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TOPOLOGY OPTIMIZATION OF STRUCTURES SUBJECTED TO DYNAMIC LOADS USING EQUIVALENT STATIC LOADS

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Abstract. *Topology optimization in the time domain of structures subjected to time-dependent loads is usually computationally expensive, starting with the large number of time-dependent analyses that are required. Furthermore, the computational cost to evaluate the gradients of the response is significantly high and requires a large storage space. In this paper, instead of solving the original problem directly, we solve a sequence of static response optimization problems with many load cases. This approach, called the Equivalent Static Loads Method (ESLM), generates static loads that produces a similar response in comparison to the same response that the dynamic analysis does. The optimization of two cantilever beam models under dynamic forces is presented as a validation of the ESL implementation.*

Keywords: *Structural optimization, equivalent static loads, multiple loading condition, topology optimization.*

1. INTRODUCTION

According to the nature of the imposed load, the structural optimization can be based on the static response or the dynamic response of the structure. However, the most of existing structures in the real world are subjected to dynamic loads such as wind, sea waves, and/or vibration of rotating machines, and the use of static loads can not give accurate solutions. For example steel towers for wind turbines, pier and bridges (Chopra, 1995; Clough and Penzien, 1993; Tong, 2010).

Optimization in the time domain of structures subjected to time-dependent loads demands large computational cost because a dynamic analysis as well as the sensitivities is required during each iteration. So, this kind of analysis is limited for a small structures and simplified models with a few degrees of freedom (Choi and Park, 2002). To solve this problem, the Equivalent Static Loads (ESL's) method has been investigated for several problems in the past (Kang *et al.*, 2001; Choi *et al.*, 2005; Park, 2011).

Considering a full evaluation of the dynamic equation, there is the work by Behrou and Guest (2017) where the dynamic response was calculated using the Newmark-β integration method. Other researches also have a significant contribution in relation to this study, such as Kang *et al.* (2001), Choi and Park (2002) and Zhao and Wang (2016).

In this contribution, we use the Equivalent Static Loads based on displacements to represent a dynamic load and apply it in a structural topology optimization as a way to reduce the computational costs achieving the optimization objectives without damaging the final structural design.

2. TOPOLOGY OPTIMIZATION

Topology optimization consists of finding the most efficient distribution of material in a previously specified domain region, satisfying the design constraints (Bendsøe and Sigmund, 1999). In the analyzed problems here, the objective function to be minimized is the compliance, which is equivalent to maximize the stiffness. The Finite Element Method (FEM) is used for the structural analysis.

In this paper, topology optimization uses the density method. A continuous variable ρ, $0 \leqslant \rho \leqslant 1$ is introduced, resembling a density of material. The Solid Isotropic Material with Penalization (SIMP) (Zhou and Rozvany, 1991; Bendsøe, 1989)) is used to recover the binary nature of the problem. In computations, a small lower bound is usually imposed, in order to avoid a singular FEM problem, when solving the equilibrium equations. At each iteration the structure layout is updated using the optimality criteria method.

2.1 Topology optimization of structures considering many independent load cases

Practical structural designs typically involve many load cases. In the topology optimization setting, the objective function can be defined as a sum of the compliance under each load case. For a set of m given load cases $\mathbf{F}_i, i = 1, \ldots, m$, the optimization problem can be written as,

$$\begin{cases} \text{min} & \phi_s(\mathbf{u}(\mathbf{b})) = \sum_{i=1}^{m} \mathbf{F}_i^T \, \mathbf{u}_i(\mathbf{b}), \\ \text{s.t.:} & \sum_{e=1}^{n} V_e - V_{max} \leqslant 0 \\ & 0 < b_{min} \leqslant b_e \leqslant b_{max}, \quad e = 1, ..., n \\ \text{with} & \mathbf{K}(\mathbf{b})\mathbf{u}_i(\mathbf{b}) = \mathbf{F}_i, \quad i = 1, ..., m, \end{cases} \tag{1}$$

where $\mathbf{b}$ is the vector of design variables, $\mathbf{F}$ and $\mathbf{u}$ denotes the vector of external forces and displacements, respectively. V_e and V_{max} correspond to volume of each element and the maximum volume allowed. $\mathbf{K}$ represents the stiffness matrix of the system and n is the number of elements which the structure is discretized. The sensitivity of the objective function, calculated by the adjoint method, is expressed as

$$\frac{d\phi_s}{db_j} = -\sum_{i=1}^{m} \mathbf{u}_i^T \frac{\partial \mathbf{K}_i}{\partial b_j} \mathbf{u}_i. \tag{2}$$

2.2 Topology optimization of structures considering transient loads

The optimization problem for structures subjected to transient loads can be posed as

$$\begin{cases} \text{min} & \phi_d(\mathbf{u}(\mathbf{b}), t) = \int_{t_0}^{t_f} \mathbf{F}^T(t)\mathbf{u}(t)\mathrm{d}t \\ \text{s.t.:} & \sum_{e=1}^{n} V_e - V_{max} \leqslant 0 \\ & 0 < b_{min} \leqslant b_e \leqslant b_{max}, \qquad e = 1, \ldots, n \\ \text{with} & \mathbf{M}(\mathbf{b})\ddot{\mathbf{u}}(t) + \mathbf{C}(\mathbf{b})\dot{\mathbf{u}}(t) + \mathbf{K}(\mathbf{b})\mathbf{u}(t) = \mathbf{F}(t), \qquad t = t_0, ..., t_f, \end{cases} \tag{3}$$

where ϕ_d is the dynamic compliance, $\mathbf{M}$ and $\mathbf{C}$ are the mass and damping matrices, and $\mathbf{u}$, $\dot{\mathbf{u}}$ and $\ddot{\mathbf{u}}$ are the displacement, velocity and acceleration vectors. The equation of motion, last line of Eq. 3, is discretized in time and solved by the implicit integration method Newmark-β, which computes the structural response at time $t + \Delta t$ as

$$\mathbf{M}(\mathbf{b})\ddot{\mathbf{u}}_{t+\Delta t} + \mathbf{C}(\mathbf{b})\dot{\mathbf{u}}_{t+\Delta t} + \mathbf{K}(\mathbf{b})\mathbf{u}_{t+\Delta t} = \mathbf{F}_{t+\Delta t} \tag{4}$$

The displacement, velocity and acceleration at time $t_0 = 0$, denoted by $\mathbf{u}_0$, $\dot{\mathbf{u}}_0$, $\ddot{\mathbf{u}}_0$, respectively, are known. The solution is obtained for l equals time intervals, i.e., $\Delta t = t_f/l$, as summarized in Table 1. The dynamic compliance ϕ_d can be rewritten as

$$\phi_d(\mathbf{u}(\mathbf{b})) = \sum_{i=1}^{l} \mathbf{F}_i^T \, \mathbf{u}_i(\mathbf{b}) \tag{5}$$

and the derivative of ϕ_d with respect to the design variable b_j can be obtained by

$$\frac{d\phi_d}{db_j} = \sum_{i=1}^{l} \mathbf{F}_i^T \frac{\partial \mathbf{u}_i}{\partial b_j}. \tag{6}$$

For the calculation of the dynamic compliance sensitivity, Eq. 6, is necessary to obtain the displacements derivatives with respect to the design variable b_j for each time step. The direct method of calculating the response sensitivity is obtained by differentiating dynamic equation of motion to obtain

$$\mathbf{M}\frac{\partial \ddot{\mathbf{u}}(t)}{\partial b_j} + \mathbf{C}\frac{\partial \dot{\mathbf{u}}(t)}{\partial b_j} + \mathbf{K}\frac{\partial \mathbf{u}(t)}{\partial b_j} = \frac{\partial \mathbf{F}(t)}{\partial b_j} - \frac{\partial \mathbf{M}}{\partial b_j}\ddot{\mathbf{u}}(t) - \frac{\partial \mathbf{C}}{\partial b_j}\dot{\mathbf{u}}(t) - \frac{\partial \mathbf{K}}{\partial b_j}\mathbf{u}(t), \qquad t = t_0, ..., t_f, \tag{7}$$

The Newmark-β used to solve Eq. 4 can be used to solve Eq. 7. Note that we need to solve as many dynamic equations as the number of design variables which makes such approach not efficient for topology optimization where we have a large number of design variables. An efficient alternative is the Adjoint variable method, not addressed in this paper.

Table 1. Step-by-step solution using Newmark-β integration method

A - Initial calculations:
1− Form stiffness matrix $\mathbf{K}$, mass matrix $\mathbf{M}$, and damping matrix $\mathbf{C}$. 2− Define vectors of initial displacement $\mathbf{u}_0$ and initial velocity $\dot{\mathbf{u}}_0$ and calculate initial acceleration $\ddot{\mathbf{u}}_0$: $\ddot{\mathbf{u}}_0 = \mathbf{M}^{-1}\left(\mathbf{F}_0 - \mathbf{C}\dot{\mathbf{u}}_0 - \mathbf{K}\mathbf{u}_0\right)$ 3− Select time step Δt and parameters γ and β and calculate integration constants c_0, c_1, c_2, c_3, c_4 and c_5 $c_0 = \frac{1}{\beta \Delta t^2}$ $\quad c_1 = \frac{\gamma}{\beta \Delta t}$ $\quad c_2 = \frac{1}{\beta \Delta t}$ $c_3 = \frac{1}{2\beta} - 1$ $\quad c_4 = \frac{\gamma}{\beta} - 1$ $\quad c_5 = \Delta t\left(\frac{\gamma}{2\beta} - 1\right)$ 4− Form effective stiffness matrix $\bar{\mathbf{K}} = \mathbf{K} + c_1\mathbf{C} + c_0\mathbf{M}$
B - For each time step:
1− Calculations of predictors: $\tilde{\mathbf{a}} = c_0\mathbf{u}_t + c_2\dot{\mathbf{u}}_t + c_3\ddot{\mathbf{u}}_t$ $\tilde{\mathbf{v}} = c_1\mathbf{u}_t + c_4\dot{\mathbf{u}}_t + c_5\ddot{\mathbf{u}}_t$ 2− Calculate effective loading : $\bar{\mathbf{F}} = \mathbf{F}_{t+\Delta t} + \mathbf{M}\tilde{\mathbf{a}} + \mathbf{C}\tilde{\mathbf{v}}$ 3− Solve for displacements at time $t + \Delta t$: $\mathbf{u}_{t+\Delta t} = \bar{\mathbf{K}}^{-1}\bar{\mathbf{F}}$ 4− Calculate velocities and accelerations at time $t + \Delta t$: $\dot{\mathbf{u}}_{t+\Delta t} = c_1\mathbf{u}_{t+\Delta t} - \tilde{\mathbf{v}}$ $\ddot{\mathbf{u}}_{t+\Delta t} = c_0\mathbf{u}_{t+\Delta t} - \tilde{\mathbf{a}}$

3. EQUIVALENT STATIC LOADS

The equivalent static load method was introduced by Choi and Park (2002) and Kang et al. (2001), where static response optimization problem Eq. 1 is successively solved to obtain the solution of the transient problem Eq. 3. The flowchart in Fig. 1 shows how ESL method is used in the topology optimization. Note that the process starts with the initial design variables $\mathbf{b}^k = \mathbf{b}^0$, considering cycle $k = 0$. Then the dynamic equilibrium equation, presented in last line of Eq.(3), is solved and the equivalent static loads are obtained according to the Eq. (8).

$$\mathbf{F}_{ESL}(t) = \mathbf{K}(\mathbf{b})\mathbf{u}(t) = \mathbf{F}(t) - \mathbf{M}(\mathbf{b})\ddot{\mathbf{u}}(t) - \mathbf{C}(\mathbf{b})\dot{\mathbf{u}}(t) \tag{8}$$

The analysis time has been divided in Δt size time steps and each one of these discrete intervals will correspond to a one load case. Thus, the time variable t is replaced by s representing the number of load cases, i.e., the number of ESL sets. The topology optimization is performed with $\mathbf{b}^k$, and $\mathbf{F}$ becomes equal to the ESL vector $\mathbf{F}_{ESL}$ and the optimization problem to be computed is similar to the problem of Eq. (1). One may note that the number of ESL sets s is the same as the time steps l used in dynamic analysis. After topology optimization, the minimized design variables $\mathbf{b}^*$ is checked through the OC-type update scheme within `PolyTop` (Talischi *et al.*, 2012). If the convergence has not been achieved, the design variables are updated and a new cycle $k + 1$ needs to be performed with $\mathbf{b}^{(k+1)} = \mathbf{b}^*$.

4. NUMERICAL EXAMPLES

The topology optimization of analyzed problems in this paper was performed using the `PolyTop` program (Talischi *et al.*, 2012) developed in MATLAB with some modifications. Regarding loading conditions, the option to use multiple load cases has been added in `PolyTop`. Furthermore, a function using the Newmark-β integration method to calculate the dynamic response in time domain has been created. This same function also calculates the sensitivity of the objective

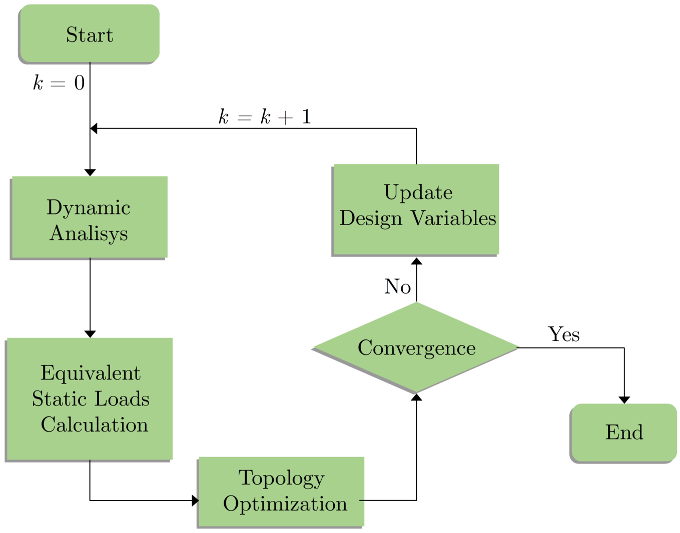


Figure 1. Flowchart optimization process using ESL, where k represents cycles of optimization using ESL

function and the design variables. In addition, the mass matrix, the damping matrix and its derivatives calculations was also added to the `PolyTop` routine.

The final topology was reached using filter of radius and SIMP model with continuation executed with penalty parameter p increased from 1 to 3 and increments of 0.5. For each value of penalty parameter a maximum number of 20 iterations is allowed. When $p = 3$, a maximum of 100 iterations is allowed.

In a first approach, we analyze the models as a static problem submitted to two different load case condition: first the optimized structure is subjected to a single load case and thereafter the optimization is performed considering two load cases. To proceed with structural analysis taking into account the dynamic behavior it was used the Newmark-β integration method. Finally, ESL procedure is used for optimization of the two cantilever beam. For the both examples, some analyzes were performed considering cases described in Tab. 2

Table 2. Analyzed cases.

	Description
Initial	Material uniformly distributed in the structure domain
Case 1	Optimization with static analisys and one load case
Case 2	Optimization with static analisys and two load cases
Case 3	Optimization considering dynamic analisys
Case 4	Optimization using ESL's

4.1 Cantilever Beam 1

In this first example, we analyze the cantilever beam submitted to a load condition as shown in Fig. 2 and its and properties are presented in Tab. 3. Initially, we investigate this problem using the maximum values of dynamic forces F_1 and F_2 shown in Fig. 2 that corresponds to $F_{1max} = F_{2max} = 1.0 \times 10^4\ N$. The designs obtained after topology optimization are illustrated in Fig. 3(a) and Fig. 3(b).

It is important to emphasize the need to performing the topological optimization taking into account that, for loads occurring at different points of the structure, even if simultaneously, it should be considered as different load cases. This is because the obtained topology does not have a good performance in dynamic analysis. In other words, the topology may not reflect the physical interpretation of the model. In Fig. 3(a), it is noticed that there is no material distributed in the domain region from de support to the middle, because the stress are zero in this region leading to the absence of material. This result shows that the stress due the load tends to cancel each other when the objective function does not consider the summation of the responses as two independent loads. The topology shown in Fig. 3(b) better displays the physical interpretation of the problem.

Table 3. Parameters for the Cantilever beam 1.

	parameter	value	units
Radius	r_{min}	0.01375	m
Young's modulus	E_0	2.0×10^{11}	Pa
Poisson's ratio	ν	0.33	-
Beam length	L	0.9	m
Beam thickness	$thick$	0.01	m
Time	t_f	π	s
Time step size	Δt	$\pi/20$	s
Maximum allowable volume	V_{max}	0.35	-

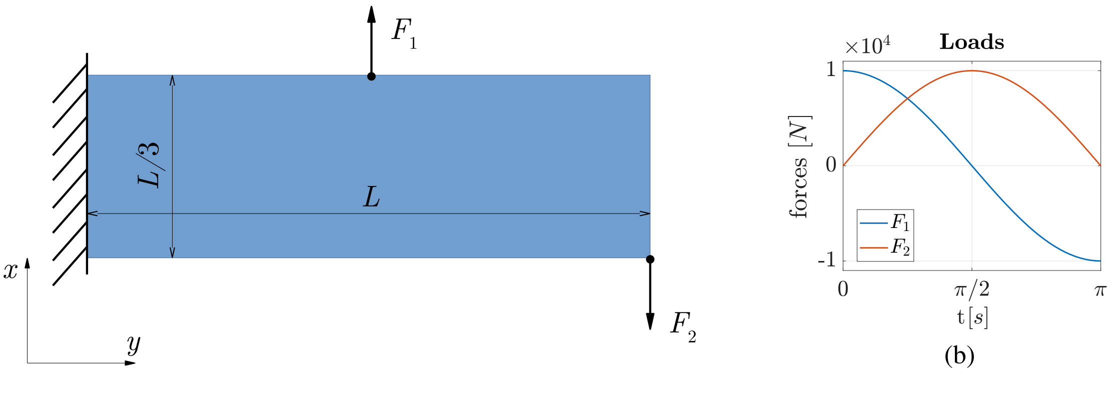


Figure 2. Cantilever beam 1: (a) analyzed model and (b) applied loads.

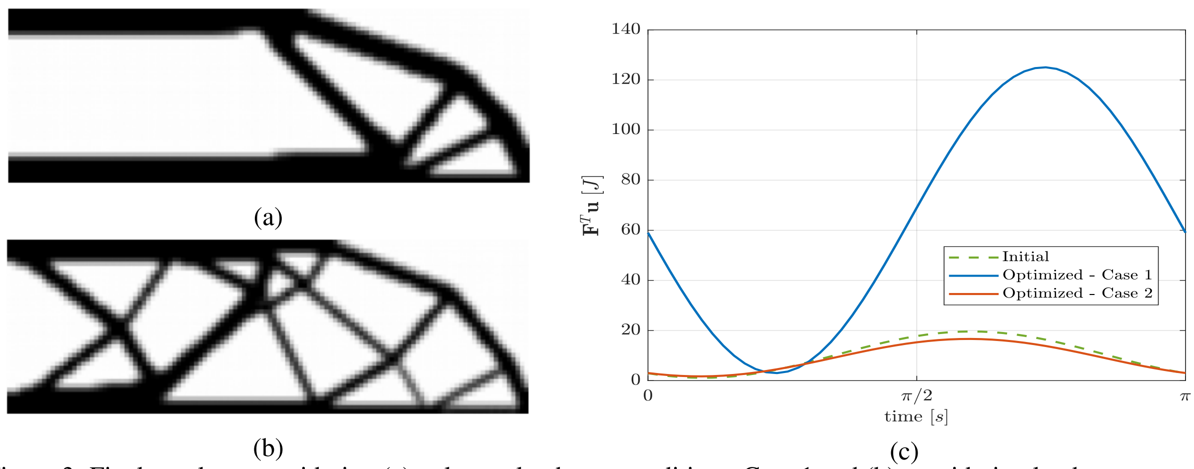


Figure 3. Final topology considering (a) only one load case condition - Case 1 and (b) considering load cases separately - Case 2. In (c) its illustrated comparison among objective function obtained in each analyzed cases.

For this example, the topology optimization based on the dynamic response of the structure is depicted in Fig. 4(a) and Fig. 4(b) depicts the final topology of the beam using the equivalent static loads. From a visual evaluation, we may state that ESL method provide a very good approximation of the dynamic loading case.

As displayed in Fig. 4, the final designs in Case 3 and Case 4 are in a good agreement. The Fig. 4(c) show us that a objective function was minimized after topology optimization and Tab. 4 summarizes the objective function values for each analyzed case.

4.2 Cantilever Beam 2

This second model was published in Zhao and Wang (2016) and also corresponds to a cantilever beam, but with the loads applied in different directions and points, as shown in Fig. 5. The properties of this model are shown in Tab. 5. Such as previously performed, we investigate the static problem for this model with the maximum values of F_1 and F_2 that

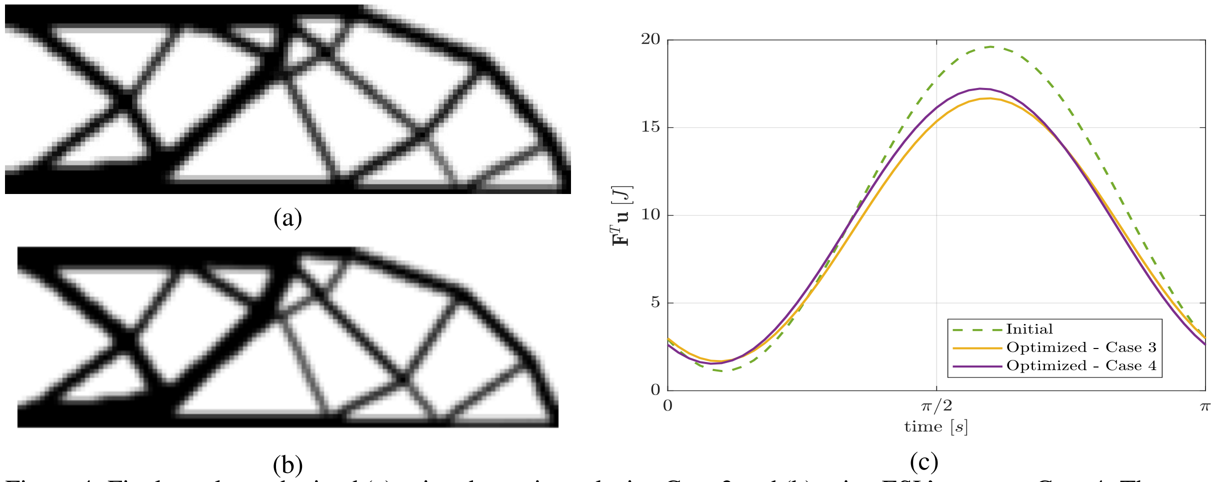


Figure 4. Final topology obtained (a) using dynamic analysis - Case 3 and (b) using ESL's vector - Case 4. The comparison among objective function calculated in each analyzed case are shown in (c).

Table 4. Objective function minimized values for Cantilever beam 1.

	Initial	**Case 1**	**Case 2**	**Case 3**	**Case 4**
Compliance	1036.60	6402.00	917.52	916.64	937.13

corresponds to $F_{1max} = 2.0 \times 10^3\ N$ and $F_{2max} = 1.0 \times 10^4\ N$, respectively (see Fig. 5(b)). Figure 6(a) illustrates the topology if only one load case were considered and Fig. 6(b) depicts the final topology when the two forces are considered like two load cases separately.

Table 5. Parameters for the Cantilever beam 2.

	parameter	value	units
Radius	r_{min}	0.20	m
Young's modulus	E_0	2.0×10^{11}	Pa
Poisson's ratio	ν	0.3	-
Beam length	L	12	m
Beam thickness	$thick$	0.01	m
Time	t_f	0.5	s
Time step size	Δt	0.025	s
Maximum allowable volume	V_{max}	0.50	-

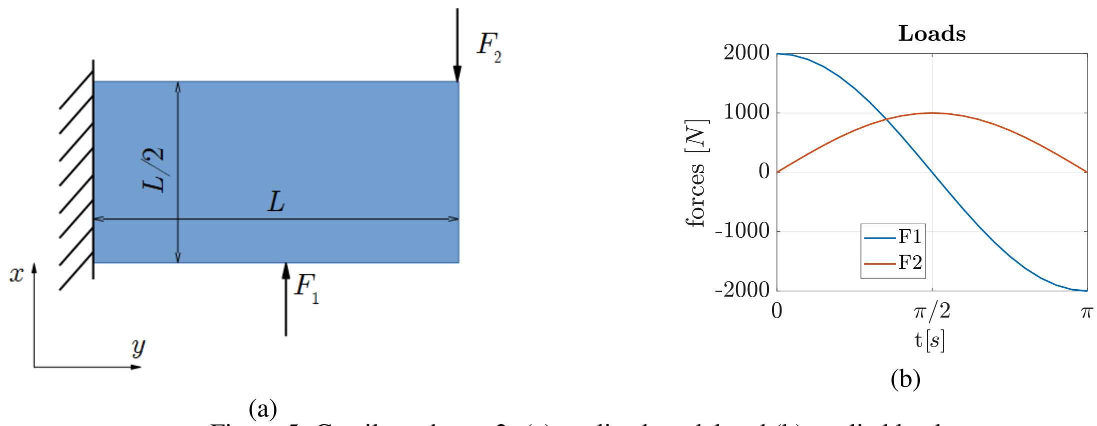


Figure 5. Cantilever beam 2: (a) analized model and (b) applied loads.

In this work topology optimization considering the dynamic response was not performed for this Cantilever beam

2 model. According to the results obtained for the first model (Cantilever beam 1) and shown in the section 4.1, it is already known that this type of analysis has a huge computational cost. However, the final ESL topology shown in Fig. 8 illustrates a good correlation with the design shown in Fig. 7 published by Zhao and Wang (2016) using dynamic analysis for this same model. The minimized values of the objective function are shown in Tab. 6, highlighting how Case 1 (considering all the external forces in only one load case) performs very poorly when the dynamics load are considered.

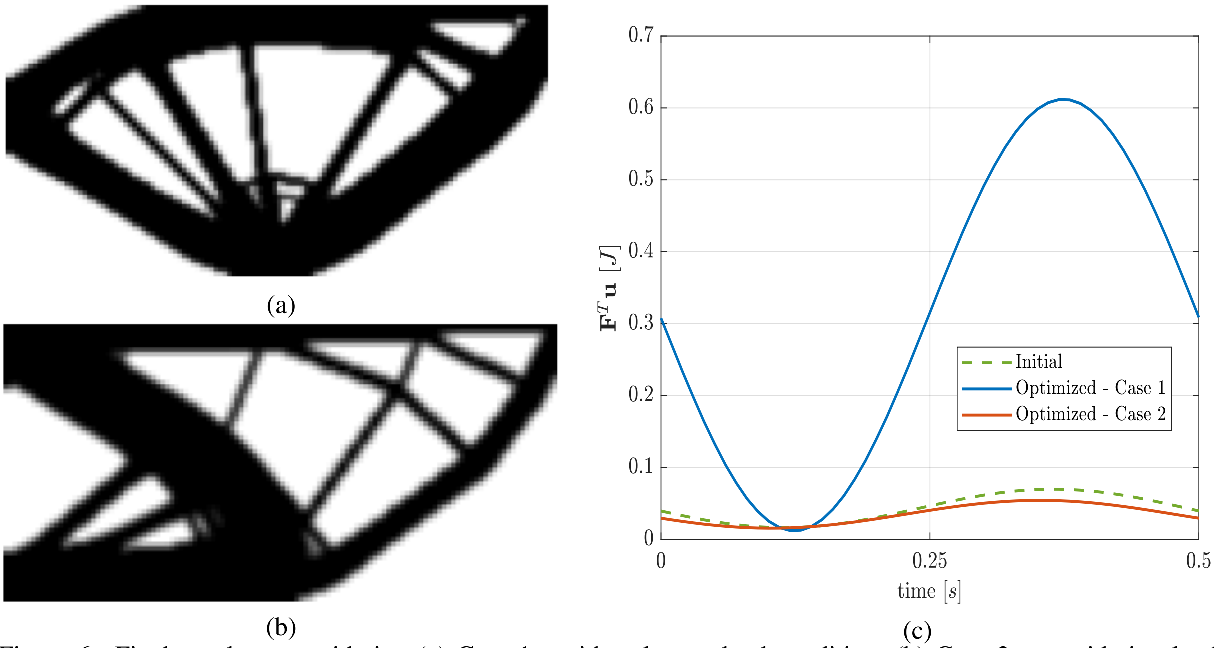


Figure 6. Final topology considering (a) Case 1 - with only one load condition, (b) Case 2 - considering load cases separately and (c) comparison between objective functions.

Figure 7. Topology (a) static design and (b) design considering dynamic loads. Figures published in Zhao and Wang (2016)

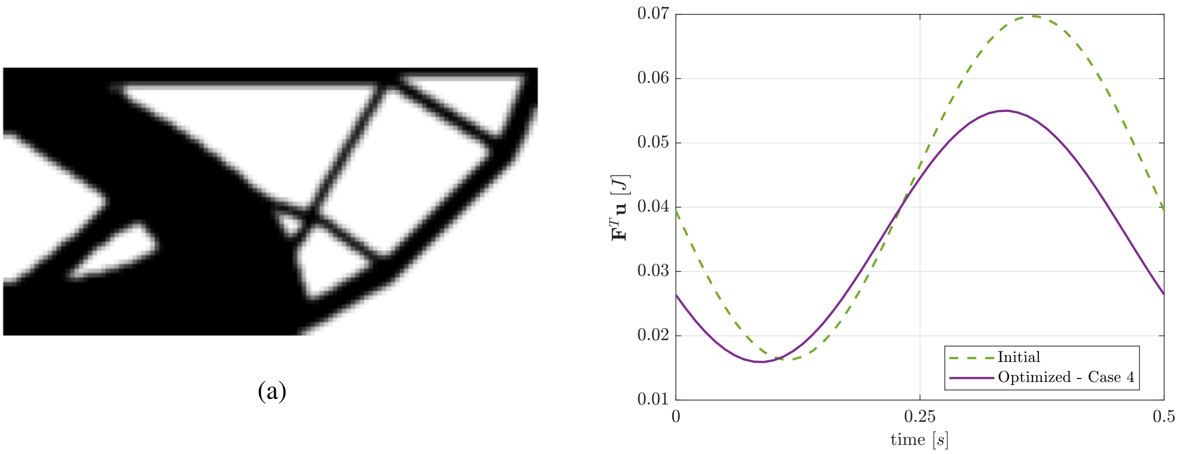
Table 6. Compliance after topology optimization for Cantilever beam 2.

	Initial	**Case 1**	**Case 2**	**Case 3**	**Case 4**
Compliance	2.15	15.59	1.78	–	1.77

5. FINAL REMARKS

The topology optimization of structures subjected to dynamic loads problem was successfully solved using the equivalent static load method. The equivalent static loads, defined from a linear dynamic analysis, were used as external loads, and a sequence of linear static response optimization was performed. The final topologies obtained using the ESL method are almost the same as the ones obtained with the traditional method.

The optimized structures for the static designs when the loads are applied simultaneously lead to a very sensitive designs with respect the dynamic loads, as illustrated in Figs. 3(c) and 6(c). On the other hand, an improved dynamic



(a) (b)

Figure 8. (a) Final topology considering ESL's and (b) objective function after topology optimization using ESL - Case 4.

behavior for both the dynamic and ESL cases can be observed (see Figs. 4(c) and 8(b)). This fact reinforces the importance of a proper problem formulation.

Although we are considering only linear dynamic analysis in this work, the equivalent loads method can still be used when the structure presents a nonlinear response: a sequence of static response optimization problems is solved, in which the cost of the sensitivity analysis optimization is much smaller than the dynamic response optimization. Another advantage of the ESL method is that we can use commercial computer-aided engineering (CAE) tools as black boxes.

6. ACKNOWLEDGMENTS

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8. RESPONSIBILITY NOTICE

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