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OPTIMAL LINEAR CONTROL APPLIED IN A ENERGY HARVESTING DYNAMIC SYSTEM WITH PERIODIC EXCITATION

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Abstract. *Generally, active control in dynamic systems is used to reduce vibrations by modifying system behavior to what is desired in design issues. However, the present work of research is precisely the use of vibrations to generate electrical energy, in such a way that the vibration becomes a desired phenomenon. In this way, we intend to use the Optimal Control via Linear Quadratic Regulator (LQR), aiming at the greater transduction of vibrational energy to electric, performing a detailed analysis of the stability of the resulting system and the effects that the control technique provides to the dynamic system studied. In the project, the analyzed system is a bimodal mass-spring-damper governed by a system of linear differential equations, with piezoelectric coupling-mechanic who suffers an external periodic excitation. It is part of the direct continuation of the research project, funded by the São Paulo Research Foundation (FAPESP), Grant No. 2017/03829-7.*

Keywords: *Dynamic Systems, Optimal Control, Quadratic Linear Regulator.*

1. INTRODUCTION

Demand for electricity has grown over the years due to the high rate of world population growth. In this scenario, the search for sustainable sources of energy grows in scientific and research environments. One of the modalities of sustainable energy is biomass that exerts great environmental impacts. Another example of energy is nuclear fusion, which also has drawbacks in terms of high cost and handling risks. Aiming, therefore, a sustainable, safe and efficient source of energy, techniques have been developed receiving the designation of Energy Harvesting. The techniques employed aim at the capture of energy from nature as small temperature gradients, small mass movements and also by means of vibration.

The Energy Harvesting is a promising alternative for low power electronic devices. In addition, electronic devices are becoming more efficient in relation to energy consumption, and some remote sensors can already operate with up to 100 μW (Tang *et al.*, 2013).

The focus of this research project is the vibration-based mechanical energy pickups, considering that for many applications with remote sensors and for structural health/integrity monitoring, the luminous energy (from solar source) and thermal energy, are not available at the site of their application and need for uptake (Youngsman *et al.*, 2010; Schlichting *et al.*, 2013). In addition, vibration-based energy pickups tend to have low maintenance requirements and can be used in hostile environments, which are often configured as needed for sensor allocation (Challa *et al.*, 2008).

The first models of vibration-based energy harvesters have responses considered efficient only if the frequency of excitation vibration coincides with the natural frequency of the excited system, giving rise to the phenomenon known as resonance (Wu *et al.*, 2006; Sari *et al.*, 2008; Eichhorn *et al.*, 2009; Van Blarigan *et al.*, 2012; Tang *et al.*, 2013). The natural frequency of a body is given by $\bar{\omega} = \sqrt{k/m}$, where k is the rigidity of the material and m is its mass (Liu *et al.*, 2008). Then the resonance occurs when the external excitation frequency (ω) is equal to the natural frequency of the body. However, most of the vibration of the environment has a wide frequency range (Harne and Wang, 2013; Tang *et al.*, 2013) and behaves randomly, for example, the vibration of civil construction structures subjected to wind action (Jung *et al.*, 2013) and consequently the effect of resonance is very limited.

Taking the researches developed in the area and, in order to increase the efficiency of energy harvesting in vibrational systems, several solutions have been proposed, one of which is known as multimodal (Ferreira *et al.*, 2016). The multimodal solution proposes the use of different beams to take advantage of the natural oscillation frequencies of the system by maintaining it in extended resonance over a larger range of excitation frequency.

A parallel solution to improve the efficiency of energy harvesting systems is the utilization of controllers. As is known, the vibration from the environment has aleatory behavior variating in a large frequency band and presenting low power. In a recent work (Wang and Inman, 2012) is presented that the most promising solution for improving the efficiency of energy harvesting systems based on piezoelectricity with active control is the Optimal Control.

Considering all works cited until here, this research project proposes the application of control theory via Linear Quadratic Regulator to a linear system of energy harvesting with piezoelectric coupling subjected to periodic excitation and with unstable/chaotic initial parameters and conditions. The control will be applied in order to make feasible the construction of an unstable model and, later, to compare the effectiveness in the electric power capture of the system with and without control.

2. METHODOLOGY

Based on the works of Chavarette (2013) and Ferreira *et al.* (2016), Fig. (1) presents the model of energy harvester system. The system is composed by two masses (m_1 and m_2) which are coupled with their respective springs (k_1 and k_2) and dampers (ζ) and, there is a piezoelectric pickup (ν), responsible for the transduction of mechanical into electric energy. The stiffness changes for each mass and for each length of each beam, generating different natural frequencies. The damping for each beam it isn't changed, due to the material be the same. Due to the exogenous excitation, there is a vibrational response of the system, causing a deformation in piezoelectric ceramic, generating an electric current that is conducted for a resistive circuit (Erturk and Inman, 2011).

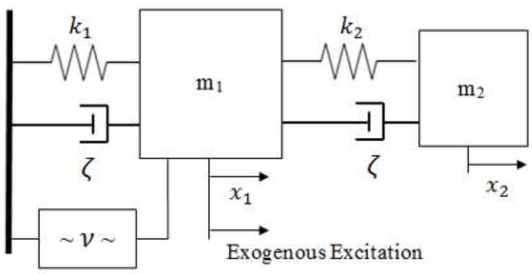

Figure 1. Model of the energy harvester system (Ferreira *et al.*, 2016)

A non-dimensional mathematical description by mass, proposed by (Erturk and Inman, 2011), that governs the system showed in Fig. (1) is presented by the differential equations in Eqs. (1) - (3), in which ζ is the damping by mass rate, χ is the piezoelectric-mechanic coupling rate, Λ is the reciprocal rate of the capacitive charge time constant, k_1 and k_2 are the stiffness rates and K is the piezoelectric-electric coupling rate. The external periodic excitation is described by an acceleration rate f and by an angular frequency rate ω. The time dependents variables are x for the beam displacement and ν for the voltage output rate (Ferreira *et al.*, 2016).

$$\ddot{x}_1 + 2\zeta\dot{x}_1 + k_1 x_1 - k_2\left(x_1 - x_2\right) - \chi\nu = f cos\left(\omega t\right) \tag{1}$$

$$\ddot{x}_2 + 2\zeta\dot{x}_2 - k_2\left(x_1 - x_2\right) = 0 \tag{2}$$

$$\dot{\nu} + \Lambda\nu + \mathrm{K}\left(\dot{x}_1 - \dot{x}_2\right) = 0 \tag{3}$$

Isolating the temporal variables and defining a space of states (in which $x_1 = y_1$, $\dot{y}_1 = y_2$, $x_2 = y_3$, $\dot{y}_3 = y_4$ and $\nu = y_5$), we obtain the differential equations showed in Eqs. (4) - (8) which describes the dynamic system behavior.

$$\dot{y}_1 = y_2 \tag{4}$$

$$\dot{y}_2 = -2\zeta y_2 - k_1 y_1 + k_2\left(y_1 - y_3\right) + \chi y_5 + f cos\left(\omega t\right) \tag{5}$$

$$\dot{y}_3 = y_4 \tag{6}$$

$$\dot{y}_4 = -2\zeta y_4 + k_2\left(y_1 - y_3\right) \tag{7}$$

$$\dot{y}_5 = -\Lambda y_5 - \mathrm{K}\left(y_2 - y_4\right) \tag{8}$$

In Eq. (9) it can be seen the matrix representation of the Eqs. (4) - (8), in which $\mathbf{y} \in \mathbb{R}^n$ is a state vector, $\mathbf{A} \in \mathbb{R}^{n\times n}$ is a matrix of parameters (Jacobian matrix) and $\mathbf{f} \in \mathbb{R}^n$ is the exogenous excitation.

$$\dot{\mathbf{y}} = \mathbf{A}\mathbf{y} + \mathbf{f}(t) \Rightarrow \begin{bmatrix} \dot{y}_1 \\ \dot{y}_2 \\ \dot{y}_3 \\ \dot{y}_4 \\ \dot{y}_5 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ k_2 - k_1 & -2\zeta & -k_2 & 0 & \chi \\ 0 & 0 & 0 & 1 & 0 \\ k_2 & 0 & -k_2 & -2\zeta & 0 \\ 0 & -\mathrm{K} & 0 & \mathrm{K} & -\Lambda \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \end{bmatrix} + \begin{bmatrix} 0 \\ f\cos(\omega t) \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{9}$$

The Optimal Linear Control used in this project is based in works of Rafikov and Balthazar (2005) and Chavarette *et al.* (2005). We have a controlled system given by Eq. (10), in which $\mathbf{y} \in \mathbb{R}^n$ is a state vector, $\mathbf{A} \in \mathbb{R}^{n\times n}$ is a matrix of parameters (Jacobian Matrix), $\mathbf{B} \in \mathbb{R}^{n\times m}$ is a constant matrix which defines which space variables will be controlled and $\mathbf{u} \in \mathbb{R}^m$ is the control vector, whose solution has the form presented in Eq. (11), in which $\mathbf{K}$ is the state feedback vector.

$$\dot{\mathbf{y}} = \mathbf{A}\mathbf{y} + \mathbf{B}\mathbf{u} \tag{10}$$

$$\mathbf{u} = -\mathbf{K}\mathbf{y} \tag{11}$$

Putting Eq. (11) on Eq. (10), we obtain the new controlled system presented in Eq. (12).

$$\dot{\mathbf{y}} = (\mathbf{A} - \mathbf{B}\mathbf{K})\mathbf{y} \tag{12}$$

Defining $\mathbf{A}, \mathbf{B}, \mathbf{Q}$ and $\mathbf{R}$ as constant matrices, the positive definite matrix $\mathbf{P}$ is obtained solving the non-linear algebraic equation of Riccati, given by Eq. (13) and so we can obtain the feedback vector $\mathbf{K}$ as shown in Eq. (14).

$$\mathbf{P}\mathbf{A} + \mathbf{A}^T\mathbf{P} - \mathbf{P}\mathbf{B}R^{-1}\mathbf{B}^T\mathbf{P} + \mathbf{Q} = 0 \tag{13}$$

$$\mathbf{K} = R^{-1}\mathbf{B}^T\mathbf{P} \tag{14}$$

With the feedback vector $\mathbf{K}$, the system has been controlled by the Linear Quadratic Regulator. A complement analysis that can be taken is to get the eigenvalues of Jacobian Matrix for the original system ($\mathbf{A}$) and for the controlled system ($\mathbf{A} - \mathbf{B}\mathbf{K}$). By Second Lyapunov's Method, if at least one of eigenvalues has positive real part, the system can be considered unstable; but if all eigenvalues has negative real part, the system can be considered stable.

The generated non-dimensional power given in RMS (Φ) is calculated dividing the voltage output rate square (ν^2) by the electric resistance rate (Ψ), as shown in Eq. (15). In this project it has been used an electric resistance rate $\Psi = 0.1$ as presented by Erturk and Inman (2011) and Ferreira *et al.* (2016).

$$\Phi = \frac{\nu^2_{\text{rms}}}{\Psi} \tag{15}$$

3. RESULTS AND DISCUSSIONS

The Tab. (1) presents the unstable parameters and initial conditions used for this project, they were settled arbitrary looking for best results in energy harvesting. The angular frequency rate (ω) is a study parameter that will be used to evaluate how the power generation will be influenced by it.

Table 1. Unstable parameters and initial conditions.

ζ	χ	K	Λ	f	k_1	k_2	$y_1(0)$	$y_2(0)$	$y_3(0)$	$y_4(0)$	$y_5(0)$
0.02	0.05	0.05	0.05	0.08	0.09	0.07	0.1	0	0.1	0	0

Using the parameters in Tab. (1) it is possible obtain the Jacobian Matrix ($\mathbf{A}$) and, for the control, it was chosen the $\mathbf{Q}$, $\mathbf{B}$ and R matrices, all them presented in Eq. (16). We were looking for values that could stabilize the system rapidly, starting with initial conditions showed in Tab. (1).

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ -0.01 & -0.04 & -0.08 & 0 & 0.04 \\ 0 & 0 & 0 & 1 & 0 \\ 0.08 & 0 & -0.08 & -0.04 & 0 \\ 0 & -0.08 & 0 & 0.08 & -0.07 \end{bmatrix}, \quad Q = \begin{bmatrix} 5 & 0 & 0 & 0 & 0 \\ 0 & 10 & 0 & 0 & 0 \\ 0 & 0 & 5 & 0 & 0 \\ 0 & 0 & 0 & 5 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \quad R = [1] \tag{16}$$

Finally, Eq. (17) presents the feedback vector obtained by solving Eqs. (13) and (14) using the matrices showed in Eq. (16).

$$K = [1.3660 \quad -5.2823 \quad 2.4983 \quad 7.7929 \quad -0.5328] \tag{17}$$

3.1 Power generation in function of the angular frequency rate

Solving the systems obtained by Eq. (9) and Eq. (12) computationally, by 4^{th} Order Runge-Kutta method (numerical integrator), it was obtained a graph, Fig. (2), that compares the power generated by the systems without and with LQR control for different external excitation angular frequencies rates. We can see the resonance band only for the non-controlled system and, as expected (Ferreira *et al.*, 2015), the power generated in this area was bigger for the non-controlled system in comparison with controlled system.

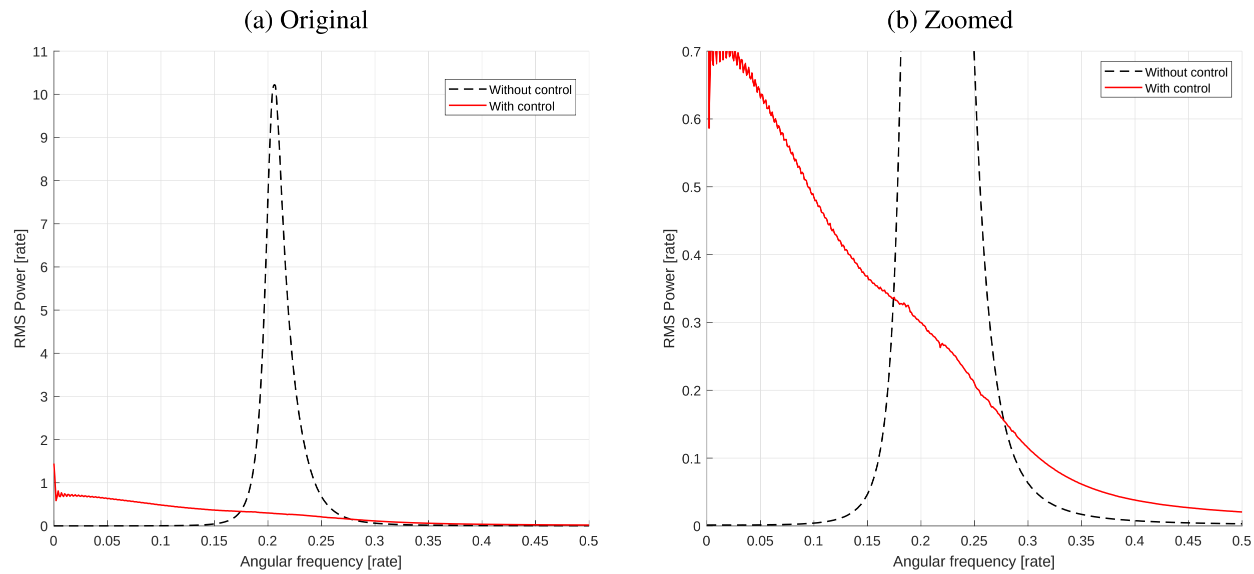

Figure 2. RMS power in function of external periodic excitation angular frequency

3.2 Influence of the variation of the angular frequency rate

It was chosen three different areas: before, during and after resonance band. For each one of them, it was generated phase portraits of displacement, velocity and voltage in function of time sample. Firstly, it will be presented the displacement and velocity phase portraits with their due discussions.

The Fig. (3) shows the displacement and velocity phase portraits for both masses considering $\omega = 0.1$.

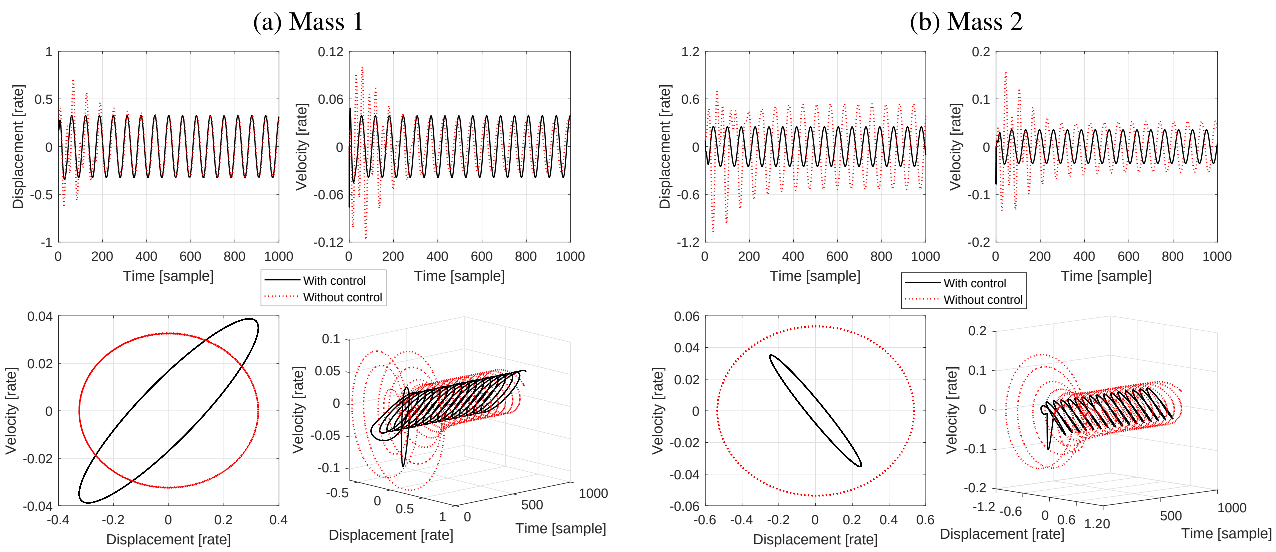

Figure 3. Phase portraits of both masses for $\omega = 0.1$

It can be seen that the transient period for the controlled system is faster when compared to the system without control. For Mass 1 we had a small amplification of velocity and displacement for the system with control. For Mass 2 we had a significant attenuation of velocity and displacement for the system with control.

The Fig. (4) shows the displacement and velocity phase portraits for both masses considering $\omega = 0.206$.

(a) Mass 1 (b) Mass 2

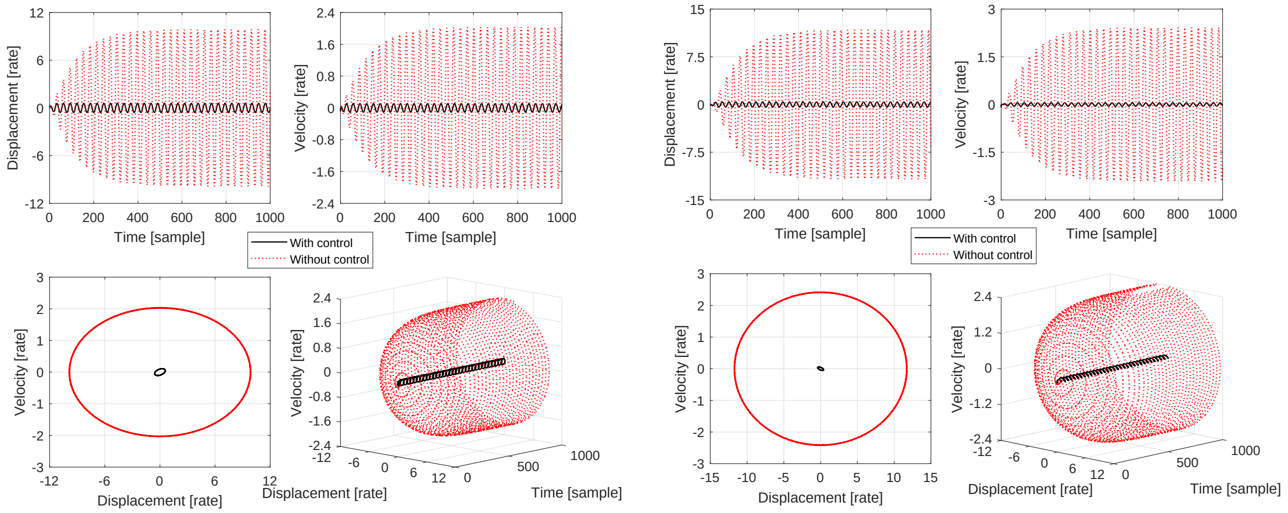

Figure 4. Phase portraits of both masses for $\omega = 0.206$

It can be seen that the transient period for the controlled system is much faster when compared to the system without control. For both masses we had a very significant attenuation of velocity and displacement for the system with control.

The Fig. (5) shows the displacement and velocity phase portraits for both masses considering $\omega = 0.35$.

(a) Mass 1 (b) Mass 2

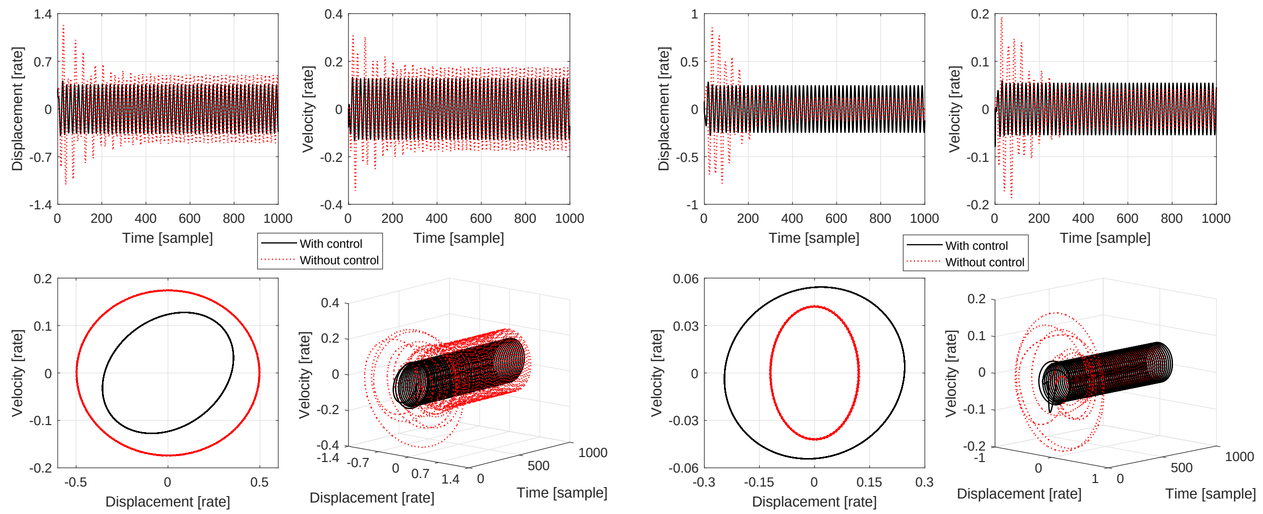

Figure 5. Phase portraits of both masses for $\omega = 0.35$

It can be seen that the transient period for the controlled system is faster when compared to the system without control. For Mass 1 we had an amplification of velocity and displacement for the system with control. For Mass 2 we had an attenuation of velocity and displacement for the system with control.

Analyzing the Fig. (3), Fig. (4) and Fig. (5) we can conclude that the LQR is efficient for all the regions when we are considering the orbit reductions for displacement and velocity. Now, it is needed analyze the voltage rate in function of time, because voltage is the variable that will be used to power generation of the system: the greater the amplitude of the voltage, the better for the energy harvesting.

Considering that for the three situations the Linear Quadratic Regulator was efficient for displacement and velocity rates, we must to found the situation that we have greater uptake looking for the best power generation. The phase portraits for the voltage can be seemed in Fig. (6).

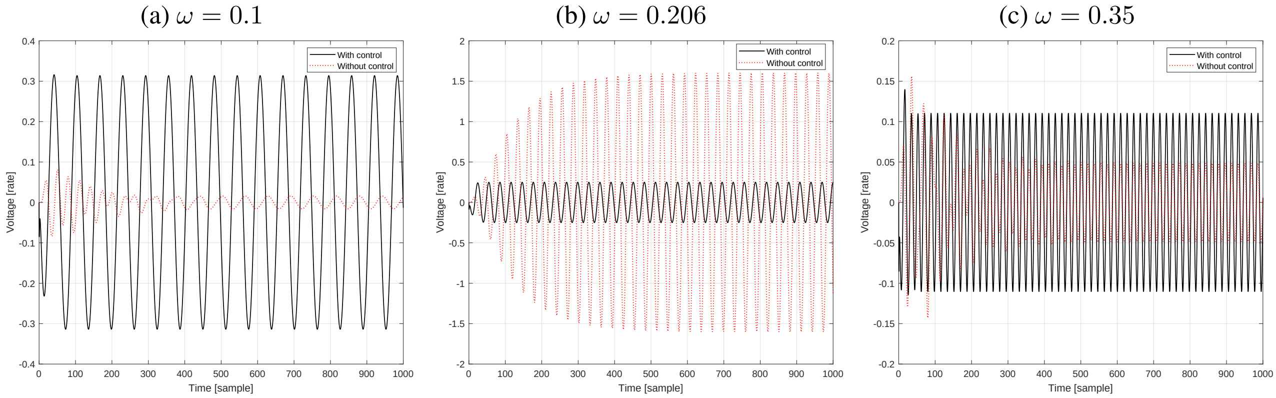

Figure 6. Voltage in function of time for three different angular frequencies rates

We can verify that the controller maximized the energy harvesting before and after resonance band areas, improving the efficiency of the dynamic system. The phase portraits presented on Fig. (6) is a complement for the graph presented on Fig. (2), since the power generation is directly dependent on the voltage.

3.3 Stability analysis by Second Lyapunov's Method

The Eqs. (18) - (20) presents the eigenvalues of original (without control) Jacobian Matrix. Considering that the real part of two first eigenvalues is positive, the system can be considered unstable by Second Lyapunov's Method.

$$\lambda_{1,2} = 0.1187 \pm j0.2551 \tag{18}$$

$$\lambda_{3,4} = -0.1588 \pm j0.2566 \tag{19}$$

$$\lambda_5 = -0.0699 + j0 \tag{20}$$

The Eqs. (21) - (24) presents the eigenvalues of controlled Jacobian Matrix ($\mathbf{A} - \mathbf{BK}$). Considering that the real part of all eigenvalues is negative, the system can be considered stable by Second Lyapunov's Method, which means that the control was efficient.

$$\lambda_1 = -5.0483 + j0 \tag{21}$$

$$\lambda_2 = -0.6449 + j0 \tag{22}$$

$$\lambda_{3,4} = -0.0910 \pm j0.2347 \tag{23}$$

$$\lambda_5 = -0.1170 + j0 \tag{24}$$

4. CONCLUSIONS

With the results obtained, it can be concluded that the Optimal Control via Linear Quadratic Regulator is only efficient, considering the power generation, when the system is outside the resonance region, evidenced by the Fig. (2) and complemented by Fig. (6).

The control will always be advantageous for any studied angular frequency since it allows the construction of the model, because the system becomes stable, and allows the operation for any angular frequency rate of the exogenous excitation as shown on phase portraits on Fig. (3), Fig. (4) and Fig. (5).

Another fact to be observed, based on Fig. (2) it is noted that, the lower the angular frequency rate, the higher power is generated. This is explained by the distance from the resonance area, a zone that makes difficult the work performed by the controller.

5. ACKNOWLEDGMENTS

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