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ANALYSIS AND ELIMINATION OF REDUNDANT CONSTRAINTS IN CLAMPING DEVICES

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Abstract. *This work presents a methodology to analyze redundant constraints in clamping devices. These important devices are largely used in manufacturing processes, however the existing devices may present redundant constraints which may hinder the manufacturing or assembly of these devices. Hence, this work presents a method to evaluate and eliminate redundant constraints in clamping devices. These devices are introduced and their reconfigurability discussed. A model for the friction constraints between the clamping device and the fixed object is presented. Finally, a case study is presented.*

Keywords: *Clamping devices, Mechanisms, Overconstraints, Matroid theory*

1. INTRODUCTION

Clamping devices are employed in several manufacturing processes, specially in processes that great forces or pressures must be absorbed, such as machining processes and injection moulding. Clamping is a fixture device employed to hold a piece during a manufacturing process. Nowadays there are several kinds of clampings applied in engineering field. Costa *et al.* (2017) reviewed many types of clamping devices and classified them according to the mechanism structures, considering the mobility and the number of loops.

Clamping devices have at least two operation modes, opened and closed modes, therefore clampings can be considered as reconfigurable mechanisms, according to the reconfigurable principles listed by Kuo *et al.* (2009). During the opened mode, the clamping must have the freedoms to allow closing and adjustment, although not all commercial clampings have the adjustment freedom. While in closed mode, the clamping must fix the piece disallowing any displacement in relation to the fixed link. The forces that the clamping device imposes in the fixed piece must be enough to fix the piece in the clamping structure without deforming the piece.

Normally, the mechanisms of clamping devices have redundant constraints, which are those constraints whose elimination would not increase the mobility of the mechanism (Reshetov, 1982). Mechanisms without redundant constraints are classified as self-aligning mechanisms. Although this class has special features, self-aligning mechanisms are obscure to the major part of designers (Blanding, 1999). The main strengths of self-aligning mechanisms are related to the assembling processes. A closed self-aligning mechanism can be assembled following any sequence of parts and when the last part must be assembled, normally the most critical stage of assembly, the assembled structure provides the necessary degrees of freedom to assemble the last part. While mechanisms with redundant constraints must have greater manufacturing accuracy to allow the alignment of the parts to assemble, thus in case of problems some parts are discarded. In addition to facilitating assembly, self-aligning mechanisms can reduce manufacturing costs, as they are able to decrease the required accuracy, fabrication and assembly time, and losses by bad manufacturing.

Based on this, the present work models statically a commercial clamping device and uses a method to eliminate the redundant constraints on similar devices. Section 2 presents the clamping device KIPP K0660, it is the case study of this work. The operation and structure of the device is discussed, as well as the friction constraints. Section 3 reviews Davies'

method and apply it to the case study. Section 4 uses a method to analyze redundant constraints of the clamping device studied, enumerate all the mechanisms without redundant constraints and select a set of feasible mechanisms. Section 5 presents and discusses the results obtained from the case study.

2. CLAMPING DEVICE KIPP K0660

The clamping device used in this work is the toggle clamping device called KIPP 0660, shown in Fig. 1. The KIPP 0660 is a simple toggle clamping widely used in the manufacturing industry, specially in plastic injection molding and woodworks. The device has a spindle to adjust the clamping for different piece thickness. The device also has an internal lock based on singularity which protects the system from opening while fixing pieces.

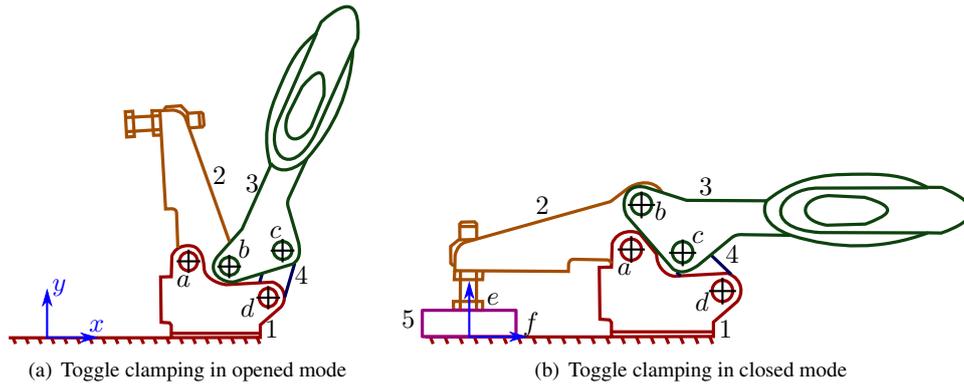


Figure 1: Two operation modes of a toggle clamping.

The toggle clamping device has two modes of operation: the opened and the closed modes. Fig. 1(a) shows the clamping in the first mode while Fig. 1(b) shows the clamping device in the closed mode. During the opened mode the clamping is not fixing any piece, and the kinematic chain is related to a four-bar mechanism (Fig. 2(a)). This kinematic chain has four joints, four links and one mobility. It is important to consider the mobility of the kinematic chain in this stage during the elimination of the redundant constraints.

During the closed mode the clamping is holding a piece (Fig. 1(b)). The piece is positioned above a surface such as a table, normally it is the same surface on which the clamping device is fixed. The contact between the piece and the table is considered as a planar pair, this contact is labelled by the letter f in Fig. 1(b). The contact between the piece and the clamping is made by the interaction between the link 2 and the piece. This contact is considered as a planar pair and is labeled by the letter e in Fig. 1(a). The kinematic chain during the closed stage has six joints, five links and two loops, Fig 2(b). The mobility of this kinematic chain must be equal to zero to the clamping fulfill its objective, to hold a piece. Although the mobility of the kinematic chains seems correct, when we consider the actual mechanism with constraints, the mobility may alter due to the presence of redundant constraints. The mobility of the clamping considering every constraint present will be discussed in Section 3. Before this discussion, it is important to examine the friction constraints existing between the clamping device and the object held by it.

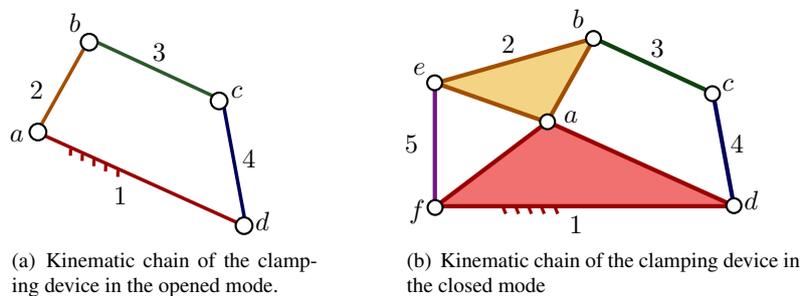


Figure 2: Kinematic chains of the clamping device in both operating modes.

2.1 FRICTION CONSTRAINTS

In this section the friction constraints between the gripper of the clamping device and the free object are reviewed. When the clamping device is in the closed mode, the device is holding an object. This interaction leads to the generation of constraints between the clamping device and the fixed object. These constraints can be divided into two groups: contact and friction forces and moments. These constraints are shown in Fig. 3.

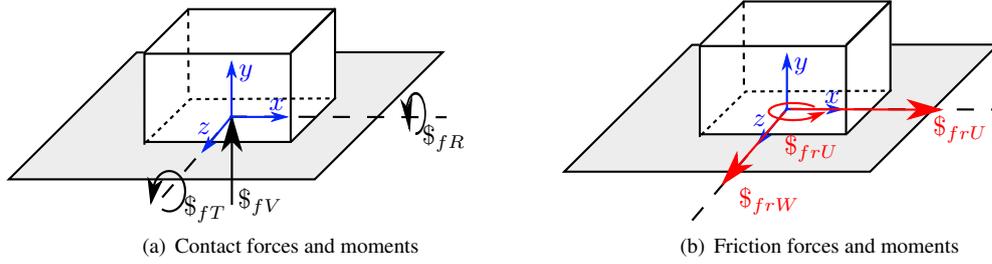


Figure 3: Constraints between the gripper and the fixed body.

The contact forces, Fig. 3(a), are due to the surface interaction between the object and table that creates a planar joints. The three constraints are a force along the y -axis and two moments around the axes x and z , respectively. These constraints are those that hinder three movements of the object. The friction constraints are the constraints that hinder the movement of the fixed body by means of friction, *i.e.*, that prevents movement of the fixed body in the directions that there is not any body or surface preventing that movement. The force along the y -axis created by the joint e and the contact between the table and object creates three friction constraints, Fig. 3(b). These constraints are two forces along the axes x and z , and a moment around the axis y . These six constraints are necessary to fix the object completely in relation to the table.

The friction forces and moment are proportional to the perpendicular force in relation to the table imposed by the clamping on the object. The friction forces and moment are not always imposed to the system, they are limit values of the reaction forces imposed by the machining forces. So, as higher the machining forces are, higher frictional forces are necessary, otherwise the object will move in relation to the table. When evaluating the constraints of a clamping device, both contact and friction constraints need to be considered.

3. DAVIES' METHOD AND THE STATIC MODEL OF THE KIPP K0660

In this work, we create a static model of the clamping device KIPP K0660 using Davies' Method. This method is based on *Graph theory*, *Screw theory* and *Kirchoff's cut-set law* and has been used to model several types of systems from humanoid robots (Toscano *et al.* (2018)) to hospital beds (Artmann *et al.* (2019)) and long combination vehicles (Moreno *et al.* (2018)).

Here, each constraint j from the coupling i is modelled as a wrench $\$_{i,j}$. The subscript j is related to the mode of constraint, thus if j is equal to R , S or T , the constraint is a moment around the axis x , y or z , respectively. But, if j is equal to U , V or W , the constraint is related to a force along the axis x , y or z , respectively.

All constraints of each joint of the mechanism are organized into the unit action matrix $[\hat{\mathbf{A}}_{\mathbf{D}}]_{\lambda,C}$, where C is the number of constraints and λ is the order of the screw system of the mechanism, which was considered $\lambda = 6$ in this work. The matrix $[\hat{\mathbf{A}}_{\mathbf{D}}]_{6,29}$ related to the clamping KIPP 0660 is organized. This system has 29 constraints, where 26 are contact constraints and three are friction constraints. The matrix $[\hat{\mathbf{A}}_{\mathbf{D}}]_{6,29}$ is shown in Eq. 1:

$$[\hat{\mathbf{A}}_{\mathbf{D}}]_{6,29} = \begin{bmatrix} \overbrace{\$_{aR} \ \$_{aS} \ \$_{aU} \ \$_{aV} \ \$_{aW}}^{\text{Coupling a}} & \overbrace{\$_{bR} \ \$_{bS} \ \$_{bU} \ \$_{bV} \ \$_{bW}}^{\text{Coupling b}} & \overbrace{\$_{cR} \ \$_{cS} \ \$_{cU} \ \$_{cV} \ \$_{cW}}^{\text{Coupling c}} & \dots \\ \dots & \overbrace{\$_{dR} \ \$_{dS} \ \$_{dU} \ \$_{dV} \ \$_{dW}}^{\text{Coupling d}} & \overbrace{\$_{eR} \ \$_{eT} \ \$_{eV}}^{\text{Coupling e}} & \overbrace{\$_{fR} \ \$_{fT} \ \$_{fV}}^{\text{Coupling f}} & \overbrace{\$_{frS} \ \$_{frU} \ \$_{frW}}^{\text{Friction Constraints}} \end{bmatrix} \quad (1)$$

The kinematic chain from Fig. 2(b) is transformed into a graph, where the constraint joints are now represented by edges and links are represented by vertices. The edges are the constraints belonging to each coupling. For example, the edge corresponding to a revolute joint around the z -axis will have five edges related to three forces (x , y and z axis) and two moments (x and y axis). This graph is called of action graph. The action graph related to the clamping K0660 has 29 edges distributed in six joints, this action graph is shown in Fig. 4

The joints a , b , c and d have five constraints, all of which are related to the constraints of a revolute pair. Joints e and f have three constraints each, related to the constraints of a planar pair. Figure 4(a) shows the action graph related to these contact constraints. However, the contact between the object (link 5) and table (link 1) generates three more friction constraints. Figure 4(b) shows how these constraints are disposed into the action graph.

In Figure 4, the cut-sets from the action graph are shown in red. These cut-sets are used to create the cut-set matrix $[\mathbf{Q}_{\mathbf{A}}]_{k,C}$ (Davies (2006)). Using the matrices $[\mathbf{A}_{\mathbf{D}}]_{\lambda,C}$ and $[\mathbf{Q}_{\mathbf{A}}]_{k,C}$, we create the network unit action matrix $[\hat{\mathbf{A}}_{\mathbf{N}}]_{\lambda,C}$. Eq. 2 shows how the matrix $[\hat{\mathbf{A}}_{\mathbf{N}}]_{k,\lambda,C}$ is arranged.

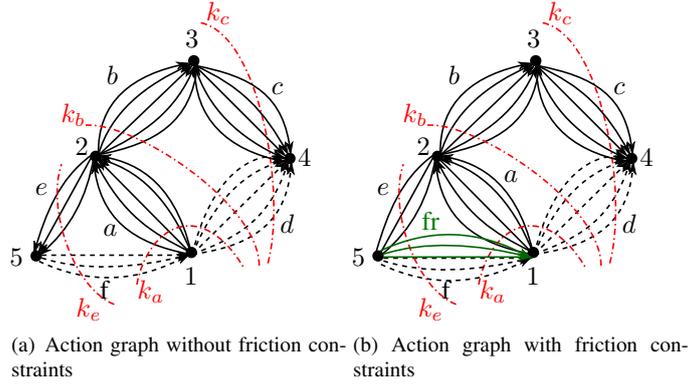


Figure 4: Action graph of the K0660 clamping device in the closed mode.

$$[\hat{\mathbf{A}}_{\mathbf{N}}]_{\lambda k, C} = \begin{bmatrix} [\hat{\mathbf{A}}_{\mathbf{D}}]_{(\lambda, C)} [\mathbf{Q}_1]_{C, C} \\ [\hat{\mathbf{A}}_{\mathbf{D}}]_{(\lambda, C)} [\mathbf{Q}_2]_{C, C} \\ \vdots \\ [\hat{\mathbf{A}}_{\mathbf{D}}]_{(\lambda, C)} [\mathbf{Q}_k]_{C, C} \end{bmatrix}_{\lambda k, C} \quad (2)$$

In Eq. 2, $[\mathbf{Q}_i]_{C, C}$, $i = 1, 2, \dots, k$ are diagonal matrices created from the i row of matrix $[\mathbf{Q}_{\mathbf{A}}]_{k, C}$, and k is the number of cut-sets of the action graph. The action graph of the clamping device KIPP0660 has four cut-sets, Fig 4(b). In this way, the matrix $[\mathbf{Q}_{\mathbf{A}}]_{4, 29}$ has four lines and 29 columns related to 29 constraints. So, applying the matrices $[\hat{\mathbf{A}}_{\mathbf{D}}]_{6, 29}$ and $[\mathbf{Q}_{\mathbf{A}}]_{4, 29}$ to Eq. 2 the matrix $[\hat{\mathbf{A}}_{\mathbf{N}}]_{4 \times 6, 29}$ is arranged. Using matrix $[\hat{\mathbf{A}}_{\mathbf{N}}]_{\lambda k, C}$ in the fundamental cut-set law, we can solve the static model of the clamping device, Eq. 3.

$$[\hat{\mathbf{A}}_{\mathbf{N}}]_{\lambda k, C} [\Psi]_C = [0]_{\lambda k} \quad (3)$$

In Eq. 3, Ψ is the action screws with unknown magnitudes imposed by the couplings. The number of redundant constraints C_N present in the mechanism can be calculated by the difference between the number of constraints C and the rank of matrix $[\hat{\mathbf{A}}_{\mathbf{N}}]_{\lambda k, C}$, as shown in Eq. 4.

$$C_N = C - \text{rank}([\hat{\mathbf{A}}_{\mathbf{N}}]_{\lambda k, C}) \quad (4)$$

The matrix $[\hat{\mathbf{A}}_{\mathbf{N}}]_{4 \times 6, 29}$ has 29 columns and rank equal to 24, the number of redundant constraints is evaluated applying these value to Eq. 4. In this case the clamping K0660 during the closed stage has five redundant constraints, $C_N = 29 - 24 = 5$. With the number C_N of redundant constraints we can evaluate the mobility of the mechanisms by the *Modified Grübler-Kutzbach Criterion* (Huang *et al.* (2009)):

$$F_N = \lambda(n - j - 1) + \sum_{i=1}^j f_i + C_N \quad (5)$$

where F_N is the mobility of the system, n is the number of links, j is the number of joints and $\sum f_i$ is the sum of degrees of freedom of the joints. Applying the Eq. 5 to the clamping K0660 during the closed stage:

$$F_N = 6(5 - 6 - 1) + 10 + 5 \Rightarrow F_N = 3 \quad (6)$$

The sum of degrees of freedom of the joints to this case was considered as: $\sum f_i = f_a + f_b + f_c + f_d + f_e + f_f = 1 + 1 + 1 + 1 + 3 + 3 = 10$. Note that the friction constraints were not considered in this evaluation and the three degrees of freedom found are disabled by the friction constraints, so the system do not have mobility if the friction constraints are sufficient to react against the machining forces.

The clamping device KIPP K0660 has five redundant constraints. The elimination of these redundant constraints creates self-aligning mechanisms kinematically equivalent to the clamping K0660 (seed mechanism). The enumeration of new self-aligning clamping devices based on the KIPP K0660 will be discussed in the next section.

4. REDUNDANT CONSTRAINT ANALYSIS METHOD

The clamping device with redundant constraints KIPP K0660 was selected and analyzed by means of Davies' method in Section 3. The redundant constraints and mobility of the clamping device were discussed. Now, Matroid theory is employed to enumerate all possible self-aligning mechanisms, while the design requirements will be used to select a set of feasible self-aligning mechanisms. Fig. 5 presents a flowchart of the method employed in this work.

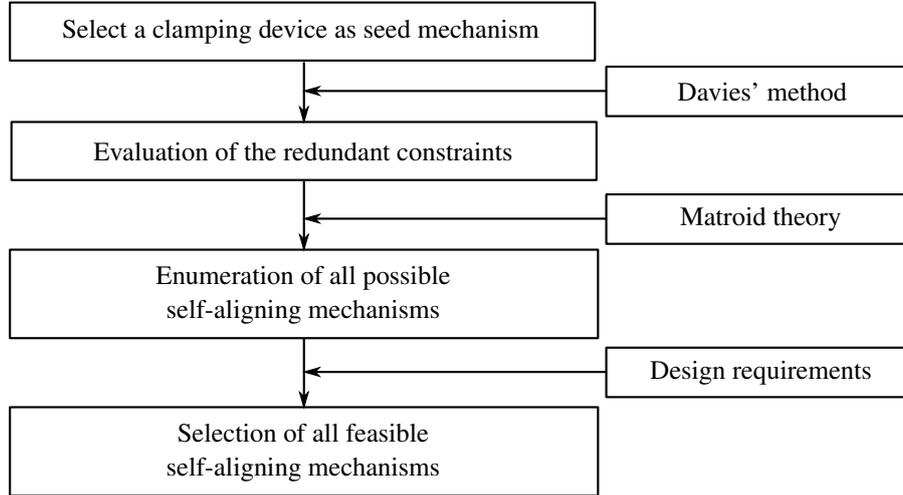


Figure 5: Type synthesis of self-aligning mechanisms.

This type synthesis method is summarized in the following steps: Using the network unit action matrix $[A_N]$ we can calculate all forces and moments present in the clamping device. However, we are more interested in the existence of redundant constraints in the device. This can be done by means of matroid theory (Carboni *et al.* (2017), Barreto *et al.* (2018)). A linear matroid \mathcal{M}_{A_N} is created on the real field \mathbb{R} defined by matrix $[A_N]$. The bases of the matroid \mathcal{M}_{A_N} are new self-aligning mechanisms created from the seed mechanism of the KIPP K0660 clamping device. A basis B of matroid \mathcal{M}_{A_N} represents a set of constraints which are linear independent among them. This set of constraints has the same number of the rank of matrix $[\hat{A}_N]$ (Carboni *et al.* (2017)). In the case of KIPP K0660 the bases are sets of 24 constraints which are linear independent among them. Each basis B has a cobasis B^* created from the complement of B . A cobasis B^* represents a set of redundant constraints. Eliminating the redundant constraints of a cobasis B^* of the seed mechanism, a new self-aligning mechanism is derived. For the KIPP K0660 the cobases are sets of five redundant constraints. The matroid \mathcal{M}_{A_N} has 1,510 cobases, hence 1,510 self-aligning mechanisms kinematically equivalent to the seed mechanism exist.

From the self-aligning mechanisms enumerated by matroid \mathcal{M}_{A_N} not all are viable to be manufactured. So, we now need to select a set to take further in the project design phases. We selected the set of self-aligning mechanisms using the method presented by Artmann *et al.* (2019), where the cobases of matroid \mathcal{M}_{A_N} are employed.

According to Artmann *et al.* (2019) the cobases are arranged into the Cobases Binary Matrix $[N]_{\mu,C}$, where μ is the number of cobases of matroid \mathcal{M}_{A_N} , and C is the number of constraints of the seed mechanism. The elements $n(i,j)$ of matrix $[N]_{\mu,C}$ are defined by:

$$n(i,j) = \begin{cases} 1 & \text{if the constraint } j \text{ is in the cobasis } i \\ 0 & \text{otherwise} \end{cases} \quad (7)$$

Before using the method, the designer needs to establish design requirements which are important during the type synthesis. With the design requirements decided, they are transformed in selection criteria to be applied to the matrix $[N]_{\mu,C}$. This procedure will deliver the sets of self-aligning mechanism which comply with the criteria. The intersection among these sets will create another set K_F :

$$K_F = K_1 \cap K_2 \cap \dots \cap K_\alpha \quad (8)$$

K_F corresponds to the set of feasible self-aligning mechanisms. The cobases present in K_F comply with all criteria, hence comply to all decided design requirements. In order to guide the replacement of constraints by degrees of freedom, three design requirements were defined for the clamping device KIPP K0660:

- (i) The friction constraints and the constraints relative from the contact between table and object (joint f) cannot be eliminated;

- (ii) The revolute joints (a), (b), (c) and (d) can be considered as spherical or cylindrical joints due to clearances;
- (iii) The contact between the clamping and object (joint e) can be transformed in a point pair.

The design requirement (i) was established because the six constraints, $\$_{fR}$, $\$_{fT}$, $\$_{fV}$, $\$_{frS}$, $\$_{frU}$ and $\$_{frW}$, are responsible to fix the object to the table. The design requirement (ii) was defined because allowing an axial or radial clearance, the revolute pair can be considered as a cylindrical or spherical pair, respectively. Fig 6 shows examples of these clearances.

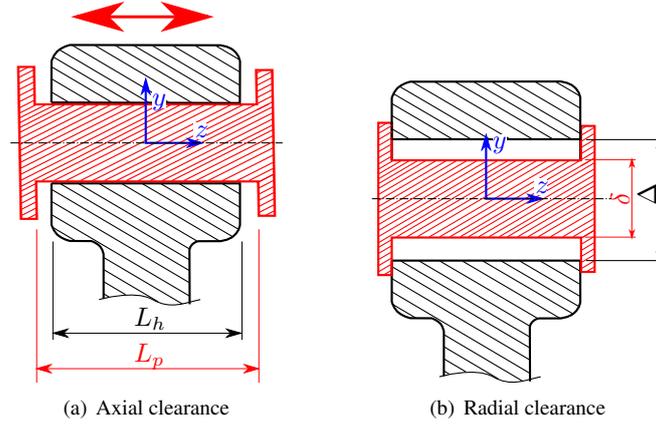


Figure 6: Clearances applied to the revolute joints. (a) Axial clearance which transforms the revolute in a cylindrical joints. (b) radial clearance which transforms the revolute in a spherical joints.

The kinematic chain of the clamping KIPP K0660 during closed mode has two loops, Fig 2(b). The loop 1 is present in both operation modes. The structure of loop 1 is related to a four-bar mechanism which has three redundant constraints Reshetov (1982), *i.e.* three constraint which can be removed without increasing the mobility of the four-bar mechanism. The entire structure has five redundant constraints, and loop 1 cannot have more than three constraints replaced by degrees of freedom. So two constraints of the others joints e and f must be replaced by degrees freedom. The design requirement (i) established that joint f cannot be modified, therefore two constraint must be replaced in joint e, now this joint, originally planar, must be converted in a point pair.

The three design requirements were converted in three selection criteria. Each selection criterion is then applied to the matrix $[N]_{1510,29}$ selecting the rows which the elements are complying with the criterion. Thus, the design requirement (i) was transformed in the following Criterion 1:

$$\{\forall i = 1, 2, \dots, 1510 | B_i^* \in K_1 \Leftrightarrow \{n(i, 24) = n(i, 25) = n(i, 26) = n(i, 27) = n(i, 28) = n(i, 29) = 0\}\} \quad (9)$$

In other words, the cobasis i will be selected to the subset K_1 only if the elements $n(i, 24)$, $n(i, 25)$, $n(i, 26)$, $n(i, 27)$, $n(i, 28)$ and $n(i, 29)$ are equal to zero. These elements are related to the constraints $\$_{fR}$, $\$_{fT}$, $\$_{fV}$, $\$_{frS}$, $\$_{frU}$ and $\$_{frW}$. In this way, the cobases selected by Criterion 1 do not have these elements, *i.e.* these constraints are still in the self-aligning mechanisms selected by Criterion 1. Table 1 and 2 shows the criteria 2 and 3 respective to design requirements (ii) and (iii).

Criterion 2 is related to the design requirement (ii), where the revolute joints a, b, c and d can be considered as a cylindrical or spherical pair, Fig 6. Joint a is originally a revolute joint represented by the wrenches $\$_{aR}$, $\$_{aS}$, $\$_{aU}$, $\$_{aV}$, $\$_{aW}$. The difference between a revolute pair and a cylindrical pair is the absence of the constraint of translation along the rotation axis of the cylindrical joint. For joint a, this constraint is represented by the wrench $\$_{aW}$. If only this wrench of joint a is present in the cobasis, this constraint was replaced by a degree of freedom, consequently the joint a was transformed into a cylindrical pair in the self-aligning mechanism represented by this cobasis. So, for a cobasis to be selected by this criterion, the elements of matrix $[N]_{1510,29}$ respective to the wrenches $\$_{aR}$, $\$_{aS}$, $\$_{aU}$, $\$_{aV}$ must be equal to zero and the element respective to $\$_{aW}$ must be equal to one.

Now, the difference between a revolute pair and a spherical pair is the absence of moment constraints in the spherical pair. The moment constraints of the revolute joint a are represented by the wrenches $\$_{aR}$, $\$_{aS}$. The revolute joint a was transformed into a spherical pair if only these wrenches of joint a are present in the cobasis. So the self-aligning mechanism represented by this cobasis has the revolute joint a replaced by a spherical pair. For a cobasis to be selected by this criterion, the elements of matrix $[N]_{1510,29}$ respective to the wrenches $\$_{aU}$, $\$_{aV}$, $\$_{aW}$ must be equal to zero and the element respective to the wrenches $\$_{aR}$, $\$_{aS}$ must be equal to one.

Table 1: Sets of constraints and respective binary conditions.

Criterion 2 - K_2			
Constraints	Condition	Constraints	Condition
$\{\$_{aR}, \$_{aS}, \$_{aU}, \$_{aV}\} \Rightarrow$	$n_{(i,j)} = 0$	$\{\$_{aW}\} \Rightarrow$	$n_{(i,j)} = 1$
OR			
$\{\$_{aU}, \$_{aV}, \$_{aW}\} \Rightarrow$	$n_{(i,j)} = 0$	$\{\$_{aR}, \$_{aS}\} \Rightarrow$	$n_{(i,j)} = 1$
OR			
$\{\$_{aR}, \$_{aS}, \$_{aU}, \$_{aV}, \$_{aW}\} \Rightarrow$	$n_{(i,j)} = 0$		
AND			
$\{\$_{bR}, \$_{bS}, \$_{bU}, \$_{bV}\} \Rightarrow$	$n_{(i,j)} = 0$	$\{\$_{bW}\} \Rightarrow$	$n_{(i,j)} = 1$
OR			
$\{\$_{bU}, \$_{bV}, \$_{bW}\} \Rightarrow$	$n_{(i,j)} = 0$	$\{\$_{bR}, \$_{bS}\} \Rightarrow$	$n_{(i,j)} = 1$
OR			
$\{\$_{bR}, \$_{bS}, \$_{bU}, \$_{bV}, \$_{bW}\} \Rightarrow$	$n_{(i,j)} = 0$		
AND			
$\{\$_{cR}, \$_{cS}, \$_{cU}, \$_{cV}\} \Rightarrow$	$n_{(i,j)} = 0$	$\{\$_{cW}\} \Rightarrow$	$n_{(i,j)} = 1$
OR			
$\{\$_{cU}, \$_{cV}, \$_{cW}\} \Rightarrow$	$n_{(i,j)} = 0$	$\{\$_{cR}, \$_{cS}\} \Rightarrow$	$n_{(i,j)} = 1$
OR			
$\{\$_{cR}, \$_{cS}, \$_{cU}, \$_{cV}, \$_{cW}\} \Rightarrow$	$n_{(i,j)} = 0$		
AND			
$\{\$_{dR}, \$_{dS}, \$_{dU}, \$_{dV}\} \Rightarrow$	$n_{(i,j)} = 0$	$\{\$_{dW}\} \Rightarrow$	$n_{(i,j)} = 1$
OR			
$\{\$_{dU}, \$_{dV}, \$_{dW}\} \Rightarrow$	$n_{(i,j)} = 0$	$\{\$_{dR}, \$_{dS}\} \Rightarrow$	$n_{(i,j)} = 1$
OR			
$\{\$_{dR}, \$_{dS}, \$_{dU}, \$_{dV}, \$_{dW}\} \Rightarrow$	$n_{(i,j)} = 0$		

The last possibility of Criterion 2 for joint a is that this joint is not modified *i.e.* no constraints are replaced by degrees of freedom. For a cobasis to be selected by this criterion, the elements of matrix $[N]_{1510,29}$ respective to the wrenches $\$_{aR}, \$_{aS}, \$_{aU}, \$_{aV}, \$_{aW}$ must be equal to zero. The same criteria are applied in parallel to joints b, c and d .

Table 2: Sets of constraints and respective binary conditions.

Criterion 3 - K_3			
Constraints	Condition	Constraints	Condition
$\{\$_{eV}\}$	$n_{(i,j)} = 0$	$\{\$_{eS}, \$_{eT}\}$	$n_{(i,j)} = 1$

Criterion 3 is related to the design requirement (iii), where the joint e must be transformed in a point pair. Joint e is represented by the wrenches $\$_{eV}, \$_{eS}, \$_{eT}$, while a point pair is represented by a singular constraint force, represented in this case by the wrench $\$_{eV}$. In this case, a cobasis will be select by Criterion 3 if the elements of matrix $[N]_{1510,29}$ respective to the wrench $\$_{eV}$ are equal to zero and the elements respective to the wrenches $\$_{eS}, \$_{eT}$ are equal to one. The three criteria were applied in the binary cobases matrix $[N]_{1510,29}$ and the results are discussed in next section.

5. RESULTS AND DISCUSSIONS

The selection criteria were transformed from the design requirements of KIPP K0660 and then applied to the binary cobases matrix $[N]_{1510,29}$. Criterion 1 selected 570 cobases, thus the subset K_1 represents 37,7% of entire set of cobases \mathcal{B}^* . The seed mechanism has 570 self-aligning mechanisms that maintain the constraints of friction and those which represent the contact between table and object. Remember that, this self-aligning mechanisms are derived from and kinematically equivalent to the seed mechanism.

Criterion 2 selected 108 cobases, thus the subset K_2 represents 7,1% of entire set of cobases \mathcal{B}^* . The seed mechanism has 108 self-aligning mechanisms that do not modify the revolute joints or transform it in cylindrical or spherical pair due to clearances. Criterion 3 selected 112 cobases, thus the subset K_3 represents 7,4% of entire set of cobases \mathcal{B}^* . The seed mechanism has 112 self-aligning mechanisms that transform the planar pair of joint e in a point pair.

The intersection between the three subsets creates the final subset K_F :

$$K_F = K_1 \cap K_2 \cap K_3 \quad (10)$$

this subset has twelve cobases. In other words, the seed mechanism has twelve respective self-aligning mechanisms that comply with the three design requirements established. The type of the joints of the selected self-aligning mechanisms were organized in Table 3, where the twelve mechanisms are listed and the type of the joints are represented by the letters C , Po , F , R . The letter C means that the joint of the mechanism is a cylindrical pair, Po means point pair, F means planar pair and R means revolute pair. The number of friction constraints presents in each selected self-aligning mechanism is also shown in the column of friction.

Table 3: Mechanisms selected and the joints.

Selected Self-aligning Mechanisms	Joints						Friction
	a	b	c	d	e	f	
Mechanism 1	C	R	R	S	F	Po	3
Mechanism 2	S	C	R	R	F	Po	3
Mechanism 3	S	R	C	R	F	Po	3
Mechanism 4	S	R	R	C	F	Po	3
Mechanism 5	C	S	R	R	F	Po	3
Mechanism 6	R	S	R	C	F	Po	3
Mechanism 7	C	R	S	R	F	Po	3
Mechanism 8	R	R	S	C	F	Po	3
Mechanism 9	R	S	C	R	F	Po	3
Mechanism 10	R	C	R	S	F	Po	3
Mechanism 11	R	R	S	C	F	Po	3
Mechanism 12	R	R	C	S	F	Po	3

Figure 7 presents an example of a new self-aligning mechanism created using the methodology proposed herein, this mechanism is the Mechanism 1 from Table 3. The mechanism in Fig. 7 has five constraints eliminated from four joints: a , d and e . Joint a had one constraints removed and became a cylindrical joint, while joint d had two constraints removed and became spherical joints. Finally, joint e also had two constraint removed, from a planar pair the joint became a point pair. The joints b , c and f remained with the same constraints of the seed mechanism.

This new self-aligning mechanism is kinematically identical to the KIPP K0660 clamping, but it do not have redundant constraints. It can facilitate the assembling of the clamping and also facilitate the fixing of the objects. The clamping has a structure of a four-bar mechanism while it is not fixing objects. The removal of the redundant constraints do not affected the operation of the clamping during the opened mode, seeing that three redundant constraint were removed from this loop that still has one degree of freedom.

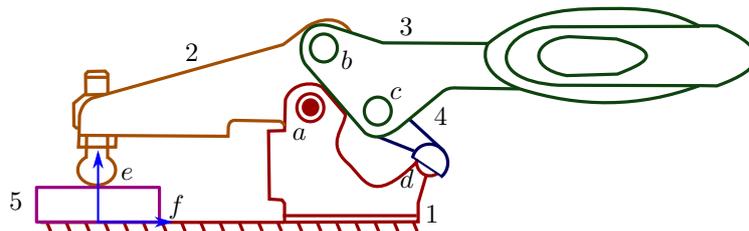


Figure 7: New self-aligning mechanism based on the KIPP K0660.

6. CONCLUSION

This work developed an analysis about the the clamping device KIPP K0660 aiming to evaluate and eliminate the redundant constraints of the mechanism. To achieve these tasks, Davies' method was applied to the clamping device from the constraint modelling by means of screw theory. The friction constraints were also considered and modeled. This mechanism has two operation modes and it presents five redundant constraints and zero degree of freedom during the closed mode. During the opened mode, the mechanism presents three redundant constraints and one degree of freedom. A matroid was created from the matrix $[\hat{A}_N]_{24,29}$ and it enumerated 1,510 cobases which are related to self-aligning mechanisms. Three design requirements were established to select a set of feasible self-aligning mechanisms. The design requirement were converted into selection criteria and were used to evaluate the cobases, where each selection criteria created a respective subset. The intersection among the three subsets created the final subset K_F . K_F is constituted by twelve cobases *i.e.* twelve viable self-aligning mechanisms according to the design requirements established.

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