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FORMATION CONTROL OF UNCERTAIN NONHOLONOMIC ROBOTS USING A ROBUST CONTROLLER

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Abstract. *This work addresses the formation control of nonholonomic robots using a robust controller. By dividing the robot model into dynamic and kinematic parts which has the nonholonomic constraints, the control strategy uses a cascaded scheme that has the signal of a kinematic tracking control as a reference for a robust dynamic tracking controller. The dynamic controller uses an extension of Active Disturbance Rejection Control (ADRC), called the modified ADRC, which is robust to external perturbations and parametric uncertainties. Simulations results confirm the efficiency of proposed control strategy.*

Keywords: *formation control, nonholonomic robots, robust control and cascaded strategy.*

1. INTRODUCTION

Formation control of multi-agent systems has received significant attention due to its wide variety of applications. In concern to formation control of mobile robots, the main objective is to control each agent using neighbor information in a decentralized control strategy. In this framework, most of the existing results deal with holonomic mobile robots (Pereira *et al.*, 2009; Tanner *et al.*, 2007). However, in practical applications, many mobile robots have to satisfy nonholonomic constraints, where the control design is quite involved. For the other hand, another challenger problem is the control of systems with uncertain dynamics parameters and under external disturbances(Li *et al.*, 2016), (Selfridge and Tao, 2016). The objective of this paper is the formation and tracking control of nonholonomic mobile robots using an extension of Active Disturbance Rejection Control (ADRC) (Han, 1998; Madoński *et al.*, 2015), called modified ADRC. The main characteristic of the ADRC method is that it possesses simpler implementation and has good robustness properties against external disturbance, unmodeled dynamics and parametric uncertainties. However, as can be seen from the recent works involving ADRC paradigm (Guo *et al.*, 2017; Xia *et al.*, 2018), the plant control gain is considered to be known. However, it is known that, in several practical applications, the control gain depends on the plant dynamic parameters. Then, the central idea of the modified ADRC is to introduce a constant gain block in series with the plant output and also a dynamical block in parallel with them, in order to generate a modified input/output plant with known control gain. Thus, with this structural change in the plant, only the knowledge of the plant control gain signal is necessary (Zachi *et al.*, 2019).

In order to consider the robots nonholonomic behavior and its dynamics, a cascaded control strategy is implemented along with the modified ADRC. For each agent, the cascaded strategy combines the modified ADRC, which takes into account the uncertain nonlinear dynamic, and a kinematic control, which takes into account the nonholonomic behavior, leading to an overall globally stable system. To verify effectiveness of the proposed formation control strategy, simulation results are included.

This paper is organized as follows: In Section 2, the basic method of ADRC is discussed. The Extended State Observer (ESO), used to estimate the uncertain dynamics of robots, is presented. The modified ADRC is discussed in Section 3. In Section 4, the formation control strategy is explained, where the cascaded control structure is presented. Finally, in Section 5, the results of the simulation and the final conclusions are shown.

2. The ADRC basic method

Consider the control design for a general class of n-th order dynamical systems (plants) described by

$$\begin{aligned} y^{(n)} &= f\big(Y, d(t), h(t)\big) + bu(t)\,, \\ Y(t) &= [y,\ \dot{y},\ \cdots,\ y^{(n-1)}]^T\,, \end{aligned} \tag{1}$$

in which $y(t) \in \mathbb{R}$ is the output variable, $u(t) \in \mathbb{R}$ is the input variable, $d(t) \in \mathbb{R}$ is an external disturbance, $Y \in \mathbb{R}^n$ is the system state vector, $b \in \mathbb{R}$ is a constant, that will be denoted by the input *control gain*, and $h(t) \in \mathbb{R}$ represents nonlinear function of the system. In this work, we use the notation $y^{(n)}$ to represent the n-th order time derivative of $y(t)$. The function $f(Y, d(t), h(t))$ in (1) is usually known in the literature as the plant *total disturbance* term. In order to simplify notation, we will henceforth represent the function $f(Y, d(t), h(t))$ by $f(t)$. The control objective is to force the plant output $y(t)$ to track a desired and bounded trajectory $y^*(t) \in \mathbb{R}$. From input/output point of view, the plant represented by (1) can be considered as a n-th order integrator system with an input $u(t)$, an output $y(t)$ and an input disturbance $f(t)$.

By defining the output error as

$$e(t) \ = \ y(t) \ - \ y^*(t)\,, \tag{2}$$

and assuming the availability of all plant signals and coefficients, a state-feedback control law $u(t)$ that would perform the desired output tracking, could be chosen as:

$$\begin{aligned} u(t) &= u^*(t) = \left(\frac{1}{b}\right)\left[-\lambda^T\sigma_p \ + \ y^{(n)*}\right]\,, \\ \lambda &:= [1,\ \lambda_{(n-1)}, \cdots, \lambda_0]^T\,, \\ \sigma_p &:= \left[f(t),\ e^{(n-1)}, \cdots, \dot{e},\ e\right]^T\,, \end{aligned} \tag{3}$$

where the real constants $\lambda_{(n-1)}, \cdots, \lambda_0$ are the coefficients of a n-th order monic and stable polynomial ($q > 0$) given by:

$$(s+q)^n = s^n + \lambda_{(n-1)}s^{(n-1)} + \cdots + \lambda_0\,. \tag{4}$$

In such ideal situation, the closed loop dynamics would results exponentially stable and governed by:

$$e^{(n)} + \sum_{k=0}^{(n-1)} \lambda_k e^{(k)} \ = \ 0\,. \tag{5}$$

However, in general applications, the total disturbance term $f(t)$ may involve some uncertain coefficients and/or may represent a combination of external disturbances, non measurable signals and/or unmodeled dynamics of the plant. In such cases, the control law (3) can not be directly computed. In order to overcome this drawback, an Extended State Observer (ESO) is designed to estimate the function $f(t)$. The basic procedure is to define the plant state vector from (1) including $f(t)$ as an additional state variable, that is:

$$X(t) = \begin{bmatrix} Y(t) \\ \\ f(t) \end{bmatrix} = \begin{bmatrix} y \\ \dot{y} \\ \vdots \\ y^{(n-1)} \\ f(t) \end{bmatrix}. \tag{6}$$

Assuming that $f(t)$ is differentiable, the plant state-space representation assumes the following form:

$$\begin{aligned} \dot{X} &= \underbrace{\begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix}}_{A} X + \underbrace{\begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ b \\ 0 \end{bmatrix}}_{B} u + \underbrace{\begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}}_{\Gamma} \dot{f}(t), \\ y &= \underbrace{[1\ 0\ \cdots\ 0]}_{C} X\,. \end{aligned} \tag{7}$$

Note that the pair (A, C) is always *observable*. Thus, assuming, initially, a known control gain b, it is possible to design a full-order ESO as follows:

$$\begin{cases} \dot{\hat{X}} = A\hat{X} + Bu + \bar{L}e_y \,, \\ \hat{y} = C\hat{X} \,, \end{cases} \tag{8}$$

where $\hat{X} \in \mathbb{R}^{(n+1)}$ represents the estimated state vector, $e_y := (y - \hat{y})$ is defined as the output estimation error and $\bar{L} = [\bar{L}_1 \; \bar{L}_2 \; \cdots \; \bar{L}_{(n+1)}]^T \in \mathbb{R}^{(n+1)}$ is the vector of the observer adjustable gains that are usually chosen as the coefficients of the stable polynomial

$$(s + w_0)^{(n+1)} = s^{(n+1)} + \bar{L}_1 s^n + \cdots + \bar{L}_{(n+1)} \,, \tag{9}$$

in which $s = -w_0$ $(w_0 > 0)$ is defined as the observer characteristic roots of multiplicity $(n+1)$. Then, a realizable version of the state feedback control law (3), according to (6) and (8), can be chosen as:

$$\begin{aligned} & u(t) = \left(\frac{1}{b}\right)\left[-\lambda^T \hat{\sigma} + y^{(n)*}\right] \,, \\ & \lambda := [1, \; \lambda_{(n-1)}, \; \cdots \,, \; \lambda_0]^T \,, \\ & \hat{\sigma} := \left[\hat{X}_{(n+1)}, \; \left(\hat{X}_{(n)} - y^{(n)*}\right), \; \cdots \,, \; (\hat{X}_1 - y^*)\right]^T \,. \end{aligned} \tag{10}$$

Defining the ESO state estimation error as

$$e_x = \begin{bmatrix} e_{x1} \\ e_{x2} \\ \vdots \\ e_{x(n+1)} \end{bmatrix} = \begin{bmatrix} y - \hat{X}_1 \\ \dot{y} - \hat{X}_2 \\ \vdots \\ f(t) - \hat{X}_{(n+1)} \end{bmatrix} \,, \tag{11}$$

then, from (35) and (8), its dynamics yields

$$\begin{aligned} & \dot{e}_x = \underbrace{(A - LC)}_{\bar{A}} e_x + \bar{B}\dot{f}(t) \,, \\ & e_y = \bar{C}e_x \,, \\ & \bar{B} = [0, \cdots 0, \; 1]^T \,. \end{aligned} \tag{12}$$

The output error dynamics, which is obtained after replacing the control law (10) in the system (1), is governed by:

$$e^{(n)} + \sum_{k=0}^{(n-1)} \lambda_k e^{(k)} = \lambda^T e_x \,, \tag{13}$$

which was obtained after manipulating $\hat{X}$ from (11). For analysis purpose, let us define the output tracking error vector as

$$e_p = [e, \dot{e}, \cdots, e^{(n)}]^T \,. \tag{14}$$

Adopting the *controllable canonical form* for writing the state space representation of the closed loop full error system composed by (12) and (13), we obtain:

$$\dot{e}_x = \bar{A} \; e_x + \bar{B}\dot{f}(t) \,, \tag{15}$$

$$\dot{e}_p = A_e \; e_p + \lambda^T e_x \,, \tag{16}$$

with

$$A_e = \begin{bmatrix} 0 & 1 & \cdots & 0 \\ 0 & 0 & 1 & \cdots \\ \vdots & \vdots & \vdots & \vdots \\ -\lambda_0 & -\lambda_1 & \cdots & -\lambda_{(n-1)} \end{bmatrix} \,. \tag{17}$$

In (Zheng *et al.*, 2009, 2012), the convergence proofs for the ESO error system (12) is established. In fact, Theorem 1 stated in Zheng *et al.* (2009) demonstrates that, for a bounded $\dot{f}$ and a finite time instant $T_1 > 0$, the ESO steady-state errors tend to residual sets η_i around origin with constant upper bounds given by:

$$|e_{xi}(t)| \leq \eta_i , \quad \forall t \geq T_1 , \tag{18}$$

$$\eta_i = \mathcal{O}\left(\frac{1}{w_0^l}\right) , \quad i = 1, \cdots , (n+1) . \tag{19}$$

in which l is some positive integer. A complete proof for (18) and (19) can be found in (Zheng *et al.*, 2009, 2012). Following a similar developing, in (Zachi *et al.*, 2019) it is show that the tracking error norm $||e(t)||_\infty$ tends to the same residual set defined by (18), but with a limited convergence time $T_2 > T_1$.

Note that the ESO design (8) and the control law (10) are both dependent on the exact knowledge of the plant control gain b. Such design feature has been addressed in several ADRC schemes by assuming that b is completely or partially known a priori (Han, 1998; Gao *et al.*, 2001; Han, 2009; Madoński and Herman, 2011; Zhu *et al.*, 2014; Xue *et al.*, 2015; Xia *et al.*, 2016). This drawback is addressed in the next section in which is explained the modified ADRC (Zachi *et al.*, 2019), witch does not require the exact knowledge of b.

3. Modified ADRC

Consider the class of dynamical systems defined by (1). For analysis purpose, let us detach the system linear part from $f(t)$ so that it can appear explicitly in the plant representation, as follows

$$y^{(n)} = \underbrace{a^T Y + g(t)}_{f(t)} + bu(t) , \tag{20}$$

$$\begin{aligned} a &= [-a_0,\ -a_1,\ \cdots ,\ -a_{(n-1)}]^T , \\ Y(t) &= [y,\ \dot{y},\ \cdots ,\ y^{(n-1)}]^T , \\ g(t) &= d(t) + h(t) . \end{aligned} \tag{21}$$

In (20), $a \in \mathbb{R}^n$ denotes the system constant parameters, whose elements can either positive, negative or null. The main idea of modified ADRC is to perform a structural transformation on the original system (20), in particular concerning the input/output behavior, in order to obtain a new dynamical system with advantageous format. For this end, we introduce an adjustable constant gain β in series with the plant output error and a linear, stable filter Q_0 in parallel with them, as shown in Fig. 1, where $\beta = K_0 sgn(b)$ and $sgn(.)$ is the signal function. The positive design constant $\gamma \in \mathbb{R}$ is chosen such that $(s+\gamma)^n = s^n + \alpha_{(n-1)} s^{(n-1)} + \cdots + \alpha_0$ results

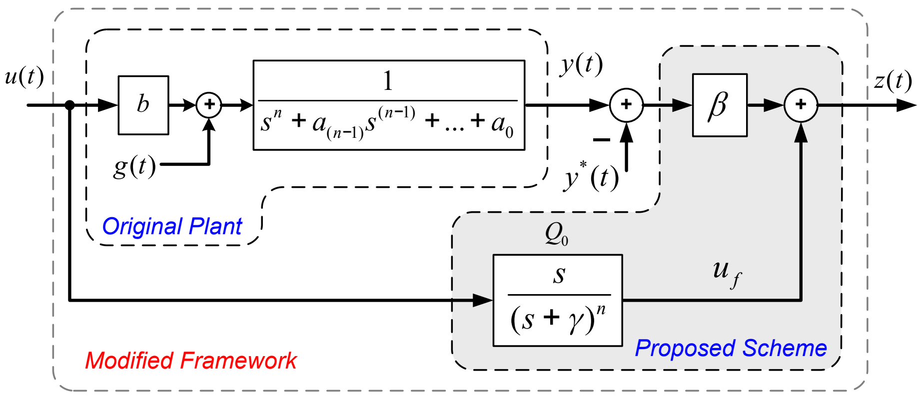

Figure 1. Block diagram of the proposed solution.

in a stable polynomial. Based on the configuration of Fig. 1, the new output error can be written as:

$$z(t) = \beta e(t) + u_f(t) , \tag{22}$$

$$e(t) = y(t) - y^*(t) , \tag{23}$$

$$u_f^{(n)} = -\alpha^T \sigma_u + \dot{u} . \tag{24}$$

$$\alpha = [\alpha_0,\ \alpha_1,\ \cdots ,\ \alpha_{(n-1)}]^T , \tag{25}$$

$$\sigma_u = [u_f,\ \dot{u}_f,\ \cdots ,\ u_f^{(n-1)}]^T . \tag{26}$$

By differentiating (22) n times, the dynamics of the new output error variable $z(t)$, with $b_p = \beta b$, will be given by:

$$z^{(n)} = \beta \underbrace{[a^T Y + g(t) + bu(t) - y^{(n)*}]}_{e^{(n)}} + u_f^{(n)}, \quad (27)$$

$$z^{(n)} = \beta[a^T Y + g(t) - y^{(n)*}] - \alpha^T \sigma_u + b_p u(t) + \dot{u}(t), \quad (28)$$

Detaching u_f from Eq. (22), we have that

$$u_f^{(i)} = z^{(i)} - \beta e^{(i)}, \quad (i = 1, \cdots, n). \quad (29)$$

Then, by replacing (29) into (28), and also using (14) and (25), we obtain:

$$z^{(n)} + \alpha^T Z(t) = \beta[a^T Y + g(t) - y^{(n)*}] + \beta\alpha^T e_p + b_p u(t) + \dot{u}(t) \quad (30)$$

$$Z(t) = [z, \dot{z}, \cdots, z^{(n-1)}]^T. \quad (31)$$

For writing the new plant description using the ADRC formalism, as done in (1), we define a new generalized disturbance function $\Omega(t)$ as

$$\Omega(t) = \beta[a^T Y + g(t) - y^{(n)*}] + \beta\alpha^T e_p + b_p u(t), \quad (32)$$

which reduces (28) to:

$$z^{(n)} + \alpha^T Z(t) = \Omega(t) + \dot{u}(t). \quad (33)$$

By comparing (33) to (1), it is possible to verify that the original problem of output tracking, which is associated with the error $e(t)$ (23), is now redefined in terms of the new output error $z(t)$. It is important to notice that the new control input $\dot{u}$ has unitary coefficient, meaning that the controller and ESO designs can be carried out without requiring the exact value of parameter b (20). Since $\Omega(t)$ is not available, now the basic ADRC design procedures can be addressed in the next section.

3.1 Extended State Observer (ESO) design

For (33), consider the following state variable definitions

$$\zeta(t) := \left[\zeta_1, \zeta_2, \cdots, \zeta_{(n+1)}\right]^T = [z(t), \dot{z}(t), \cdots, \Omega(t)]^T. \quad (34)$$

Assuming that $\Omega(t)$ is differentiable, the plant state-space representation of (33), in companion form, can be written as:

$$\begin{aligned} \dot{\zeta} &= A_m \zeta + B_\zeta \dot{u} + \Gamma \dot{\Omega}(t), \\ z(t) &= C\zeta. \end{aligned} \quad (35)$$

with

$$A_m = \begin{bmatrix} 0 & 1 & \cdots & 0 & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ -\alpha_0 & -\alpha_1 & \cdots & -\alpha_{(n-1)} & 1 \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix}, \quad (36)$$

$$B_\zeta = [0\ 0\ \cdots\ 0\ 1\ 0]^T, \quad \Gamma = [0\ 0\ \cdots\ 0\ 1]^T,$$

$$C = [1\ 0\ \cdots\ 0].$$

Since the pair (A_m, C) is always observable, a full-order ESO for (35)-(36) is then designed as follows:

$$\begin{cases} \dot{\hat{\zeta}} = A_m \hat{\zeta} + B_\zeta \dot{u} + L e_z, \\ \hat{z} = C\hat{\zeta}, \end{cases} \quad (37)$$

where $\hat{\zeta} \in \mathbb{R}^{(n+1)}$ represents the estimated state vector, $e_z := (z - \hat{z})$ is defined as the output estimation error and $L = [L_1\ L_2\ \cdots\ L_{(n+1)}]^T \in \mathbb{R}^{(n+1)}$ is the vector of the observer gains defined by

$$det[sI - (A_m - LC)] = (s + w_0)^{(n+1)}. \quad (38)$$

By defining the ESO state error as

$$e_\zeta = \zeta - \hat{\zeta}\,, \tag{39}$$

the ESO state error dynamic can be compute from (35), (36) and (37), resulting in:

$$\begin{cases} \dot{e}_\zeta = \underbrace{(A_m - LC)}_{\bar{A}_m} e_\zeta + \Gamma\,\dot{\Omega}(t)\,, \\ e_z = Ce_\zeta\,. \end{cases} \tag{40}$$

3.2 Control design

By observing the new plant description in (33), we note that its homogeneous part (i.e., with $\Omega(t) + \dot{u} \equiv 0$) is stable, that was achieved after the introduction of the filter Q_0 in (24). Therefore, in order to compensate the disturbance term $\Omega(t)$, forcing the new error $z(t)$ to tend to zero, we propose the control law

$$\dot{u} = -\,\hat{\zeta}_{(n+1)}\,. \tag{41}$$

Note, from (34) and (37), that $\hat{\zeta}$ is the estimative of $\Omega(t)$.

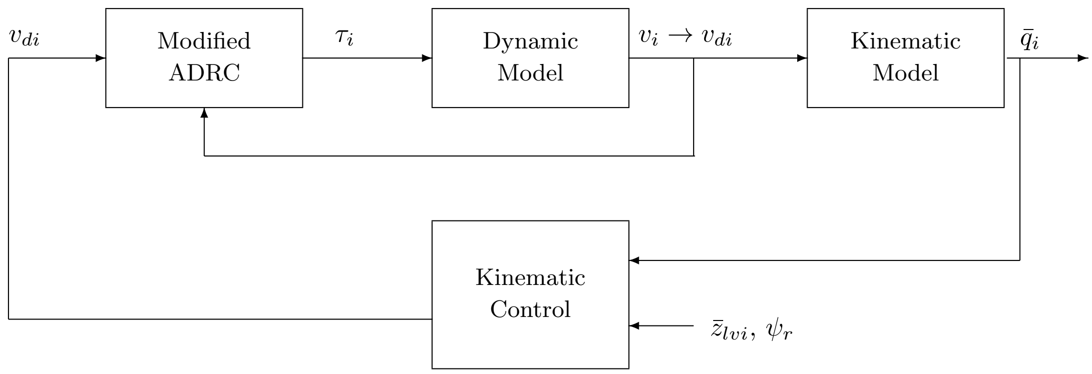

Figure 2. Cascaded strategy

In (Zachi *et al.*, 2019), it is shown that the control law (41) assures that: (i) the tracking error $e(t)$ tends to a residual set which can be reduced by increasing γ and (ii) $e(t) \to 0$ if the reference y^* is a constant.

4. Formation Control Strategy

Consider a set of N mobile robots described by

$$\begin{aligned} \dot{\omega}_{i_1} &= \overbrace{\frac{m_{12}}{m_{11}}\dot{\omega}_{i_2} + \frac{C}{m_{11}}\omega\omega_{i_2}}^{\Omega_{i_1}} + \tfrac{1}{m_{11}}\tau_{i_1} \\ \dot{\omega}_{i_2} &= \overbrace{\frac{m_{12}}{m_{11}}\dot{\omega}_{i_1} - \frac{C}{m_{11}}\omega\omega_{i_1}}^{\Omega_{i_2}} + \tfrac{1}{m_{11}}\tau_{i_2} \end{aligned} \tag{42}$$

$$\overbrace{\begin{bmatrix} \dot{x}_i \\ \dot{y}_i \\ \phi_i \end{bmatrix}}^{\dot{q}_i} = \frac{r_i}{2}\begin{bmatrix} \cos(\phi_i) & \cos(\phi_i) \\ \sin(\phi_i) & \sin(\phi_i) \\ b_i^{-1} & -b_i^{-1} \end{bmatrix}\begin{bmatrix} \omega_{i_1} \\ \omega_{i_2} \end{bmatrix} \tag{43}$$

with $i = 1 \cdots N$, ω_{i_1} and ω_{i_2} being the wheels angular velocities, x_i, y_i the position, ϕ_i the orientation with respect to an inertial coordinate system, τ_{i_1}, τ_{i_2} the control torques applied to the robot wheels and m_{11}, m_{12},

C, b_i and r_i some robot parameters. Note that the robot mathematical model is separated in a dynamic part, described by (42), and kinematic part, described by (43), which has the nonholonomic restrictions. As it is shown in Figure 2, where $\tau_i = [\tau_{i_1} \quad \tau_{i_2}]$, for each robot i, a kinematic tracking formation control is designed and its velocity signal v_{di} is used as reference to a modified ADRC control. Thus, as the modified ADRC assures that $v_i \to v_{di}$, the robots will have a kinematic behavior after a transitory time.

The control objective is to drive the N agents to a formation which minimizes

$$J = \sum_{i=1}^{N} J_i \quad \text{with } J_i = \sum_{i,j \in N_i}^{N} J_{ij}(\|\bar{z}_{ij}\|) + \sum_{i}^{m} J_{ri}(\|\bar{z}_i\|), \tag{44}$$

while a desired trajectory defined by $m \leq N$ virtual leaders is tracked, where $\bar{z}_{ij} = \bar{z}_i - \bar{z}_j$, $\bar{z}_i^T = [x_i \,, y_i]$ and J_{ij}, J_{ri} are defined as follows:

Definition 1 *The potential function J_{ij} is a differentiable, nonnegative function of the distance $\|\bar{z}_{ij}\|$ between agents i and j, such that (i) $J_{ij}(\|\bar{z}_{ij}\|) \to \infty$ as $\|\bar{z}_{ij}\| \to \infty$ and $\|\bar{z}_{ij}\| \to 0$; (ii) J_{ij} attains its unique minimum when agents i and j are located at a desired relative position d_{ij}.*

Definition 2 *The tracking potential function J_{ri} is a differentiable, nonnegative function of the agent position $\|z_i\|$ such that (i) $J_{ri}(\|\bar{z}_i\|) \to \infty$ as $\|\bar{z}_i\| \to \infty$; (ii) J_{ij} attains its unique minimum when the agent i is located at a desired position $\bar{z}_{ri}$.*

Thus, the used formation kinematic control law is described by

$$v_{di} = \begin{bmatrix} \omega_{d1i} \\ \omega_{d2i} \end{bmatrix} = \underbrace{\frac{1}{r_i}\begin{bmatrix} 1 & b_i \\ 1 & -b_i \end{bmatrix}}_{B_i} \begin{bmatrix} u_{di} \\ \omega_{di} \end{bmatrix} \tag{45}$$

$$\left.\begin{array}{rcl} u_{di} & = & -k_{fi}e_{fi} + \bar{u}_{ri} \\ w_{di} & = & u_{ri}k_{ri}\bar{e}_{ri} + k_{wi}e_{\psi ri} + w_{ri} \end{array}\right\} i = 1 \cdots m \qquad \left.\begin{array}{rcl} u_{di} & = & -k_{fi}e_{fi} \\ w_{di} & = & -k_{wi}e_{\psi fi} \end{array}\right\} i = m+1 \cdots n \tag{46}$$

where $e_{fi} = \nabla_{\bar{z}_i} J^T R_{li}$ is the projection of vector $\nabla_{\bar{z}_i} J^T$ in $R_{li} = [\cos(\psi_i)\sin(\psi_i)]$, the direction movement permitted by nonholonomic restrictions, $e_{\psi ri} = \sin(\psi_i - \psi_{ri})$ is the orientation error of robot i with respect to orientation of its virtual leader, $\bar{e}_{ri} = (\bar{z}_i - \bar{z}_{lvi})^T R_{ni}$ is the projection of the relative position vector of each robot, with respect to its virtual leader, in the space orthogonal to R_{li}, defined by $R_{ni} = [-\sin(\psi_i)\cos(\psi_i)]^T$, and $\bar{u}_{ri} = u_{lvi}\cos(\psi_{ri} - \psi_i)$ is the projection of the velocity vector of each virtual leader u_{lvi} in the direction of its follower robot. The each virtual leader configuration $\bar{z}_{lvi}$ is defined by $\bar{z}_{lvi}(t) = \bar{z}_r(t) + d_i(t)$ where $\bar{z}_r(t) = [\ x_r(t) \quad y_r(t)\ ]^T$ is a trajectory described by

$$\begin{bmatrix} \dot{x}_r \\ \dot{y}_r \\ \dot{\psi}_r \end{bmatrix} = \begin{bmatrix} \cos\psi & 0 \\ \sin\psi & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u_r \\ \omega_r \end{bmatrix} \tag{47}$$

being u_r, w_r linear and angular velocities common to all leaders. Besides that, $d_i(t)$ is used in order to define the formation orientation and should be chosen to obey the geometric shape defined by the potential function. It is important stress that the trajectory described by (47) obeys the nonholonomic restriction defined by (43). In Pereira *et al.* (2009), it is shown that the control law (45) assures that a set of robots with kinematic model (43) reaches a formation that minimizes the potential function (44).

For the dynamic part, the idea is to consider it as two first order systems with generalizes disturbance Ω_{i_1} and Ω_{i_2}. Then, one modified ADRC, described by control law (41) and ESO (35), is implemented to each first order system.

5. SIMULATION RESULTS

The Table 1 shows the used dynamic parameters corresponding to two different values of mass m_c and inertial moment I_w of each robot. The idea is to verify the robustness of formation and tracking control considering a variation in the dynamic parameters. The used control parameters are $\bar{L}_1 = 1960$, $\bar{L}_2 = 10^6$, $\gamma = 40$ and $K_0 = 100$ for the modified ADRC controls, $k_1 = 0.5$, $k_2 = 0.1$, $k_3 = 0.5$, $k_{fi} = 150$ and $k_{\omega_i} = 5$ for formation kinematic tracking controls. Every potential funtions have one minimal in $d_{ij} = 30.2m$ and are described by

$$J_{ij} \;=\; \tfrac{a_{ij}}{2}\|\bar{z}_{ij}\|^2 + \tfrac{b_{ij}c_{ij}}{2}\exp^{(-\|\bar{z}_{ij}\|^2/c_{ij})} \qquad J_{ri} \;=\; \|\bar{z}_i - \bar{z}_{lvi}\|^2$$

Table 1. Dynamic parameters

Dyn. par.	$m_c = 30kg$, $I_\omega = 15kgm^2$	$m_c = 45kg$, $I_\omega = 25kgm^2$
m_{11}	0.2609	0.3838
m_{12}	0.1042	0.1501
C	0.1350	0.2025

where $a_{ij} = 0.01$, $b_{ij} = 10$ and $c_{ij} = d_{ij}^2/log(b_{ij}/a_{ij})$. The initial conditions $[x, y, \theta]$ are $[280 \quad 0 \quad pi]$, $[300 \quad 300 \quad pi]$, $[-300 \quad 0 \quad -pi]$ and the formation should tracking two virtual leaders on circular trajectories with radius $R_r = 310m$. The Figures 3 and 4 shows that the reach formation tracks the desired trajectory. Besides that, it there is not a significant difference among the robots trajectories in the two figures. Then, this results show the robustness of the proposed formation control strategy.

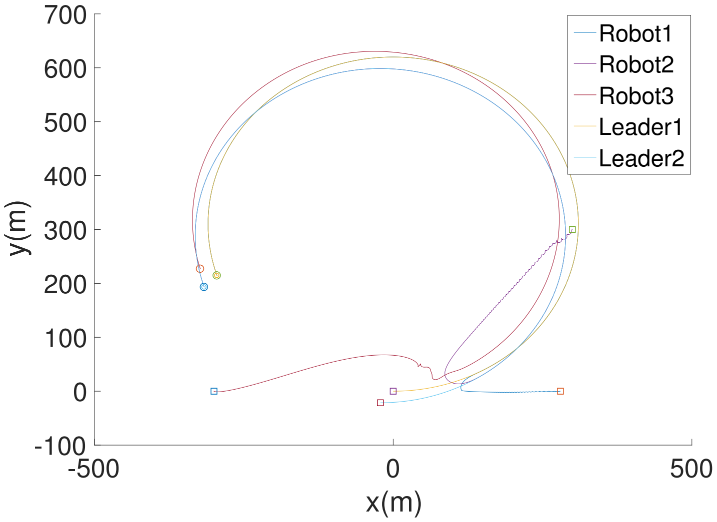

Figure 3. Robots trajectories: $m_c = 30kg/I_\omega = 15kgm^2$

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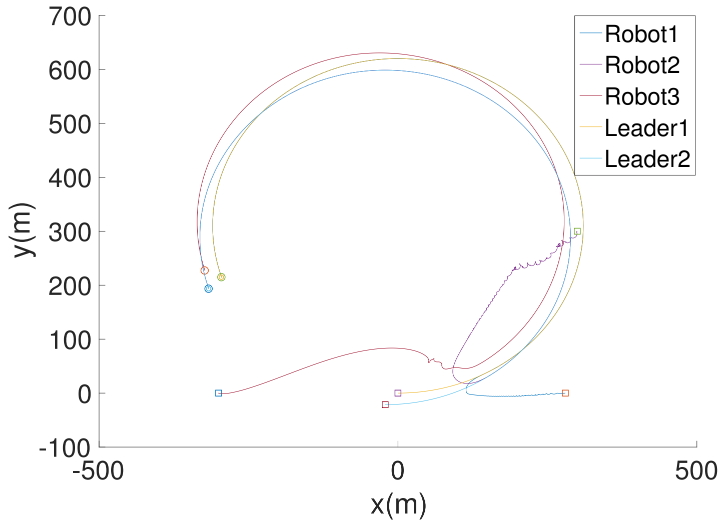

Figure 4. Robots trajectories: $m_c = 45kg/I_\omega = 25kgm^2$

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