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ON THE TRESHOLD BETWEEN VISCOUS AND INVISCID INSTABILITY ANALYSIS OF FREE JET FLOWS

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Abstract. *Hydrodynamic instabilities are present in many problems related to fluid mechanics. In aerospace engineering, the relevant applications are associated with air breathing propulsion engines such as turbojets, ramjets and scramjets, working in subsonic or supersonic state. The combustion efficiency is obtained by understanding the phenomenon of the mixing shear layer. Generally, for the free jet, the inviscid fomulation has been used to simulate the stability analysis. The numerical and experimental results are in agreement for high Reynolds number. However, there are but a few studies for low Reynolds number. In this context, the present work have been studying the minimum Reynolds number that can be considered as an inviscid formulation. The ratio between viscous and inviscid stability analysis was calculated using the matrix forming approach. It was concluded that for small Reynolds number it is necessary to use viscous equations in the simulations.*

Keywords: *Viscous stability analysis, matrix forming method, Free jet profile*

1. INTRODUCTION

Hydrodynamic instabilities are present in many problems related to fluid mechanics. In aerospace engineering, the relevant applications are associated with air breathing propulsion engines such as turbojets, ramjets and scramjets, working in subsonic or supersonic state.

The linear stability problem is an eigenvalue problem that uses, for the inviscid case, the Rayleigh's equation Rayleigh (1880) and for the viscous case the Orr-Sommeferld equation Orr (1907). These two equations are the linearized Navier-Stokes equation (LNSE) for the disturbances. Shooting method is one of the traditional ways to solve instability analyses problems and different situations have been studied with this method along the years, for example the classic problems of mixing layer Michalke (1964, 1965), planar channel Orszag (1969) and boundary layer Mack (1984). The analogous expansion for compressible flow may be found on Mendonca (2014). Mendonca *et al.* (2015) has been using extensively this compressible formulation to study the compressible binary mixing layer.

To start the shooting method one needs the contours analysis to find a good kicking to locate all the important modes. In case of free jet is necessary the cylindrical form of Navier-Stokes equations and the boundary conditions at $r = 0$ must be supplemented by the values of the derivatives of the variables in order for it to be possible to advance the solutions by a numerical step-by-step procedure. The disturbance equations are regularly singular at $r = 0$ so the Frobenius method is used to obtain power-series solutions, Lessen and Singh (1973), Mollendorf and Gebhart (1973). It can be quite hard to determine a suitable initial guess. Furthermore, for the viscous case the orthogonality is lost and the orthonormalization process is required in order to keep the linear independence of the solutions.

The matrix forming is another form to solve the LNSE. In this one, the perturbations equations are spatially discretized getting a generalized eigenvalue problem. There are in the literature a lot of suggestion to make the spacial discretization. Paredes *et al.* (2013) realized a comparison among different types of spacial discretization and showed that the high order finite difference method is a strong competitive over other discretizations of the multidimensional eigenvalue problems. The main advantage of the matrix forming is due to its simplicity and flexibility to program. It is easy to work with incompressible or compressible, viscous or inviscid, using the same code and making just slight modifications Theofilis

(2003).

Inviscid instability of circular jet with external flow was studied by Michalke and Hermann (1982). The instability properties of spatially growing axisymmetric and first-order azimuthal disturbances show that the external flow inhibits the instability of the circular jet, but increases the unstable frequency range and they concluded that the large-scale structure of jet turbulence is modified in the same manner by the external flow. Deals with the spatial viscous instability of a two-dimensional developing mixing-layer, Seo (1995) analyzed the hydrodynamic instability of the free shear layer and it was found that when the Reynolds number increases, the amplification rate approaches to that of the inviscid solutions. The spacial viscous instability of axisymmetric jets was made by Morris (1976). Numerical solutions for the spatial stability for the axisymmetric disturbance and the first azimuthal disturbance are presented. The critical Reynolds number is found for these cases. However, nothing is presented about the other azimuthal disturbances. Another work that studies the necessity of a viscous modeling for jets with high Reynolds number is Monkewitz (1988).

More recently dealing with local linear instability of axisymmetric coaxial jets with a duct wall separating the two streams, using a Reynolds number about 10^4, Talamelli and Gavarini (2006) studied how the various parameters describing this flow geometry (i.e. boundary layers thicknesses at the exit, velocity ratio, wall thickness) may influence the instability of the flow and, in particular, the convective/absolute instability transition. Results show that the absolute unstable mode is present only for a limited range of velocity ratios. Considering transverse jets, the Reynolds number varies from 1500 Mcloskey *et al.* (2002) to 3000 Karagozian (2010).

The objective of this work is do a preliminary studied of the impact of Reynolds number in relation to viscous and inviscid instability. At first, a simple case was made considering the free jet and for future the plan is to extend this work for transverse jet case. In order to know the critical Reynolds number that the problem can be treated as an inviscid problem, a spacial instability analysis for the free jet profile (considering the viscous term) was performed and the critical Reynolds number was found for a range of azimuthal modes disturbances. In other words, to which Reynolds numbers it does not influence the results for each azimuthal mode.

2. COMPUTATIONAL PROCEDURE

2.1 Linearized Navier-Stokes Equations

In order to derivate the LNSE, incompressible equations in cylindrical coordinates with constant properties was used. The non dimensional incompressible Navier Stokes equations are given by:

$$\nabla.\mathbf{u} = 0 \tag{1}$$

$$\frac{\partial \mathbf{u}}{\partial t} + u.\nabla\mathbf{u} = -\nabla p + \frac{1}{Re}\nabla^2\mathbf{u} \tag{2}$$

where $\mathbf{u} = (u, v, w)$ is the velocity vector, p the pressure and Re the Reynolds number. For cylindrical coordinates

$$\mathbf{u}.\nabla = u\frac{\partial}{\partial x} + v\frac{\partial}{\partial r} + \frac{w}{r}\frac{\partial}{\partial \theta} \quad \text{and} \quad \nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2}$$

.

The base flow have been decomposed into the steady base flow (ϕ_b) and unsteady disturbance (ϕ_d) parts, where the disturbance is assumed to Fourier decompose into complex components typically of the form:

$$\mathbf{\Phi}_b(x, r, \theta) + \epsilon\mathbf{\Phi}_d(x, r, \theta, t) = u_b(r) + \epsilon\left(\hat{\mathbf{\Phi}}(r)e^{i(\alpha x + -\omega t + m\theta)} + cc.\right) \tag{3}$$

$\mathbf{\Phi} = (u, v, w, p)$, where the axial, radial and azimuthal velocity as well as pressure disturbances u_d, v_d, w_d and p_d are described as functions of α and ω. The eigenfunctions $\hat{u}$, $\hat{v}$, $\hat{w}$ and $\hat{p}$ are the linear disturbance amplitudes.

Applying this decompositions, Eq.(3) into Eq. (1) to (2) and collecting the terms of $O(\epsilon)$ yields the LNSE.

$$i\alpha(u_b - \omega/\alpha)\hat{u} + \frac{\partial u_b}{\partial r}\hat{v} + i\alpha\hat{p} = \frac{1}{Re}\left[\frac{\partial^2\hat{u}}{\partial r^2} + \frac{1}{r}\frac{\partial\hat{u}}{\partial r} - \left(\alpha^2 + \frac{m^2}{r^2}\right)\hat{u}\right] \tag{4}$$

$$i\alpha(u_b - \omega/\alpha)\hat{v} + \frac{\partial\hat{p}}{\partial r} = \frac{1}{Re}\left[\frac{\partial^2\hat{v}}{\partial r^2} + \frac{1}{r}\frac{\partial\hat{v}}{\partial r} - \left(\alpha^2 + \frac{m^2+1}{r^2}\right)\hat{v} - i\frac{2m}{r^2}\hat{w}\right] \tag{5}$$

$$i\alpha(u_b - \omega/\alpha)\hat{w} + \frac{im}{r}\hat{p} = \frac{1}{Re}\left[\frac{\partial^2\hat{w}}{\partial r^2} + \frac{1}{r}\frac{\partial\hat{w}}{\partial r} - \left(\alpha^2 + \frac{m^2+1}{r^2}\right)\hat{w} + i\frac{2m}{r^2}\hat{v}\right] \tag{6}$$

$$i\alpha\hat{u} + \frac{\partial\hat{v}}{\partial r} + \frac{\hat{v}}{r} + \frac{im\hat{w}}{r} = 0 \tag{7}$$

2.2 Matrix Forming

The matrix forming describe the spatial derivate in LNSE creating the generalized matrix eigenvalue problem

$$\mathbf{A}.\hat{\mathbf{q}} = \sum_{k=1}^{2} \alpha^k \mathbf{B}_k.\hat{\mathbf{q}}. \tag{8}$$

This is a non linear system. However, it can be transformed into linear problem using the approach called companion matrix method, Bridges and Morris (1984). The vector $\hat{\mathbf{q}}$ given by $\hat{\mathbf{q}} = [\hat{u}, \hat{v}, \hat{w}, \hat{p}]$ is modified to the auxiliary vector $\hat{q}^* = [\hat{u}, \hat{v}, \hat{w}, \hat{p}, \alpha\hat{u}, \alpha\hat{v}, \alpha\hat{w}]$ and the new system becomes:

$$\mathbf{A}.\hat{q}^* = \alpha\mathbf{B}.\hat{q}^* \tag{9}$$

Sixth order central finite difference have been used for spatial derivative in LNSE and build the EVP. The Arnoldi method was used to decompose the matrix A and to obtain the Hessemberg matrix, where the Ritz value are calculated by Linear algebra subroutine ZGEEV of LAPACK, Anderson *et al.* (1999).

Written using matrix notation, the operators $\mathbf{A}$ and $\mathbf{B}$ become:

$$\mathbf{A} = \begin{pmatrix} \mathcal{L} & \frac{\partial u_b}{\partial r} & 0 & 0 & 0 & 0 & 0 \\ 0 & \mathcal{L} + \frac{1}{r^2 Re} & \frac{2im}{r^2 Re} & \frac{\partial(.)}{\partial r} & 0 & 0 & 0 \\ 0 & -\frac{2im}{r^2 Re} & \mathcal{L} + \frac{1}{r^2 Re} & \frac{im}{r} & 0 & 0 & 0 \\ 0 & \frac{\partial(.)}{\partial r} + \frac{1}{r} & \frac{im}{r} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \mathbf{I} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \mathbf{I} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \mathbf{I} \end{pmatrix} \mathbf{B} = \begin{pmatrix} -iu_b & 0 & 0 & -i & -\frac{1}{Re} & 0 & 0 \\ 0 & -iu_b & 0 & 0 & 0 & -\frac{1}{Re} & 0 \\ 0 & 0 & -iu_b & 0 & 0 & 0 & -\frac{1}{Re} \\ -i & 0 & 0 & 0 & 0 & 0 & 0 \\ \mathbf{I} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \mathbf{I} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \mathbf{I} & 0 & 0 & 0 & 0 \end{pmatrix} \tag{10}$$

where the operator $\mathcal{L}$ is given by $\mathcal{L}(.) = -i\omega - \frac{1}{Re}\left(\frac{\partial^2(.)}{\partial r^2} + \frac{1}{r}\frac{\partial(.)}{\partial r} - \frac{m^2}{r^2}\right)$.

3. RESULTS

The free jet problem was chosen to carry out the study of Reynolds number effect over the spatial stability analysis. The profile studied is given by :

$$u_b(r) = \frac{1}{2}(U_j + U_\infty) - \frac{1}{2}(U_j - U_\infty)\tanh\left[\frac{1}{4}\frac{R}{\theta}\left(\frac{r}{R} - \frac{R}{r}\right)\right] \tag{11}$$

where U_j is the jet core velocity, U_∞ the external flow velocity, θ the momentum boundary layer thickness of the jet shear layer and the radius R denotes the middle of the shear layer.

In order to validate the code, first the inviscid instability was considered using the Michalke's profile, Michalke and Hermann (1982). The code has been built with second, fourth and sixth order. The order verification is presented at Fig. 1 for the frequency $\omega = 0.5$ and the jet parameters $D/\theta = 20$ and $\theta = 0.1$. The axisymmetric and the first azimuthal disturbances were calculated and compared with Michalke and Hermann (1982) for the same parameters. The result is presented at Fig. 2, where the absolute error is less than 10^{-4}.

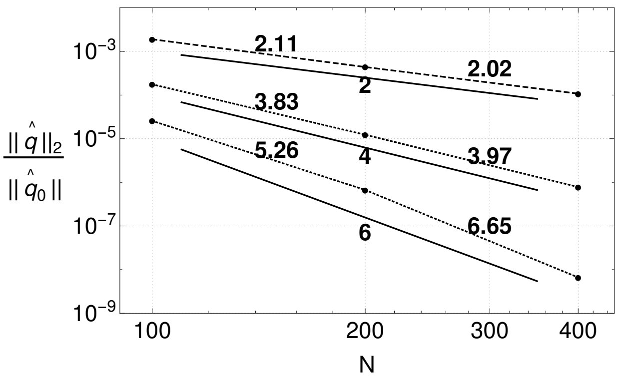

Figure 1. Order verification.

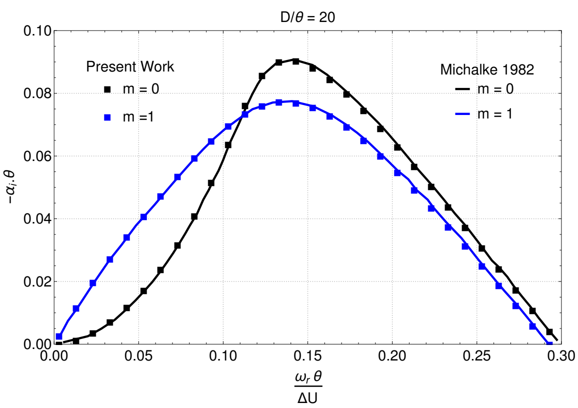

Figure 2. Spatial growth rate as function of the frequency.

The spatial viscous incompressible cylindrical matrix forming code have been built and verified by the same base flow at Eq. 11 for $\theta = 0.16$. The results were compared with Morris (1976). The Figs. 3 and 4 show the axisymmetric and azimuthal modes respectively to different Reynolds numbers. Three boundary layer thickness were simulated to both modes. In all cases, the absolute error calculated was less than 10^{-3}. A good agreement is observed even with stable frequency in the axisymmetric mode.

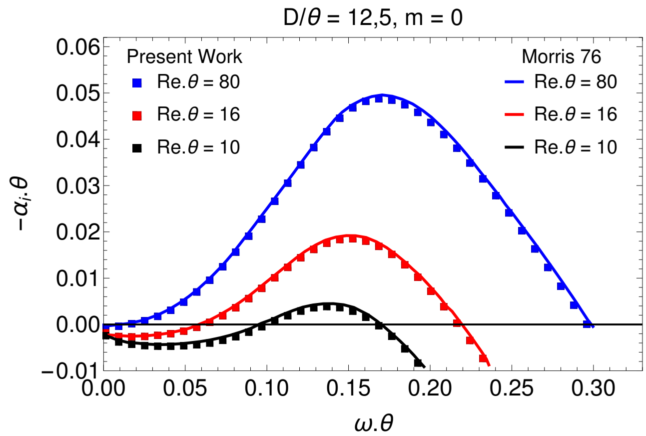

Figure 3. Axisymmetric mode: Spatial growth rate as function of the frequency.

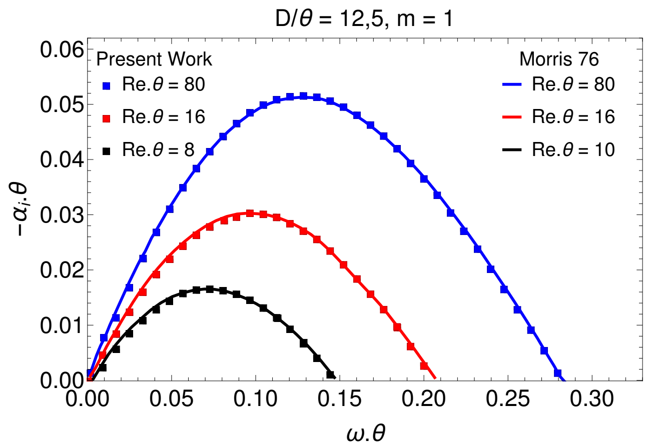

Figure 4. Azimuthal mode: Spatial growth rate as function of the frequency.

In order to study the influence of Reynolds number over the modes in free jet profile, for each Reynolds number, the maximum spatial growth rate was calculated and compared with the inviscid case. The results are shown at the Figs. 5,6 and 7 for $D/\theta = 12.5$, 20 and 30, respectively. Two dashed line were drawn to indicate the 90% and 99% for more up ratio between viscous and inviscid case. It is possible to observe that to 99% all viscous modes converge to the inviscid case at high Reynolds number, $Re = 10^4$. The same does not occur at low Reynolds number. It is possible to see that for Reynolds number less than 1000, the ratio between viscous and inviscid case diverge. This is more evident for small boundary layer thickness. For the second azimuthal mode, the effect of the Reynolds number is strong even for intermediate values between 1×10^3 to 3×10^3.

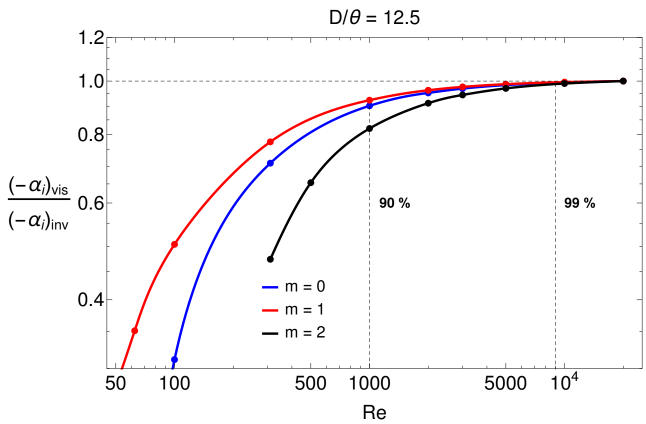

Figure 5. Spatial growth rate $-\alpha_i$ as function of the Reynolds number.

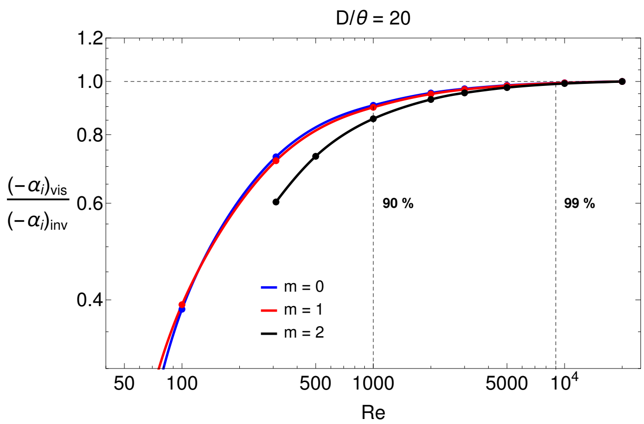

Figure 6. Spatial growth rate $-\alpha_i$ as function of the Reynolds number.

The boundary layer thickness is one of the most important parameters in the stability analysis of the free jet profile. For this reason, it has been investigated separately in each mode. The Figs. 8, 9 and 10 show the ratio between viscous and inviscid, respectively for m = 0, 1, and 2. Again, the results were obtained by the viscous maximum spatial growth rate. It is noted that the boundary layer thickness does not affect the axisymmetric mode, however, for Reynolds number less than 10^3 there is a small effect over the azimuthal modes.

For all cases, the 90% dashed line lies over around $Re = 1000$, this leads us to conclude that for simulations with low Reynolds number it is necessary to use the viscous formulation for the free jet analysis stability .

4. CONCLUSION

This work investigated the influence of the Reynolds number over the free jet stability analysis. The ratio between viscous and inviscid formulation was calculated using the matrix forming approach. Three different profiles were proposed varying the boundary layer thickness. It was possible to conclude that the ratio between viscous and inviscid formulation

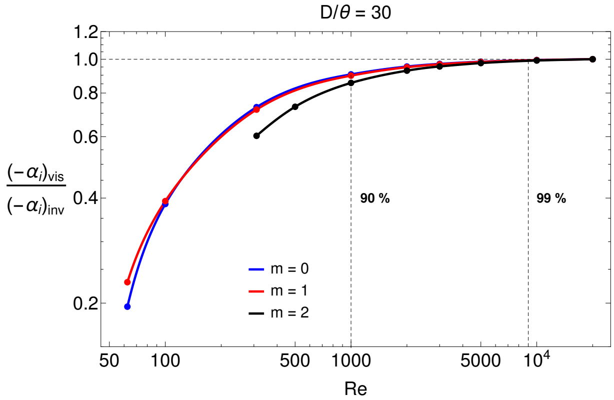

Figure 7. Spatial growth rate $-\alpha_i$ as function of the Reynolds number.

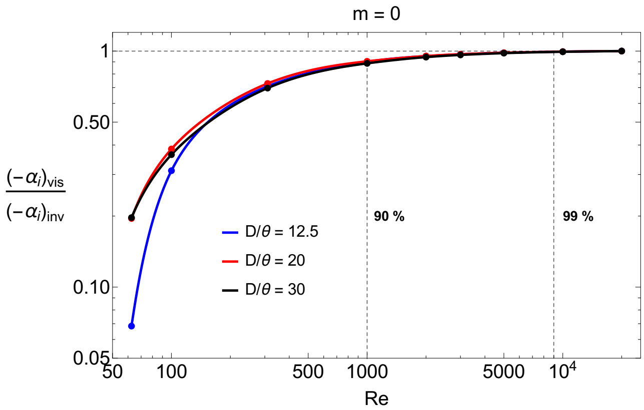

Figure 8. Spatial growth rate $-\alpha_i$ as function of the Reynolds number.

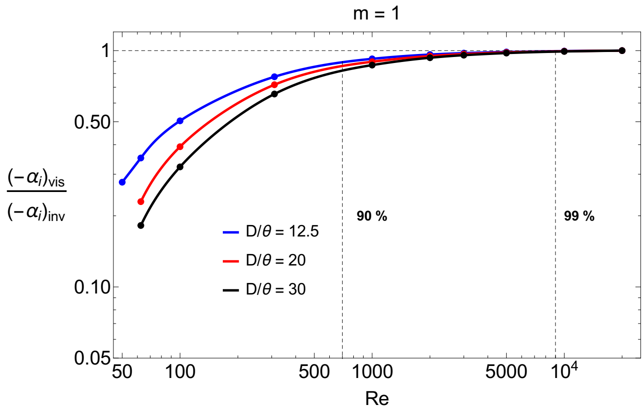

Figure 9. Spatial growth rate $-\alpha_i$ as function of the Reynolds number.

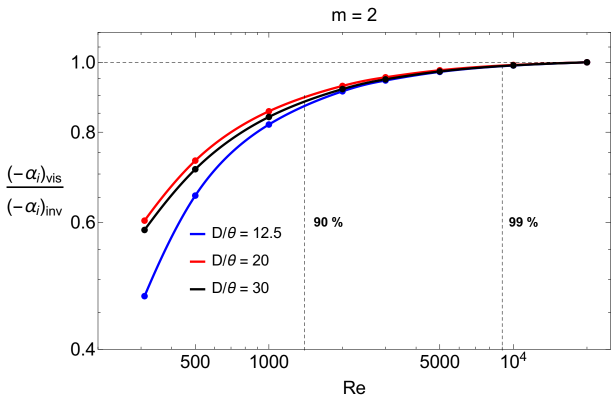

Figure 10. Spatial growth rate $-\alpha_i$ as function of the Reynolds number.

in stability analysis diverge for small Reynolds Number. The viscous dependence is stronger for the second mode. The two formulations have agreement below 90% for Reynolds number less than 10^3. It was concluded that for small Reynolds number it is necessary to used viscous LNSE in the simulations in order to predict more accurate results.

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