COBEM UBERLÂNDIA 2019

ABCM Associação Brasileira de Engenharia e Ciências Mecânicas

25th ABCM International Congress of Mechanical Engineering
October 20-25, 2019, Uberlândia, MG, Brazil

SEMI-EMPIRICAL MODELING OF SINGLE-SPEED REFRIGERATING COMPRESSORS FOR SMALL CAPACITY APPLICATIONS AIMING AT RANGE EXTRAPOLATION AND REDUCED DATASETS

Guilherme Z. Santos - guizsantos@polo.ufsc.br
Christian J. L. Hermes
POLO Laboratories, Mechanical Engineering Department, Federal University of Santa Catarina – Florianópolis, SC, Brazil

Adriano F. Ronzoni
Embraco – Joinville, Santa Catarina, Brazil

Abstract. *The present work puts forward a semi-empirical methodology to modeling the vapor compression process in a piston-cylinder system. The proposed model provides the mass flow rate and the power consumption as a function of the working pressures and the inlet temperature, making use of only 6 fitting coefficients, all with physical meaning. Due to its semi-empirical nature, the model can extrapolate the fitting range without compromising its accuracy. Catalog data of 26 conventional fixed-capacity reciprocating compressors with displacements ranging from 2 to 20 cm³ were used to validate the model. Out of all data points, 97% presented errors within the band of ±10% for the mass flow rate, while 93% of the data points for the compression power were within the same error band. Additionally, the model performance was evaluated under extrapolation and reduced data set conditions. In this regard, the model performed robustly with 98% of all data points within the ±20% error band for all cases.*
Keywords: *reciprocating compressor, semi-empirical model, catalog data, performance characteristics*

1. INTRODUCTION

Hermetic reciprocating compressors are widely used in household and light commercial refrigerators. In most appliances, the refrigerant is displaced by an alternating piston driven by a crank mechanism powered by an electric motor (Tuhovcak et al., 2016), although linear compression technology has reached the market of domestic refrigeration in the last years (Liang, 2017). Several mathematical models have been proposed for compressors used in domestic refrigerators. In general, empirical models (polynomial interpolations) are the most used (ASHRAE, 2005; AHRI, 2015), albeit they are not recommended to be applied outside the data span due to their lack of physical scales (Pottker and Melo, 2002).

On the one hand, detailed first-principles models (Prata *et al.*, 1994), besides requiring a prohibitive computational cost, particularly when simulating the performance of the compressor in the cooling system (Diniz *et al.*, 2018), do still need a considerable amount of closing parameters such as piston-cylinder clearances, dead volume, valve geometry and the physical properties of the oil, which are not easily obtained from manufacturers. On the other hand, semi-empirical models use the thermodynamics to find out the scales that govern the key phenomena, which in turn are calibrated from experimental data, demanding fewer coefficients than polynomials and allowing extrapolations without prohibitive errors, so that model can be fitted based on a smaller data set.

The present work proposes a semi-empirical model for conventional, single-speed reciprocating compressors making use 3 fitting coefficients only (3 for mass flow rate and 3 for power draw). An interpretation of the fitting coefficients meanings, and also the model extrapolation capabilities is also presented. Finally, the use of a lower set of fitting coefficients is explored and reported.

2. FORMULATION

From reciprocating compressor theory (Gosney, 1982), the compression work can be calculated as follows

$$\dot{W}_{\mathrm{id}} = \dot{m} \oint v dp \qquad (1)$$

Considering the indicated diagram of an ideal compressor with dead volume presented in Fig. 1, assuming ideal gas behavior for the refrigerant, and that the compression and expansion processes are isentropic, integration of Eq. 1 yields:

$$\frac{\dot{W}_{\text{id}}}{\dot{m}} = RT_s \frac{k}{k-1}\left[\left(\frac{p_d}{p_s}\right)^{\frac{k-1}{k}} - 1\right] \tag{2}$$

where the indices 's' and 'd' stand for suction and discharge, respectively, T is the temperature, p the pressure, k the specific heat ratio (also known as isentropic exponent) and R the specific gas constant of the refrigerant.

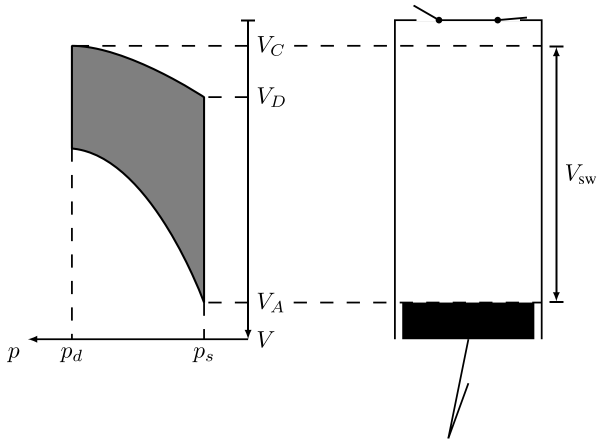

Figure 1. Schematics of a reciprocating compressor compression along with an indicated diagram

Furthermore, part of the net power input for the compressor is not used in the compression process, due to irreversibilities that take place, thereby being released as heat to the surrounding environment. Therefore, an energy balance in the compressor results in $\dot{W} = \dot{Q} + \dot{m}(hd - hs)$, where $\dot{Q}$ is the released heat. From the isentropic efficiency definition, η_s, it follows that

$$\eta_s = \frac{\dot{W}_{\text{id}}}{\dot{m}(h_d - h_s)} \tag{3}$$

Assuming that $\dot{Q} = \dot{m}q$, and applying the definition of isentropic efficiency, Eq. 2 can be re-written as:

$$\dot{W} = \dot{m}\left\{T_s \frac{R}{\eta_s}\frac{k}{k-1}\left[\left(\frac{p_d}{p_s}\right)^{\frac{k-1}{k}} - 1\right]\right\} \tag{4}$$

Considering that $R,\ k,\ q$ and η_s are fairly constant, the following semi-empirical expression is obtained:

$$\dot{W} = \dot{m}\left\{a_0 T_s \left[\left(\frac{p_d}{p_s}\right)^{a_1} - 1\right] + a_2\right\} \tag{5}$$

where the coefficients $a_{0..2}$ must be fitted based on catalog data.

The mass flow rate pumped by the compressor may be calculated from (Gosney, 1982):

$$\dot{m} = \frac{N V_{\text{sw}}}{v_s}\eta_v \tag{6}$$

where N is the compressor speed and η_v the volumetric efficiency, defined as follows:

$$\eta_v = \frac{V_A - V_D}{V_{\text{sw}}} \tag{7}$$

From the assumption of isentropic compression and expansion, it follows that $V_D = V_C\,(p_d/p_s)^{1/k}$. Defining the compressor clearance, $C = V_C/V_{\text{sw}}$, thus, $V_A = V_{\text{sw}} + V_C = V_{\text{sw}}(1 + C)$. Thus, applying such definitions to Eq. 6 and invoking once again the assumption of ideal gas, it follows that

$$\dot{m} = \frac{N p_s V_{sw}}{R T_s}\left[1 + C - C\left(\frac{p_d}{p_s}\right)^{\frac{1}{k}}\right] \tag{8}$$

Considering $C,\ k,\ R$ and V_{sw} nearly constant, the following semi-empirical expression is obtained:

$$\dot{m} = \frac{N p_s}{T_s}\left\{b_0 - b_1\left[\left(\frac{p_d}{p_s}\right)^{b_2} - 1\right]\right\} \tag{9}$$

where the coefficients $b_{0..2}$ must be fitted from experimental data.

3. RESULTS AND DISCUSSIONS

Conventional reciprocating compressors have constant volumetric displacements and operate under constant speed, such that Equations 5 and 9 can be used straightforwardly. It can be noticed that the speed N could have been embedded into coefficients $b_{0..1}$ but it was left out so that the model remains sensitive to changes in speed and therefore may be applied to variable-speed compressors as well (Santos et al., 2019).

The fitting coefficients were determined by means of the quadratic error minimization method, also known as least squares best fit. A programming code, written in python, uses the Trust Region Reflective method (Jones *et al.*, 2001) to minimize the error. The mass flow rate fitting coefficients are determined first, since the power consumption (Eq. 5) requires that the mass flow rate is known and, in order to minimize the error propagation, the error minimization for the power consumption fit is performed using the calculated mass flow rate instead of the experimental value.

The proposed model (Equations 5 and 9) was applied to 26 single-speed compressors, with displacement ranging from 2 to 20 cm^3, using calorimeter catalog data obtained at the manufacturer website. Figure 2 shows the relative error of the model for mass flow rate and compression power, classifying the results with respect to the compressor size. The results shown in Fig. 2 made use of the entire data set (18 data points: condensing and evaporating temperatures of, respectively 35, 45 and 55°C; and -35, -30, -25, -20, -15 and -10°C) to fit the model parameters.

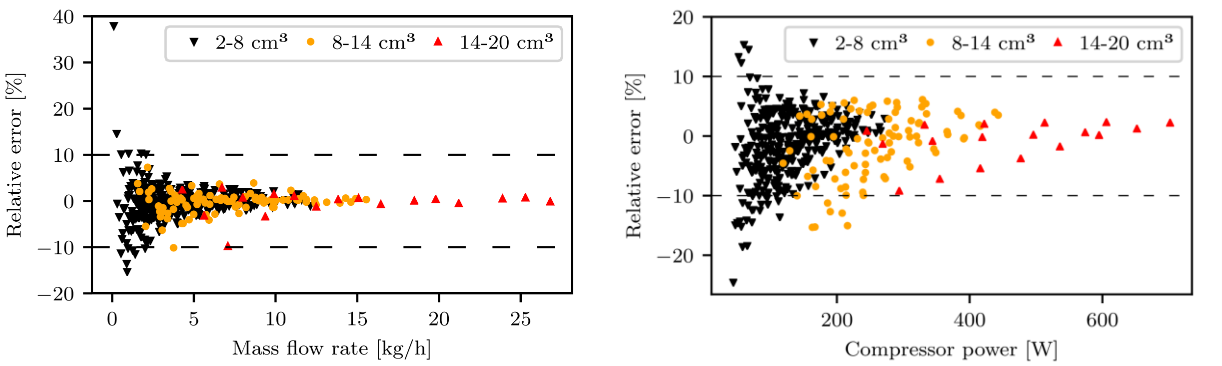

Figure 2. Percentage error of mass flow rate (left) and compression power (right) for single-speed compressors

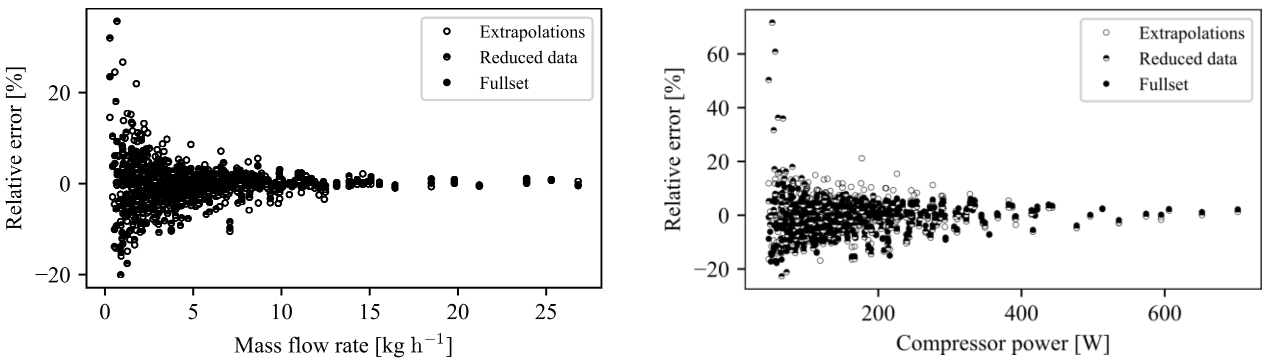

Figure 3. Percentage error of mass flow rate (left) and compression power (right) considering different data sets

Moreover, the model was also fitted with restrained data sets, aiming at evaluating its capabilities regarding (i) extrapolation of the fitting range, and (ii) accuracy when data sets are quite small but cover the entire range. To evaluate case (i) the outer envelope of the data set was not used to minimize the error although the entire data set was used for error verification. In other words, the fitting range comprises data at 45°C of condensing temperature and -30 to -15°C of evaporating temperature (4 data points). For case (ii), in turn, only the extreme points of the data set were used during the fitting process, so that the fitting range is the same as the entire data set but only 4 data points are used to obtain the model coefficients. Figure 3 shows the relative error of the model predictions for both cases, as well as when the entire data set is used.

Finally, it is possible to further reduce the number of fitting coefficients. Considering that the compressor stroke is known beforehand and that the thermodynamic properties k and R can also be calculated, the following expressions are

$$\dot{W} = \dot{m}\left\{T_s \frac{R}{a_0}\frac{k}{k-1}\left[\left(\frac{p_d}{p_s}\right)^{\frac{k-1}{k}} - 1\right]\right\} \quad (10)$$

$$\dot{m} = \frac{Np_s}{T_s}\left\{b_0 - b_1\left[\left(\frac{p_d}{p_s}\right)^{\frac{1}{k}} - 1\right]\right\} \quad (11a)$$

$$\dot{m} = \frac{NV_{sw}p_s}{T_s}\left[1 - b_0\left[\left(\frac{p_d}{p_s}\right)^{\frac{1}{k}} - 1\right]\right] \quad (11b)$$

where Eq. 10 and 11a use k calculated from inlet conditions (p_s, T_s) and Eq. 11b uses the nominal stroke of the compressor, requiring only one fitting coefficient. Figure 4 shows the predictions errors when Equations 10 and 11 are employed in comparison to the proposed model. It can be noticed, from Fig. 4, that using the calculated specific heat ratio (Equations 10 and 11a) slightly improves the mass flow rate predictions but, in turn, jeopardizes the power draw predictions. As for the one-coefficient fit, Eq. 11b, it is s clear that the model performance strongly decayed, albeit the errors may be acceptable for larger compressors (>10 cm^3) applications.

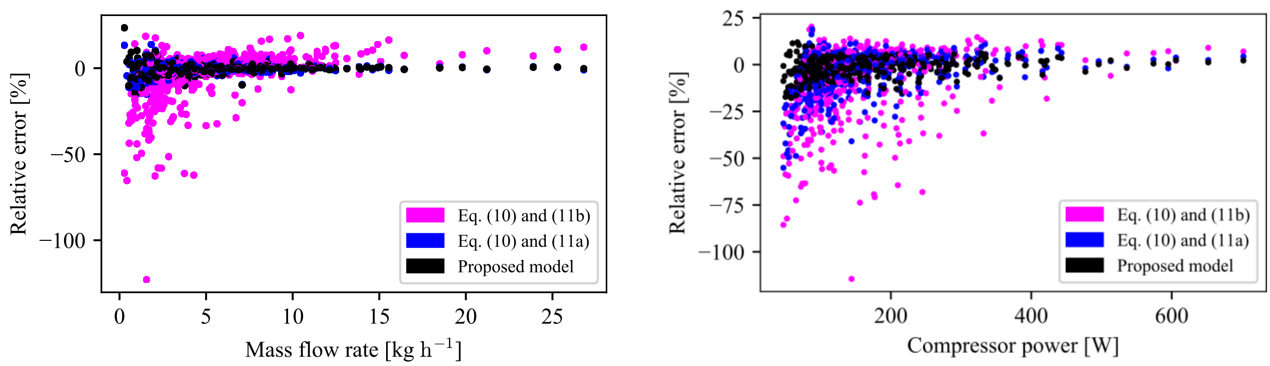

Figure 4. Percentage error of mass flow rate (left) and compression power (right) considering fewer coefficients

4. SUMMARY AND CONCLUSIONS

A semi-empirical model for the vapor compression process which the refrigerant undergoes in reciprocal compressors was developed based on the fundamental theory exposed by (Gosney, 1982). The proposed model uses only 3 fitting coefficients for each equation, 7 less than required according to the standard ASHRAE (2005), which implies a smaller number of experiments required (3 at minimum). The proposed model was validated against catalog data obtained from manufacturer of hermetic compressors available in the market. In order to assess the impact of the compressor size, a wide range of alternative single-speed (conventional) compressors sizes were considered (2 to 20 cm^3).

For the conventional compressors analyzed, 97% of the data points showed errors within ±10% error band for the mass flow rate, while 93% of the compression power data points fell within the same error thresholds. No errors higher than 20% were observed, with the exception of the compressor with smallest displacement (2.3 cm^3) which had a mass flow rate overestimation by 40% under the conditions of highest-pressure ratio. From Fig. 2 it can be noticed that higher compressor displacements lead to higher model performance (small errors), which is expected since piston-cylinder leakages, not captured by the proposed model, become more expressive as the compressor decreases in size.

The model performance was tested when subjected to extrapolations and heavily reduced data sets. In such tests, the model presented errors that did not differ significantly from the baseline (full data set fitting) for most data points, with the exception of the smallest compressor analyzed, which showed unreasonable errors of up to 70 %.

Furthermore, the use of fewer coefficients was also explored. Considering to calculate the specific heat ratio k instead of fitting it allows to drop one coefficient. By doing so, the calculated mass flow rate slightly improved, though at the cost of jeopardizing the calculated power consumption. Further reducing the number of coefficients was only possible for the mass flow rate, considering using the nominal stroke instead of fitting a coefficient for it. By doing so, a one-coefficient expression for the mass flow rate was obtained, Fig. 4 shows that the errors from the use of such expression are unreasonable for small compressors, but may be acceptable for larger ones (>10 cm^3).

As a complementary perk of the presented model, one can obtain an estimate of the compressor stroke based on the fitted coefficient b_0. Since $b_0 = V_{\rm sw}/R$, thereby $V_{\rm sw} = Rb_0$. Using this strategy, the model was also able to predict the displacement of the analyzed compressors with an RMS deviation of 8 %. It can also be observed, that parameter a_2 from Eq. 5 is strongly associated with the compressor inefficiency, as it represents the energy loss (energy not transferred to the refrigerant).

5. ACKNOWLEDGEMENTS

This work is part of a technical and scientific partnership between POLO and Embraco. Financial support from CNPq, CAPES and EMBRAPII is duly acknowledged.

6. REFERENCES

AHRI, 2015. “Performance rating of positive displacement refrigerant compressors and compressor units”. Standard 540, Air-Conditioning, Heating and Refrigeration Institute.

ASHRAE, 2005. “Methods of testing for rating positive displacement refrigerant compressors and condensing units”. Standard 23, American Society of Heating, Refrigerating and Air-Conditioning Engineers, Atlanta.

Diniz, M.C., Melo, C. and Deschamps, C.J., 2018. “Experimental performance assessment of a hermetic reciprocating compressor operating in a household refrigerator under on–off cycling conditions”. Int. J. Refrig., Vol. 88, pp. 587 – 598.

Gosney, W.B., 1982. Principles of Refrigeration. Cambridge University Press, Cambridge; New York.

Jones, E., Oliphant, T., Peterson, P. et al., 2001. “SciPy: Open source scientific tools for Python”. URL http://www.scipy.org/.

Liang, K., 2017. “A review of linear compressors for refrigeration”. Int. J. Refrig., Vol. 84, pp. 253 – 273.

Pottker, G. and Melo, C., 2002. “Ajuste de curvas para o fluxo de massa e a potência consumida em compressores. Parte i”. Relatório de pesquisa, Universidade Federal de Santa Catarina, Florianópolis, SC, Brasil.

Prata, A., Ferreira, R., Fagotti, F. and Todescat, M., 1994. “Heat transfer in a reciprocating compressor”. In International Compressor Engineering Conference at Purdue. West Lafayette, IN, USA, pp. 605 – 610.

Tuhovcak, J., Hejcik, J. and Jicha, M., 2016. “Comparison of heat transfer models for reciprocating compressor”. Applied Thermal Engineering, Vol. 103, pp. 607–615.

Santos, G.Z., Ronzoni, A.F., Hermes, C.J.L., 2019. “Performance characterization of small variable-capacity reciprocating compressors using a minimal dataset”. Int. J. Refrig., In Press.