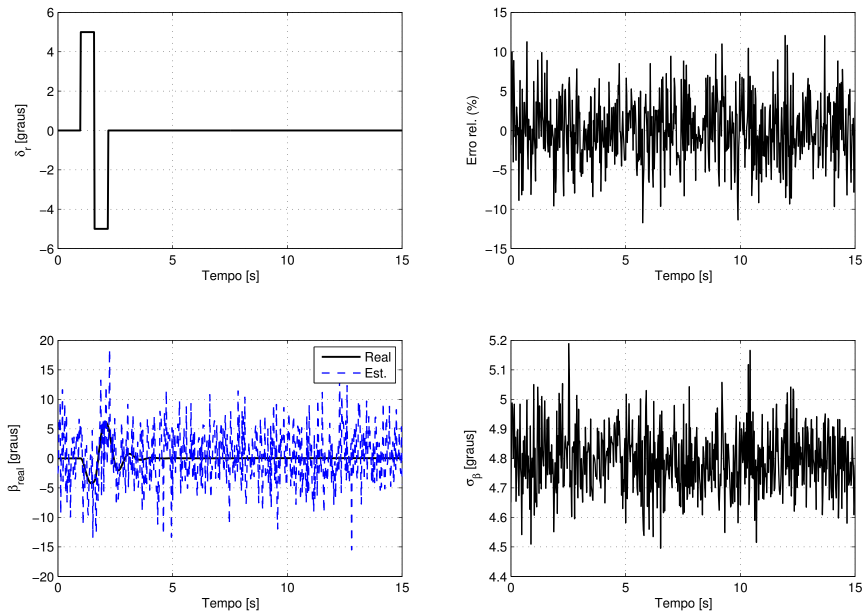

Figure 3. Dynamic response of β to a doublet input on the rudder. Estimated values with noise.

Details of the actual and estimated angles, with noise effect are shown in Fig. 4. The actual value of β is shown by the black solid line. The dashed lines are the result of adding and subtracting the standard deviation to the actual value, for each time t. Again, the "envelope" formed gives an idea of the dispersion of estimated data around the actual values. One should notice that the estimated values of β are considerably noisier than those of α. This happens due to the estimator expression in Eq. (16), in which the combination of terms results in noisier output, with respect to the α estimator.

4. CONCLUSIONS

This work has presented the formulation of a method for estimation of the aerodynamic angles of an aircraft: α and β. The angles are directly computed from estimators derived from the aircraft movement equations (Newton's Second Law), and do not require integration, Kalman filtering or knowledge of the initial conditions. This makes the method computationally efficient and well suited for small on-board computers. Estimators for the variances of α and β have also been derived. The variances provide a way of evaluating the quality of estimation and the dispersion of estimated data. Simulations were carried out in order to provide a quantitative insight to the method. The simulation results show that noise has a significant impact on the estimation process. It was also shown that the estimation of β is significantly noisier than α. For future works, filtering methods could be tested, as a means to reduce the the estimation noise. Another suggestion for future contributions could be the inclusion of non stationary effects, in order to evaluate the results of the estimation process in non-smooth manoeuvres.

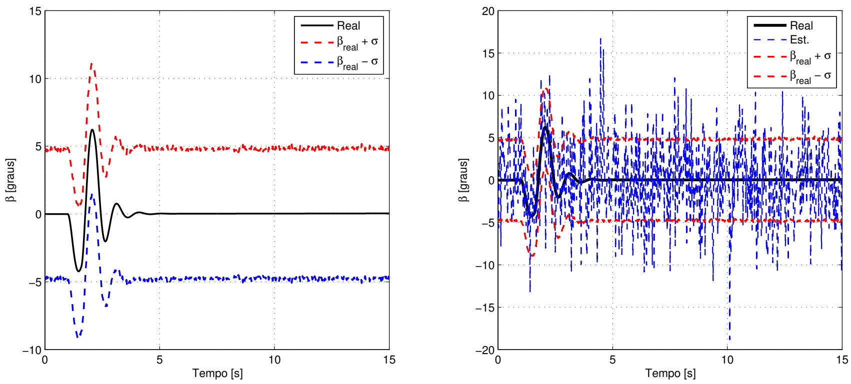

Figure 4. Estimation of β: detail of the "envelope" around the actual value, obtained by superposing the estimation of the standard deviation σ_β.

5. ACKNOWLEDGEMENTS

This study was financed in part by the Coordenação de Aperfeiçoamento de Pessoal de Nível Superior - Brasil (CAPES) - Finance Code 001, under the research grant number 88882.180826/2018-01.

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