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ACTIVE VIBRATION CONTROL APPLIED TO A COMPOSITE MATERIAL BEAM

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Abstract. *The present work proposes the active vibration control in a composite material beam using electromagnetic actuators. The advantage of the use of electromagnetic actuators is the application of the control force without the mechanical contact. The control technique used is the optimal control, specifically the linear quadratic regulator solved by linear matrix inequalities. The system identification is done using the Eigensysten Realization Algorithm/Observer/Kalman Identification, also known as ERA/OKID, where it is a method used to identify modal parameters in complex structures. From the model, the optimal controller is designed, which is responsible for determining the control effort, and the electromagnetic actuators are responsible for applying the control forces to stabilize the system. From this point on, the responses of the numerical simulation and the experimental responses of the system were obtained, in which the vibration attenuation of the system was obtained in both procedures, evaluated through displacement responses in time and frequency domain. For analyzing the robustness of the control system, a variation is considered in the dynamic matrix in the numerical and experimental tests.*

Keywords: *Active Vibration Control, Linear Quadratic Regulator, Eigensysten Realization Algorithm, Electromagnetic Actuator, Composite Material.*

1. INTRODUCTION

Beams are often employed in structures that are subject to the propagation of vibration. Such propagation can result in premature wear of the structure, or even cause problems associated to the vibration level. One of the used techniques to guarantee structural integrity and mechanical performance is the active vibration control (AVC), which in recent decades has presented great advances and new control methodologies. Thus, in industrial and production processes, control is of great importance, as these areas seek ways to optimize the performance of mechanical systems.

Among the techniques known in the active vibration control, optimal control has shown great efficiency in vibration attenuation, and successful applications in several areas, and one of the areas of great interest today is the study of composite materials, materials which have an innovative arrangement, characterized by their lightness, mechanical resistance, and the possibility of optimization of specific operating conditions. Such materials are formed by fibrous slides in different orientations, which allows the adaptation to particular applications (Reddy, 1997).

In this context, the work proposes the use of the active vibration control in a composite material beam, by means of the use of electromagnetic actuators. For the controller design, the mathematical model of the beam is required. For this, the Eigensystem Realization Algorithm / Observer/ Kalman Identification was used, which consists of a system identification technique for complex structures. The controllers were determined using linear quadratic regulator solved by linear matrix inequalities.

2. ERA/OKID

The Eigensystem Realization Algorithm/Observer/Kalman Identification, also known as ERA/OKID is an identification technique that incorporates a state observer or a Kalman filter. The ERA is used for the identification of modal parameters in systems that have complex structures.

The ERA / OKID consists of two parts, in which the ERA determines the matrices that represent the dynamic behavior of the system in state-space from the input and output data of the system and the OKID method determines the Markov parameters of the system (Abreu et al., 2012).

2.1 The OKID Techinique

OKID is used in structures that have a very small output signal decay rate, so the state observer introduces an artificial damping into the system, which anticipates the stabilization of the output signal. It can be applied to any type of system, even with damping.

With the use of the observer, mathematically the true system becomes a new one, consequently the Markov parameters will be obtained from this new system. However, OKID allows the Markov parameters of true system, and the Markov parameters of observer gain to be recovered. Therefore, OKID is able to obtain, from any type of input signal, the impulsive response of the true system, which is essential for the use of the ERA. The process of identifying a system using this method was described by Juang et al. (1993) and Juang and Phan (2001).

Considering the linear system in the state-space form in the discrete-time described in Eq. (1) and (2).

$$\{x(k+1)\} = [A]\{x(k)\} + [B]\{u(k)\} \qquad where \;\; [B]\{u(k)\} = [B_{exc}]\{i_{exc}\} + [B_{con}]\{i_{con}\} \tag{1}$$

$$\{y(k)\} = [C]\{x(k)\} + [D]\{u(k)\} \tag{2}$$

Where $[A]$, $[B]$, $[B_{exc}]$, $[B_{con}]$ and $[C]$, correspond to the dynamic matrix, the input matrix, the input matrix of excitation, the input matrix of control, and the output matrix. The vector $\{x\}$ corresponds the displacement, $\{i_{exc}\}$ to excitation current, $\{i_{con}\}$ the control current and $\{u\}$ the control vector, the integer k represents sampled time.

Considering the discrete system of Eq. (1) and Eq. (2), an impulsive input is applied over the system $u = [1 \quad 0 \quad 0 \quad \cdots \quad 0]$, under zero initial conditions, $x(0) = 0$ (Alves and Ribeiro, 2004), then results in Eq. (3), Eq. (4), Eq. (5), Eq. (6) and Eq. (7).

$$k = 0 \Rightarrow \begin{matrix} \{x(1)\} = [A]\{x(0)\} + [B]\{u(0)\} = [B] \\ \{y(0)\} = [C]\{x(0)\} + [D]\{u(0)\} = [D] \end{matrix} \tag{3}$$

$$k = 1 \Rightarrow \begin{matrix} \{x(2)\} = [A]\{x(1)\} + [B]\{u(1)\} = [A][B] \\ \{y(1)\} = [C]\{x(1)\} + [D]\{u(1)\} = [C][B] \end{matrix} \tag{4}$$

$$k = 2 \Rightarrow \begin{matrix} \{x(3)\} = [A]\{x(2)\} + [B]\{u(2)\} = [A]^2[B] \\ \{y(2)\} = [C]\{x(2)\} + [D]\{u(2)\} = [C][A][B] \end{matrix} \tag{5}$$

$$k = 3 \Rightarrow \begin{matrix} \{x(4)\} = [A]\{x(3)\} + [B]\{u(3)\} = [A]^3[B] \\ \{y(3)\} = [C]\{x(3)\} + [D]\{u(3)\} = [C][A]^2[B] \end{matrix} \tag{6}$$

$$k = \cdots \Rightarrow \begin{matrix} \{x(k+1)\} = [A]\{x(k)\} + [B]\{u(k)\} = [A]^k[B] \\ \{y(k)\} = [C]\{x(k)\} + [D]\{u(k)\} = [C][A]^{k-1}[B] = [Y_k] \end{matrix} \tag{7}$$

Where u is the excitation sign, and y the measured signal. The expression for impulsive response Y is represented by the Eq. (8).

$$[Y_0] = [D], \quad [Y_1] = [C][B], \quad [Y_2] = [C][A][B], \quad \cdots, \quad [Y_k] = [C][A]^{k-1}[B] \tag{8}$$

The sequence of matrices presented by Eq. (8) is known as Markov Parameter matrices. According to Alves (2005), Markov parameters are used as a basis for the identification of discrete time domain models, represented by the constant matrices $[A]$, $[B]$, $[C]]$ and $[D]$ Since $[Y_0] = [D]$, only arrays $[A]$, $[B]$, $[C]$ need to be determined in identification.

In practice, Markov parameters are obtained by assembling the matrix of Eq. (9), such matrix construction $[Y_k]$ is fundamental for the computational implementation of the identification method.

$$[Y_k] = \begin{bmatrix} y_k{}^{(1,1)} & y_k{}^{(1,2)} & \cdots & y_k{}^{(1,j)} \\ y_k{}^{(2,1)} & y_k{}^{(2,2)} & \cdots & y_k{}^{(2,j)} \\ \vdots & \vdots & & \vdots \\ y_k{}^{(i,1)} & y_k{}^{(i,2)} & \cdots & y_k{}^{(i,j)} \end{bmatrix}_{m\times r} \tag{9}$$

Where m and r are respectively the numbers of outputs and inputs, being $i = 1,\cdots,m$ and $j = 1,\cdots,r$, thus, each row represents the ith-output, and each column the it- input.

The input-output description of the system with zero initial conditions can be obtained from Eq. (2) recursively as Eq. (10).

$$\{y(k)\} = \sum_{i=0}^{k-1}[Y_i]\{u(k-i-1)\} + [D]\{u(k)\} \tag{10}$$

Where $[Y_i] = [C][A]^i[B]$ and $[B]$ are the Markov parameters of the system. If $[A]$ and $[C]$ are an observable pair, then there exists an observer of the form according to Eq. (11) and Eq. (12).

$$\begin{aligned} &\{\hat{x}(k+1)\} = [A]\{\hat{x}(k)\} + [B]\{u(k)\} - [M][y(k) - \hat{y}(k)] \\ &= ([A] + [M][C])\{\hat{x}(k)\} + ([B] + [M][D])\{u(k)\} - [M]\{y(k)\} \end{aligned} \tag{11}$$

$$\{\hat{y}(k)\} = [C]\{\hat{x}(k)\} + [D]\{u(k)\} \tag{12}$$

Where $[M]$ corresponds to the observer gain matrix, $\{\hat{x}\}$ the estimated state vector and $\{\hat{y}\}$ the estimated output vector.

In the case that the eigenvalues of $[A] + [\,M][C]$ are zero, the estimated state converges to the true state $x\ (k)$ after a maximum of n steps, where n is the order of the system. The input-output description of the system described is given by Eq. (13) for $k\ >\ n$.

$$\{y(k)\} = \sum_{i=0}^{n-1}[\bar{Y}_i][u(k-i-1) \quad y(k-i-1)] + [D]\{u(k)\} \tag{13}$$

Where $[\bar{Y}_i]$ is given by Eq. (14).

$$[\bar{Y}_i] = \left[[C]([A] + [M][C])^i([B] + [M][D]) - [C]([A] + [M][C])^i[M]\right] = \left[[\bar{Y}_i^{(1)}] \quad [\bar{Y}_i^{(2)}]\right] \tag{14}$$

The $[\bar{Y}_i]$ and $[D]$ are Markov parameters of the observer system.

Once the Markov parameters of the observer system are identified, the actual system Markov parameters can be calculated. The relationship between the Markov parameters of the observer system and those of the actual system is given by Eq. (15).

$$[Y_i] = [C][A]^i[B] = \left[\bar{Y}_i^{(1)}\right] + \sum_{k=0}^{i-1}\left[\bar{Y}_k^{(2)}\right][Y_{i-1-k}] + \left[\bar{Y}_i^{(2)}\right][D] \tag{15}$$

With the determination Markov parameters of the system, the state-space model of system can be derived using the ERA.

2.2 Minimum Realization of the System Model using the ERA

From the experimental data (input and output) of the system in which it is desired to obtain identification, the ERA is able to determine the matrices that represent the dynamic behavior in state-space, matrices $[A]$, $[B_{exc}]$, $[B_{con}]$, $[C]$ and $[D]$, of a linear system represented by Eqs. (1) and (2).

The matrices of dynamic behavior in state-space is determined from the system Markov parameters $[Y_i]$ obtained by OKID. The ERA algorithm starts from the formation of the Hankel matrix, according to Eq. (16).

$$[H(\ell,i)] = \begin{bmatrix} Y_i & Y_{i+1} & \cdots & Y_{i+\ell-1} \\ Y_{i+1} & Y_{i+2} & \cdots & Y_{i+\ell} \\ \vdots & \vdots & \ddots & \vdots \\ Y_{i+\ell-1} & Y_{i+\ell} & \cdots & Y_{i+2\ell-2} \end{bmatrix} \tag{16}$$

Where $[H]$ is Hankel matrix. The order of the system is determined from the singular value decomposition of $[H(\ell,0)]$ which is given by Eq. (17).

$$[H(\ell,0)] = [U][\Sigma][V]^T \tag{17}$$

Where $[U]$ and $[V]$ are unitary matrices and $[\Sigma]$ is a diagonal matrix of positive singular values $(n \times n)$, where n is the order of the system.

To proceed with the ERA algorithm it is necessary to define a $q \times \ell q$ matrix $[E_q^T]$ and a $m \times \ell m$ matrix $[E_m^T]$ made up of identity and null matrices, according to Eq. (18) and Eq. (19).

$$[E_q^T] = [I_q \quad 0_{q\times(\ell-1)q}] \tag{18}$$

$$[E_m^T] = [I_m \quad 0_{m\times(\ell-1)m}] \tag{19}$$

A discrete-time minimal order realization of the system can be written as Eq. (20), Eq. (21) and Eq. (22).

$$[\hat{A}] = [\Sigma]^{-1/2}[U]^T[H(\ell,1)][V][\Sigma]^{-1/2} \tag{20}$$

$$[\hat{B}] = [\Sigma]^{1/2}[V]^T[E_m] \tag{21}$$

$$[\hat{C}] = [E_q^T][U][\Sigma]^{1/2} \tag{22}$$

The direct influence matrix $[\hat{D}]$ can defined solving the Eq. (13).

3. LINEAR QUADRATIC REGULATOR SOLVED BY LMIS

Optimal control theory, and especially that of the linear quadratic regulator, propose the possibility of optimizing physical quantities by adopting a performance index. The Linear Quadratic Regulator (LQR), in addition to being a powerful control technique, is of great importance in multivariate dynamic control, and it is the basis for the recent development of linear MIMO (multiple input and multiple output) control systems.

For closed loop systems, LQR provides a methodology for controlling feedback gain, ensuring a good margin of stability.

The optimal control, in the present context, contributes to the minimization of the performance index leading to the optimization of the pre-defined physical quantities (Simões, 2006). The feedback control is given by Eq. (23).

$$\{u(t)\} = -[K]\{x(t)\} \qquad where \quad \{i_{con}\} = \{u(t)\} \tag{23}$$

Where $[K]$ is the controller´s gain matrix and can be determined through the minimization of the performance index represented by the Eq. (24).

$$J = \int_0^\infty \left(\{x(t)\}^T[Q_{lqr}]\{x(t)\} + \{u(t)\}^T[R_{lqr}]\{u(t)\}\right) dt \tag{24}$$

where $[Q_{lqr}]$ is a positive defined hermitian matrix (positive definite or semi-definite) or real symmetric that weights each state, and $[R_{lqr}]$ is a positive defined hermitian matrix or real symmetric that weights the energy cost of each controller (Simões, 2007).

3.1 Linear Matrix Inequalities (LMIs)

A version of the LQR solved by linear matrix inequality is illustrated in Erkus and Lee (2004). The linear matrix inequality (LMI) is an important method that encompasses several mathematical problems, its use is advantageous in control systems for the determination of controller gain, due to the possibility of assuming modal parameters involving uncertainties. Currently, it is a subject of worldwide study, in several researches that aim at different applications, such as optimal control and robust control (Van Antwerp and Braatz, 2000) (Silva et al., 2004).

Lyapunov showed that the system represented by (25) is stable if and only if there exists a positive matrix P_{lmi}, which satisfies the condition given by (26), known as Lyapunov inequality.

$$\{\dot{x}(t)\} = [A]\{x(t)\} \tag{25}$$

$$\begin{aligned} [A]^T[P_{lmi}] + [P_{lmi}](A) < 0 \\ [P_{lmi}] > 0 \end{aligned} \tag{26}$$

The LQR via LMI problem, from the minimization of the performance index, is described by the Eq. (27):

$$min_{X,P_{lmi},X_{lmi}} tr([Q_{lqr}][P_{lmi}]) + tr([X_{lmi}]) + tr([Y_{lmi}]N) + tr([N]^T[Y_{lmi}]^T) \quad (27)$$

where N corresponds the noise insertion, $[X_{lmi}]$ and $[Y_{lmi}]$ the solutions of the LMIs denoted in the matrix, and $tr()$ to the trace matrix. The Eq. (27) is subject to conditions imposed by Eq. (28) and Eq. (29):

$$[A][P] - [B][Y_{lmi}] + [P][A]^T - [Y_{lmi}]^T[B]^T + [B_w][B_w]^T < 0 \quad (28)$$

$$\begin{bmatrix} [X_{lmi}] & [R_{lqr}]^{1/2}[Y_{lmi}] \\ [Y_{lmi}][R_{lqr}]^{1/2} & [P_{lmi}] \end{bmatrix} > 0 \quad (29)$$

Where B_w is the perturbation matrix.
The controller gain is obtained through the Eq. (30).

$$[K] = [Y][P_{lmi}]^{-1} \quad (30)$$

4. METHODOLOGY

The experimental bench consists of a composite material beam is presented by Fig. 1. The composite material beam has 310 [mm] length, 59.30 [mm] width and 3 [mm] thickness and is formed of epoxy, glass fiber and 1045 steel screen.

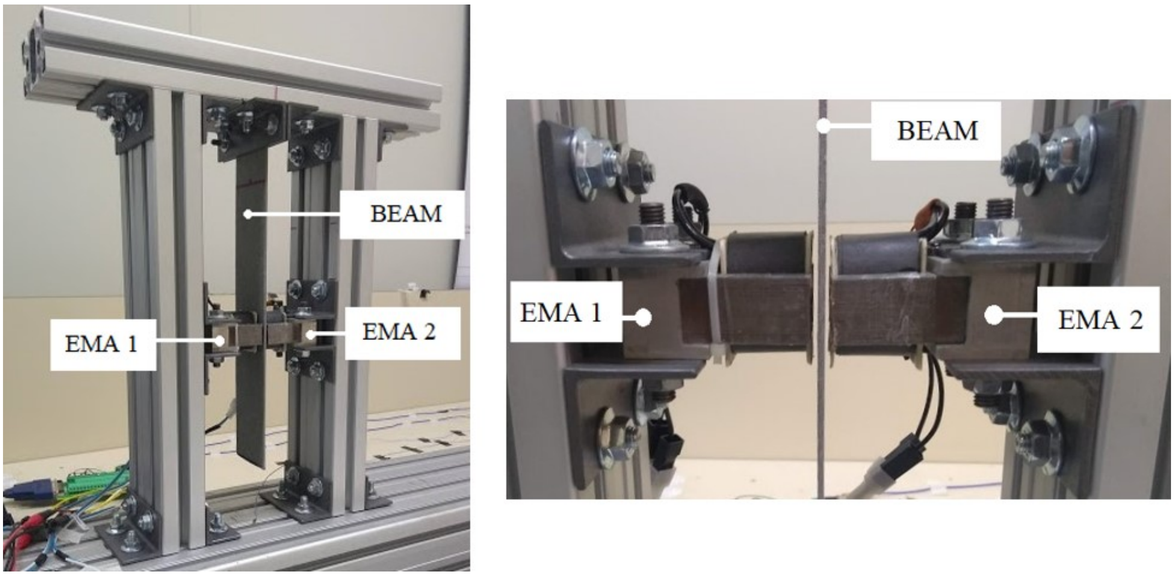

Figure 1. Experimental bench.

Two electromagnetic actuators (EMA1 and EMA2) are positioned in the structure, for the application of the experimental control force. Each actuator has a gap of 1.5 [mm] from the beam and acts separately, applying only pulling force. The design of the electromagnetic actuator was an adaptation of the model presented by Morais (2010), according to Fig. 2. In the present work a core of ferromagnetic composed of several blades is used, where in the same one is a coil composed of 250 coiled turns by copper wire, providing a direction of the flux of the magnetic field next to the composite material beam, being this one composed of screens of steel 1045. The parameters of electromagnetic actuator are shown in the Tab. 1.

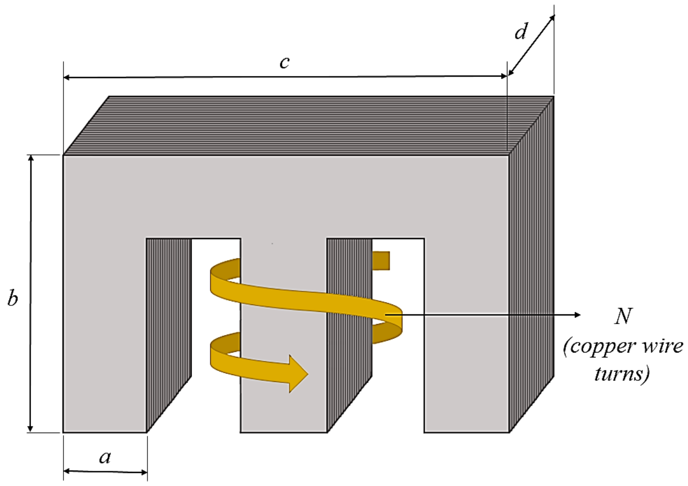

Figure 2. Model of the electromagnetic actuator (adapted by Morais, 2010).

Table 1. Parameters of electromagnetic actuator (Koroishi, 2013)

a [mm]	9.5
b [mm]	38
c [mm]	57
d [mm]	9.5
N [turns]	250

The parameters of cooper wire are shown in the Tab 2.

Table 2. Parameters of AWG24 copper wire (Koroishi, 2013)

Diameter [mm]	0.511
Area [mm²]	0.205
Resistance [Ohm/m]	0.0842
Maximum Current [A]	3.5

For the identification of the dynamic behavior matrices of the composite material beam using ERA/OKID method, 10 responses were collected from the input and output of the system. The impulse input corresponds to a current pulse of 0.5 [A] of one of the electromagnetic actuators.

The schematic form of which optimal control is used in the present work to control the composite material beam is shown in Fig. 3, where δ corresponds to the displacement, X to the modal state and i the electric current.

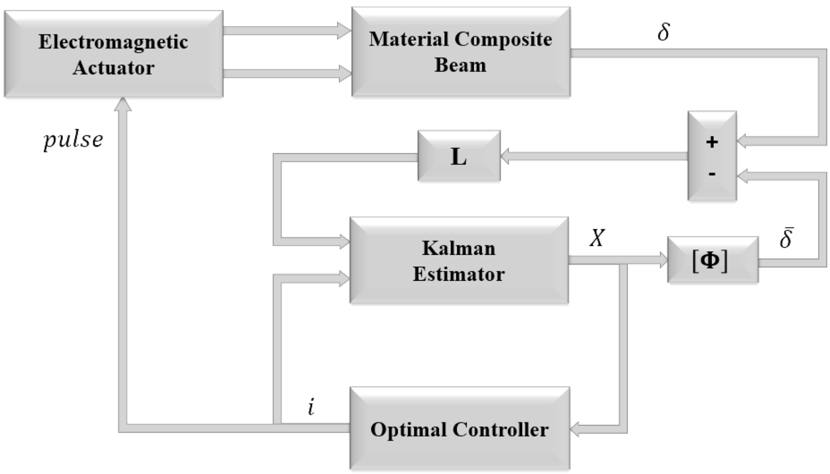

Figure 3. Schematic form of the optimal control (adapted from Koroishi et al., 2015)

In Fig 3, the $[\phi]$ corresponds the transformation matrix, where consisting the n first modes controllable and observables. The L corresponds the Kalman estimator gain matrix, where this matrix was determined by using the command lqe.m in the software Matlab®. The Kalman Estimator is represented in Eq. (31).

$$\{\dot{x}_r(t)\} = [A_r]\{x_r(t)\} + [B_r]\{u(t)\} + [L]\{\delta(t) - \bar{\delta}(t)\} \tag{31}$$

Where $\{x_r(t)\}$ is the reduced state vector, $\{x_r(t)\}$ the reduced input vector, $[A_r]$ the reduced dynamic matrix rxr, $[B_r]$ the reduced input matrix rxm, $[L]$ the gain matrix, $\delta(t)$ the displacement vector and $\bar{\delta}\ (t)$ the estimate displacement vector.

The reduced matrices presented in Eq. (31) were obtained through balanced realization, which consists of a model reduction technique and were reduced to the number of controllable and observable modes. In the balanced realization, the system represented in Eq. (1) and Eq. (2), must respects the Lyapunov equations represented by Eq. (32) and Eq. (33) and also to present $P = Q = diag\ (\sigma_1, \sigma_2 \ldots \sigma_n)$ (Meirovitch, 1990) (Assunção and Hemerly, 1992) (Zhou and Doyle, 1998).

$$[A][P] + [P][A]^T + [B][B]^T = 0 \tag{32}$$

$$[A]^T[Q] + [Q][A] + [C]^T[C] = 0 \tag{33}$$

Where $[P]$ and $[Q]$ are respectively the controllability and observability graminians, and σ_i, $i = 1, 2, \ldots, n$, are the singular values of the system $(\sigma_1 \geq \sigma_2 \geq \ldots \sigma_n \geq 0)$. The singular values are ranked by their degree of importance in the system, from the most important to the least important. Each σ_1 is associated with a state x_i of the balanced system, this value quantifies the state contribution x_i of system response. How $\sigma_1 \geq \sigma_2$, the state x_1 affects system behavior more than x_2.

For numerical simulation, in the scheme presents in Fig. 3, the composite beam described in the state-space form from the ERA/OKID identification is subjected to a current pulse from the electromagnetic actuator, thus generating a displacement δ. The Kalman estimator is used to minimize the effects of noise the input and output signals of the system, and also to estimate the states and outputs of the system. It has the function of determining the modal states required by controller. So, the optimal control determines the control effort required for the electromagnetic actuators to stabilize the system.

For the experimental control, the excitation of the composite material beam is made from the electromagnetic actuator, in which a pulse of 0.5 [A] is given to excite the beam. The equipment to perform this experimental procedure consists of an accelerometer of the PCB Piezotronics® manufacturer, positioned at the free end of the beam, in order to obtain the displacement response. The accelerometer signal is then transmitted to a signal conditioner of the 480E09 model of the Piezotronics® PCB, in order to improve the accuracy of the response obtained by the accelerometer. Also used are amplifiers of the Maxon Motor® Servo-Amplifier 4-Q-Dc model, whose purpose is the conversion of the output voltage signal from the plate to electric current which feeds the electromagnetic actuators. The data acquisition is done by the PCI-6221, from National Instrument®, which is connected to a desktop computer. An analog input is used for the accelerometer signal and two outputs, referring to the output voltages that will power the amplifiers. Ten responses of the experimental control were collected to obtain a mean of responses.

For analyzing the robustness of the control system, a variation is considered in the dynamic matrix $[A] + [\delta A]$ in the numerical and experimental tests, considering a variation from -20% to +20%.

5. RESULTS

In the identification of the system using the ERA / OKID algorithm was obtained the order model (n) equal to 60 and later the order of the model was reduced to the number of controllable and observable modes, which in the present work are the first two vibrate modes. Figures 4 and 5 show the real system and the time and frequency domain identified system.

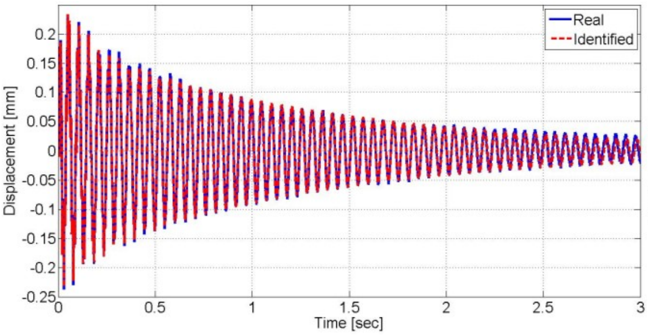

Figure 4. Response of time domain displacement (Real and Identified)

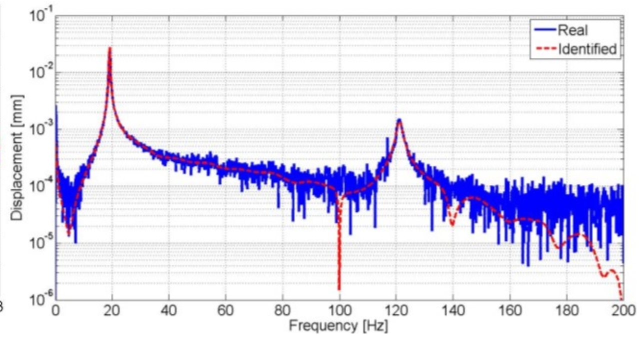

Figure 5. Response of time domain displacement (Real and Identified)

In order to analyze the efficiency of the proposed methodology, considering the optimal control technique, aiming at the vibration attenuation of the composite material beam, the reduction of the system displacement response, the frequency response and the electrical currents used by the electromagnetic actuators were analyzed. First the analysis was done through the numerical simulation and later its experimental validation was done.

The displacement responses in time domain, in numerical and experimental tests, are presented in Figs. 6 and 7.

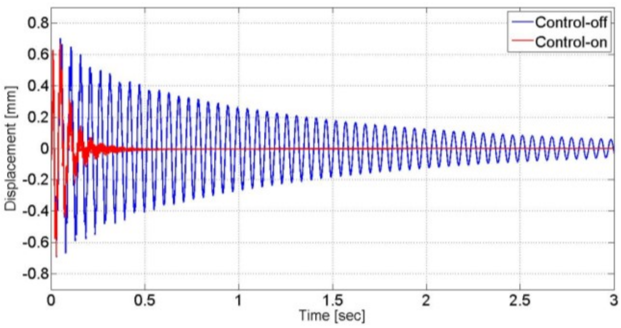

Figure 6. Numerical - Response of time domain displacement using optimal control theory

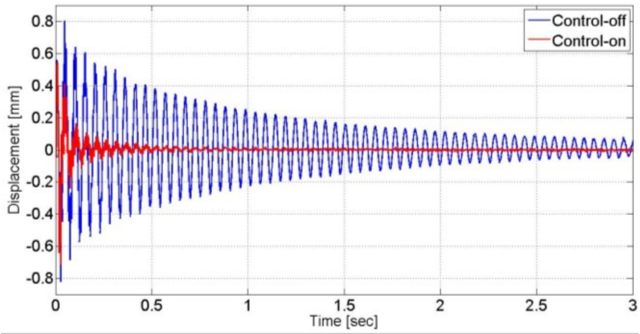

Figure 7. Experimental - Response of time domain displacement using optimal control theory.

Analyzing the numerical result, in Fig. 6 is observed the displacement attenuation in a time of 0.5 [sec]. The experimental displacement response in Fig. 7 shows the vibration attenuation at between 0.5 [sec] and 1 [sec]. Comparing with the numerical simulation response is an acceptable validation, since experimental procedure may contain external noises.

The electric current responses, in numerical and experimental tests, are presented in Figs. 8 and 9.

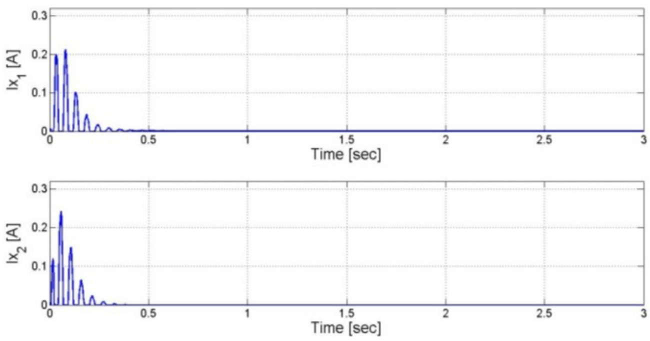

Figure 8. Numerical - Electric current of EMA1

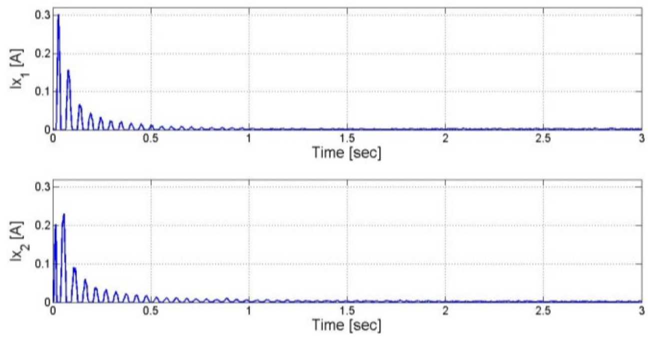

Figure 9. Experimental - Electric current of EMA2.

In Figs. 8 and 9, it was observed the accompaniment of the currents of the electromagnetic actuators along the displacement responses of Figs. 4 and 5. In both the numerical simulation and the experimental procedure, the peak currents were less than 0.3 [A].

The FRFs, in numerical and experimental tests, are presented in Figs. 10 and 11.

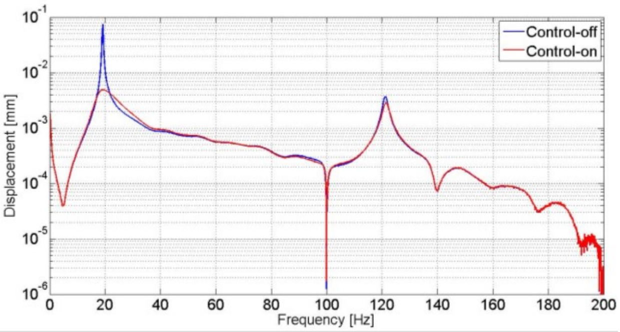

Figure 10. Numerical - Displacement in frequency domain

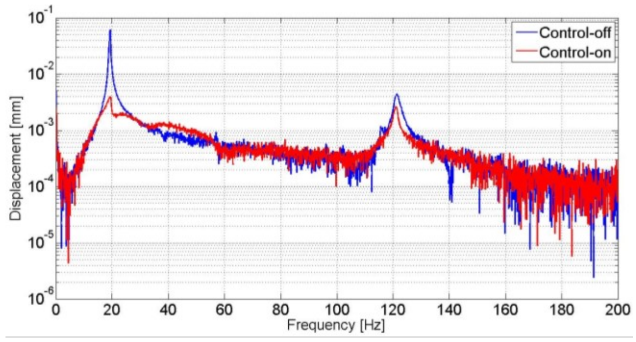

Figure 11. Experimental - Displacement in frequency domain.

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In Fig. 10 is observed the significant attenuation of the first vibration mode by about 0.0693 [mm], as well as in the experimental control of Fig. 11, where the first vibration mode attenuated by about 0.0576 [mm].

The attenuation of first vibration mode of the variation considered in the dynamic matrix $[A]$ from -20% to +20%, in the numerical and experimental tests for analyzing the robustness of the control system, is presented in Tab. 3.

Table 3. Attenuation of first mode.

Variation	Numerical [mm]	Experimental [mm]
-20%	0.067310	0,057789
-15%	0.068033	0,058521
-10%	0.068657	0,058571
-5%	0.069131	0,058878
0%	0.069348	0,058963
5%	0.069189	0,058799
10%	0.068796	0,058581
15%	0.068306	0,058137
20%	0.067777	0,057743

The results of Tab. 3 show the attenuation of the first vibration mode of the beam with a variation in the dynamic matrix $[A]$ of the system. It can be observed that even with the variation from +/- 20%, the attenuation of the beam remains satisfactory, where the attenuation of the first mode presents a variation of about 3% in the numerical simulation and about 2% in the experimental procedure, showing so the robustness of the control system.

In fact, LQR ensures good stability margins. However, the combination of LQR and Kalman Estimator, according to Doyle (1978), has no such guarantees, but despite this fact, the closed loop control system worked properly.

6. CONCLUSION

The aim of present work was the vibration attenuation in a composite material beam using electromagnetic actuators through the application of the optimal control. First, the system model was obtained through the method known as ERA / OKID, where the same from a mean of experimental data of input and output of the system, determined the matrices that represent the dynamic behavior in state-space.

Subsequently, the optimal control was designed by means of the linear quadratic regulator via linear matrix inequalities, in order to determine the control force that the electromagnetic actuators must apply in order to have the beam stabilization.

For the validation of the proposed methodology, an experimental procedure was performed, in which it presented similar results to the numerical simulation. He presented the stabilization of the beam between 0.5 to 1 *sec* and an expressive attenuation of the first vibration mode.

Also, a variation was considered in the dynamic matrix $[A]$ from -20% to +20%, in order to analyze the robustness of the control system. It was observed the attenuation of the first vibration mode in this variation, which showed a robustness of the control system. In view of this, it can be said that the proposed methodology is valid for the control of composite materials beams.

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