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# NUMERICAL STUDY OF THE SHEAR LAG EFFECTS IN STEEL ANGLES

**Ruan Richelly Santos**
**José Osvaldo Ferreira Filho**
Federal University of Minas Gerais, 6627 Antonio Carlos Ave - School of Engineering, Departament of Structural Engineering, Belo Horizonte - MG, Brazil
ruanrs@ufmg.br
josefilho@ufmg.br

**Ana Lydia Reis de Castro e Silva**
**Hermes Carvalho**
**Rodrigo Barreto Caldas**
Federal University of Minas Gerais, 6627 Antonio Carlos Ave - School of Engineering, Departament of Structural Engineering, Belo Horizonte - MG, Brazil
analydiarcs@gmail.com
hermes@dees.ufmg.br
caldas@dees.ufmg.br

***Abstract.*** *The use of steel angles is common and advantageous, mainly in structures subjected to tensile stresses. However, it is necessary to accurately define their ultimate tensile capacity to avoid structural oversizing, and guarantee safety to constructions. In connected angles with bolts, due to the nonuniform tension distribution, there is a phenomenon known as shear lag. Therefore, it is necessary to consider its impacts for the calculation of tensile resistance and, consequently, to the design. It is accomplished by the determination of a reduction factor by standardizations, as ABNT NBR 8800:2008, ANSI/AISC 360-16 and EN 1993-1-1:2005. Besides them, there are other procedures, such as another analytical formulae, and numerical simulation. As this last one is now widely spread and considered an accurate solution in the technical field, it was aimed to numerically study the shear lag effects on the behavior of steel angles bolted at one leg. Thus, finite element models were developed and validated against existing experimental data. Furthermore, numerical results were compared to the analytical ones obtained by the codes' procedures. Then, they were used to perform a parametric study in order to check the connection length effects. At the end, it was obtained a consistent set of numerical models which can be used as base for future studies.*

***Keywords:*** *steel angles, shear lag, tension members, numerical analysis, finite elements*

## 1. INTRODUCTION

The use of tension steel members is common in structural systems. These elements constitute, for example, trusses, bracings for buildings and bridges, and cables in suspension. Among their applications, it is worth pointing out that they are efficient over long spans. Furthermore, they are necessary to transfer gravitational loads from a sublevel to an upper level. Generally, these members are composed by any steel cross-sectional configuration, such as H, I, and U shapes, circular rods, and angle shapes. In Brazil, the last ones are the most used built-up configuration (Fakury et al., 2016).

Steel angles are commonly connected to other structural elements by bolting just one leg. In that situation, as some parts of the tension member are not connected, it is established nonuniform distribution of stresses. It occurs because these unconnected parts are not completely stressed, and, consequently, the connected ones become overloaded. This phenomenon is known as shear lag and influences the section in resisting tensile forces (Figure 1). If there was connection in the entire cross-section, the resistance would be greater. Therefore, it is concluded that the shear lag effects reduce the efficiency of the section. It is necessary to highlight that the magnitude of this reduction depends mainly on the length and eccentricity of the connection (Salih, 2012).

In regions of connection, the gross area is reduced because the connection usually weakens the structural elements. In this perspective, according to standards, tensile rupture in the net section is one of the ultimate limit states. So, it is important to be checked. For this verification, the determination of effective net area is fundamental. In this scenario, it is important to introduce the concepts of net area and effective net area (Segui, 2013).

In order to avoid structural collapse, two reductions are applied to gross area, $A_g$. The first one characterizes the net area, $A_n$, being accomplished due to the presence of bolts. In this sense, there are holes responsible to develop high stressed regions at the members' connection. Therefore, the area should be reduced to consider these holes. For members without holes, $A_n$ is equal to $A_g$. The second reduction is performed to take into account the nonuniform tension distribution, in other words, the shear lag effects. Thus, it is necessary to reduce the net area, aiming to consider just its effective part. For this, the reduction factor is determined and applied to the net area, resulting in the effective net area, $A_e$ (Figure 2).

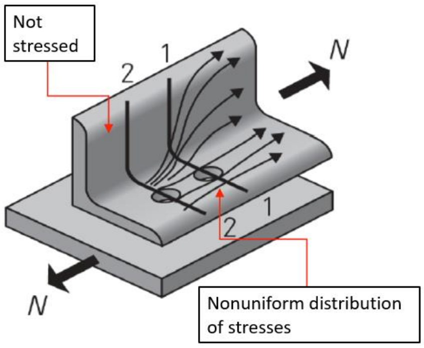


Figure 1. Shear lag phenomenon (Adapted from Fakury, 2016)

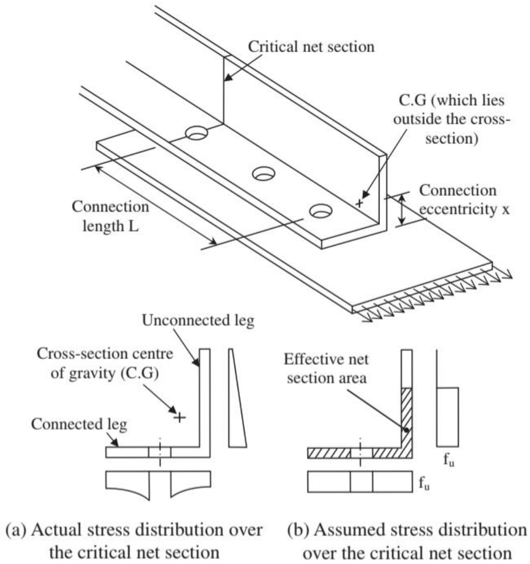


Figure 2. Gusset plate connection (Salih, 2012)

The reduction factor is responsible to consider the shear lag phenomenon. It is usually defined by standards, as NBR 8800:2008, ANSI/AISC 360-16 and EN 1993-1-1:2005. They provide different procedures, resulting in different values of factors. Thus, it is also found divergent results of effective net area. Consequently, disparities related to ultimate tensile capacity are noted. Moreover, this factor can also be obtained by numerical simulations based on Finite Element Method, and by author's formulae.

In this regard, it was aimed to numerically investigate the influence of shear lag on the behavior of steel angles connected by one leg to a gusset plate with a single row of bolts. For this, finite element models were elaborated and validated against existing experimental tests. It was also accomplished a parametric study ranging the connection length of the angles in order to check its effects in the tensile capacity.

## 2. NUMERICAL MODELS

As this paper aimed to analyze the shear lag effects on steel angles bolted at one leg, finite element analyses were conducted using *ABAQUS/Standard* to investigate this phenomenon. The numerical results validation was made through the comparison with experimental data obtained by the study of Kulak and Wu (1997), which carried out tests for steel angles subjected to axial tensile forces.

As aforementioned, the shear lag phenomenon reduces the efficiency of the cross-section, mainly due to the length and eccentricity of the connection. Then, from the validated finite element models, a parametric study was conducted as a mean to describe the impact of the connection length to the shear lag effects, and, consequently, to the steel angles tensile capacity.

### 2.1 Geometry description

According to Kulak and Wu (1997), the test efficiency between single and double angle members are generally about the same. Some previous studies, as McKibbon (1906), Davis and Boomsliter (1934) and Hebrandt and Demol (1995), reinforce the idea that the efficiency of single and double angle members is similar. Double angles are popular solutions in civil engineering structures, such as bracings, since this type of solution allows savings in connection and prevents the occurrence of buckling around the weakest axis. Thus, the numerical study developed in this paper contemplated only the analysis of double angles to evaluate the shear lag effects. A total of twelve models of double steel angles bolted at one leg were proposed, as shown in Tab. 1. The geometry of the models proposed is described in Fig. 3. For the connection, it was considered that all specimens presented holes of 22 mm for the bolts.

As a mean to study the effect of the connection length to the shear lag effects, a small parametric study was carried out. At each model defined in Tab. 1, three different types of connections were analyzed: two-bolt, four-bolt and six-bolt connections. Thus, accounting for a total of 36 numerical models.

Table 1. Specifications of the double steel angles bolted at one leg models

| Models | Angle size (mm) | Connected leg (mm) | g (mm) | $e_c$ (mm) | $A_g$ ($mm^2$) |
|---|---|---|---|---|---|
| 1 | 2L 76 x 51 x 4.8 | 51 | 22 | 24.64 | 1163.87 |
| 2 | 2L 76 x 51 x 4.8 | 76 | 44 | 11.94 | 1163.87 |
| 3 | 2L 76 x 51 x 9.5 | 51 | 22 | 26.42 | 2232.25 |
| 4 | 2L 76 x 51 x 9.5 | 76 | 44 | 13.69 | 2232.25 |
| 5 | 2L 76 x 76 x 4.8 | 76 | 44 | 20.83 | 1406.45 |
| 6 | 2L 76 x 76 x 9.5 | 76 | 44 | 22.56 | 2722.58 |
| 7 | 2L 102 x 76 x 6.4 | 76 | 44 | 31.50 | 2180.64 |
| 8 | 2L 102 x 76 x 6.4 | 102 | 64 | 18.69 | 2180.64 |
| 9 | 2L 102 x 76 x 12.7 | 76 | 44 | 33.78 | 4193.54 |
| 10 | 2L 102 x 76 x 12.7 | 102 | 64 | 21.01 | 4193.54 |
| 11 | 2L 102 x 102 x 6.4 | 102 | 64 | 27.69 | 2503.22 |
| 12 | 2L 102 x 102 x 12.7 | 102 | 64 | 29.97 | 4838.70 |

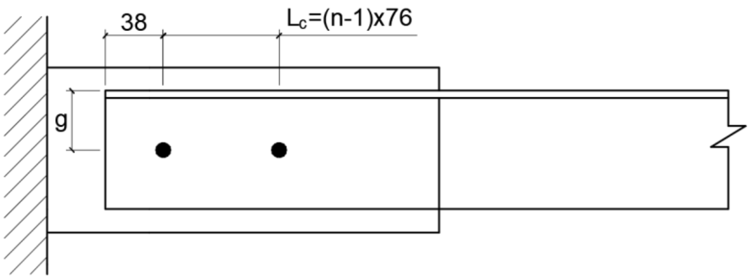


Figure 3. Geometry description of the steel angles

### 2.2 Finite element analysis

Some relevant aspects must be defined to represent the actual behavior of the elements from the numerical modelling. In this perspective, the criteria related to the material, type of analysis, mesh definitions, boundary conditions and loading are decisive to obtain accuracy of results. In this regard, this work concerned to correctly choose these aspects to reproduce the steel angles' performance. For this, a solid study was accomplished.

#### 2.2.1 Material

According to Salih et al. (2013), finite element analysis for bolted connections is expected to involve large inelastic strains. Thus, the elastic and plastic range of the steel must be considered. Therefore, for the elastic behavior, the steel was simulated as an isotropic material with elasticity modulus of 200 GPa and Poisson's ratio of 0.3. In order to obtain the plastic behavior, nominal stresses ($\sigma_{nom}$) and nominal strains ($\varepsilon_{nom}$) were converted into the corresponding true stresses ($\sigma_{real}$) and true plastic strains ($\varepsilon_{real}$). These values were based on the well-known relations presented on the literature and defined by the Eq. (1) and (2). The steel properties used for the simulations were the same as those adopted in the experiments of Kulak and Wu (1997), which used the CAN/CSA-M300W, a steel with yield strength of 300 MPa and ultimate strength of 450 MPa.

$$\varepsilon_{real} = \ln(1 + \varepsilon_{nom}) \tag{1}$$

$$\sigma_{real} = \sigma_{nom}(1 + \varepsilon_{nom}) \tag{2}$$

### 2.2.2 Type of analysis

As a mean to achieve consistent numerical approximations with the actual behavior of tension members, physical nonlinearities should be taken into account. Therefore, an analysis based on the Riks' method was conducted. As stated in Vasios (2015), this approach is a very efficient procedure in solving nonlinear system of equations when the problem exhibits one or more critical points. The method assumes that the loading is proportional. In other words, all load magnitudes vary with a single scalar parameter. The solution is viewed as the discovery of a single equilibrium path in a space defined by the nodal variables and the loading parameter. Thus, a path is traverse as far as required, demanding that the number and size of increments be chosen and limited by the user. For the numerical analysis, the standard values for the number of increments and arc lengths proposed by the finite elements software were enough to achieve the convergence of the model, and to obtain satisfactory results.

Through the analysis, a prediction of the ultimate load was obtained, and, subsequently, the reduction coefficient was found. Then, these values were verified and validated against the experimental data. The reduction coefficient ($C_t$) was obtained from the load proportionality factor (LPF), the initial stress applied by the user ($\sigma_0$), and the ultimate tensile strength of the steel ($f_u$), as presented in Eq. (3).

$$C_t = \frac{LPF \times \sigma_0}{f_u} \tag{3}$$

### 2.2.3 Mesh definition

The three dimensional, four-node shell elements with reduced integration (S4R) was employed to model the steel angles. The choice of the element with reduced integration was justified by similar results to those from the element with complete integration and shorter processing time developed. A mesh convergence study was performed to ensure the accuracy of results. For the analysis with meshes smaller than 10 mm, the computational cost was high, and the results did not show great variations. Thus, a mesh size of 10 mm was adopted, which provided the best refinement with low computational cost.

A structured mesh was adopted along the steel angle. However, in the bolted connection region, a free mesh was applied. This type of mesh provides a more flexible distribution of the elements, resulting in a better adjustment of the finite elements in the connected area. An example of the mesh application described is presented in Fig. 4.

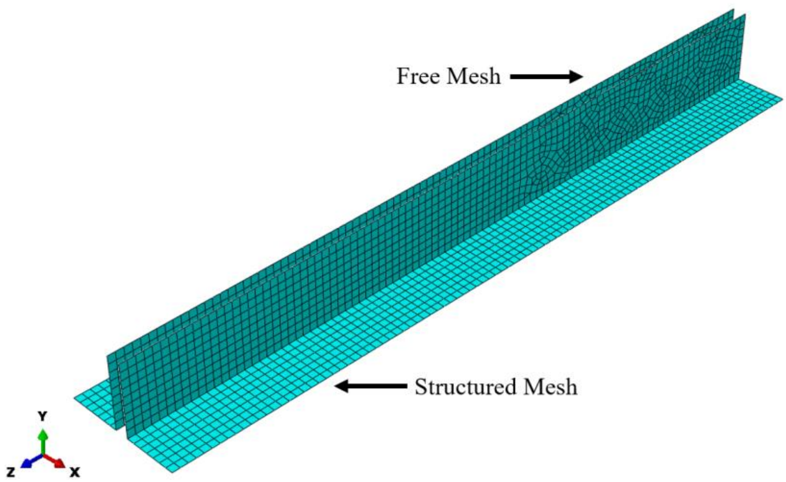


Figure 4. Mesh adopted for the double steel angles models

### 2.2.4 Boundary and loading conditions

In the definition of the geometry of bolts, a simplification was adopted. The connection regions were defined through rectangular partitions in the steel angles' leg. Then, appropriate boundary conditions were applied to represent the behavior of the bolts, in which all the translational and rotational displacements were restricted. In order to reproduce the tension stress in which steel angles are usually subjected, a concentrated load was applied to the centroid of the section. In order to apply the load, a control point was created in the centroid of the steel angle section to transfer the tension stress to the whole section. A representation of the boundary conditions and the loading employed in the numerical modelling is shown in Fig. 5.

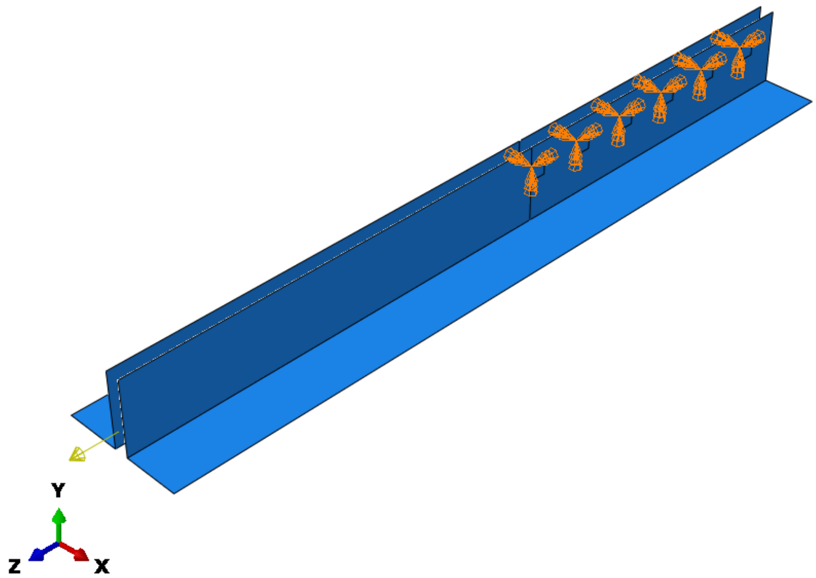


Figure 5. Boundary conditions and loading representation

### 2.3 Model validation

As a mean to guarantee coherent numerical models, the values obtained through the software Abaqus were compared with experimental tests conducted by Kulak and Wu (1997). The authors performed physical testing of single and double angle tension members to obtain the net section strength and thereby examine the shear lag effects. A total of eight different specimens of double steel angles were accomplished by the study, as described in Tab. 2.

Through the comparison between the numerical and experimental results, it was obtained an error lower than 10% for all cases, as shown in Tab. 2. The difference achieved is below the usual values of error adopted for the comparison of numerical and experimental results. In this sense, the numerical models developed are useful to represent the actual behavior of bolted steel angles. Thus, a reliable analysis of the steel angles proposed is possible.

Table 2. Validation of numerical models against tests of Kaluk and Wu (1997)

| Angle size (mm) | Connected leg (mm) | n | $C_{t,ABAQUS}$ | $C_{t,Tests}$ | Difference (%) |
|---|---|---|---|---|---|
| 2L 102 x 76 x 6.4 | 102 | 6 | 0.83 | 0.89 | 6.89 |
| 2L 102 x 76 x 6.4 | 76 | 6 | 0.82 | 0.82 | 0.41 |
| 2L 102 x 102 x 6.4 | 102 | 6 | 0.79 | 0.79 | 0.55 |
| 2L 76 x 76 x 4.8 | 76 | 6 | 0.83 | 0.82 | 1.62 |
| 2L 76 x 51 x 9.5 | 76 | 6 | 0.85 | 0.90 | 6.07 |
| 2L 76 x 51 x 4.8 | 76 | 6 | 0.84 | 0.86 | 2.12 |
| 2L 76 x 51 x 4.8 | 76 | 4 | 0.84 | 0.90 | 6.43 |
| 2L 76 x 51 x 4.8 | 76 | 2 | 0.78 | 0.72 | 8.21 |

The numerical models presented in Tab. 2 were also compared with the analytical procedures defined by the following standards: NBR 8800:2008, ANSI/AISC 360-16 and EN 1993-1-1:2005. Through this comparison is possible to define the behavior obtained through the codes' application to the shear lag effects.

## 3. RESULTS AND DISCUSSION

As aforementioned, the shear lag phenomenon reduces the capacity of the cross-section, mainly due to the length and eccentricity of the connection. From the parametric study, it was possible to define the impact of the connection length to the steel angles net section efficiency. The results of the numerical study are given by Fig. 6. An example of the stress distribution obtained is displayed by Fig. 7.

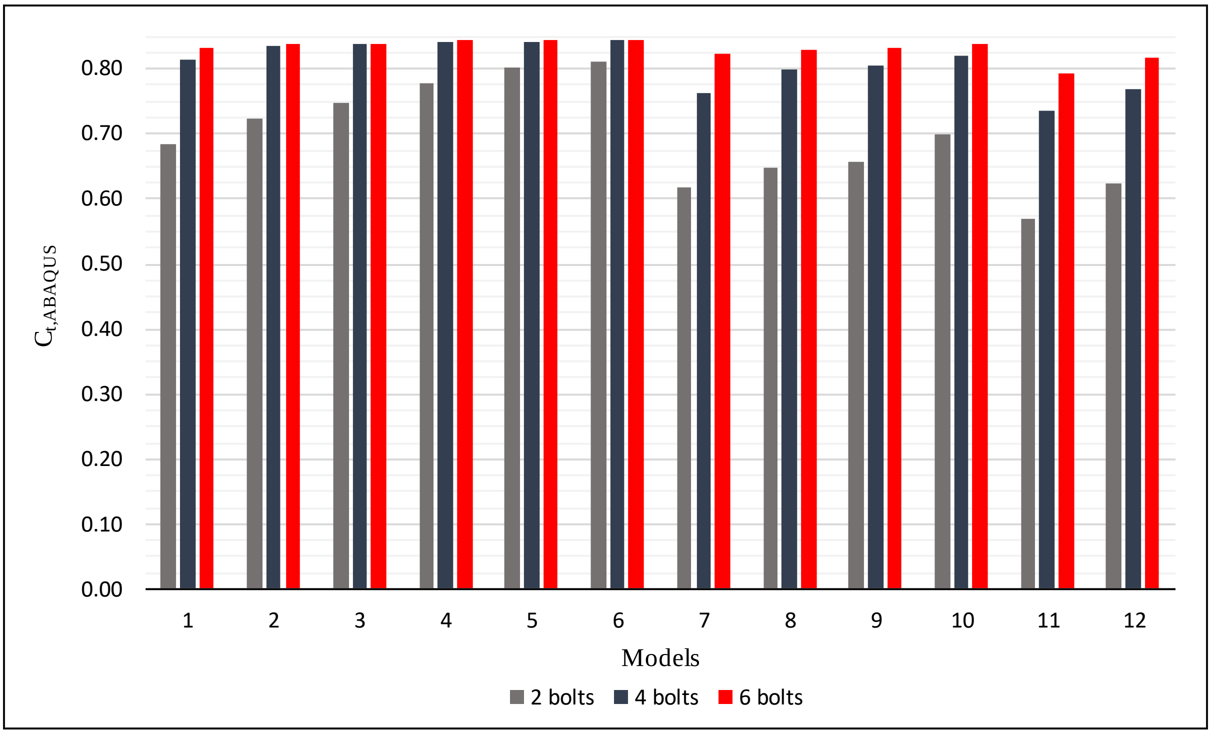


Figure 6. Results of reduction factor of the steel angles numerical study

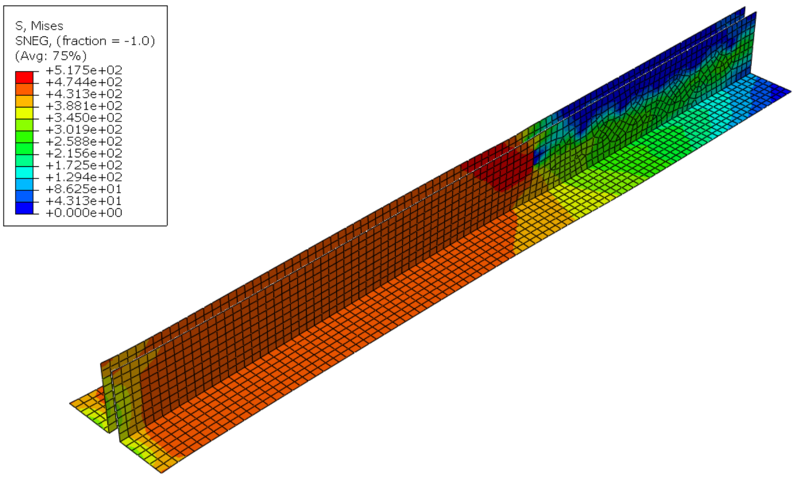


Figure 7. Stress distribution for the double steel angle

As shown in Fig. 6, the shear lag coefficient is affected by the connection length. For all cases, the results indicated that the net section efficiency increases with a higher connection length. The two-bolt connection presented a substantial reduction in efficiency when compared to the four-bolt connection. For the cases 7 to 12, which correspond to angles with equal legs of 102 mm and angles with unequal legs of 102 mm and 76 mm, the reduction of the coefficient due to the lower number of bolts ranged from 14.7% to 22.6%. Meanwhile, for the cases 1 to 6, which are represented by angles with equal legs of 76 mm and angles with unequal legs of 76 mm and 51 mm, the decrease in efficiency varied from 4% to 16%. In this sense, the impact in the coefficient is accentuated for the steel angles with greater dimensions.

The difference of the coefficient from the four-bolt to the six-bolt connection is small, especially for the Models 1 to 6. For these cases the difference is minimal, inferior to 2%. However, for the cases 7 to 12, the net section effectiveness increased slightly from the four-bolt to the six-bolt connection, ranging from 2.1% to 7.2%. Furthermore, it can be concluded that the size of steel angle has an inverse relationship with the shear lag coefficient results obtained. This conclusion is coherent, since a steel angle with longer legs indicates a higher eccentricity for the connection and, consequently, a decrease in the reduction factor values for the same connection length.

Regarding the connection length, the study indicated a direct relationship among length and the net section efficiency for double angles. However, with the increase of this length, the section effectiveness tends to converge to a specific value, as displayed in Fig. 6 for the steel angles with smaller dimensions (Models 1 to 6), which presented similar results for the connections with four and six bolts.

From the numerical models is also possible to analyze the effect of the angle disposition and the angle thickness to the tensile effectiveness of the cross-section. In this study, the angles with unequal legs were connected either by the long leg or by the short leg. For all cases, the cross-section effectiveness is higher in steel angles connected by the long leg. This result was expected, since a connection made through the long leg culminates in a lower value of eccentricity, which guarantees a better net section efficiency for the same connection length.

As demonstrated in Tab. 1, each steel angle type has two similar models except for the angle thickness. For example, the Models 1 and 3 present the same geometry and are connected by the same leg size, however, the thickness of the Model 3 is twice the thickness of the Model 1. In this sense, it was possible to determine the effect regarding the angle thickness to the reduction factor. For all cases, the models with a higher thickness presented a better value for the shear lag coefficient. The impact in the coefficient is particularly noted in the models that presented two-bolt connections, in which the efficiency ranged from 4.1% to 8.5%. For the four-bolt and six-bolt connections, in general, the difference of thickness had little effect on the net section efficiency. The numerical models presented a similar behavior regarding the effect of the angle thickness. In order to illustrate this comportment, the comparison between the Models 1 and 3 is presented in Tab. 3.

Table 3. Comparison of the Models 1 and 3 to the effects of the angle thickness

| Models | Angle size (mm) | n | $C_{t,ABAQUS}$ | Difference (%) |
|---|---|---|---|---|
| 1 | 2L 76 x 51 x 4.8 | 2 | 0.747 | 4.11 |
| 3 | 2L 76 x 51 x 9.5 | | 0.801 | |
| 1 | 2L 76 x 51 x 4.8 | 4 | 0.837 | 0.64 |
| 3 | 2L 76 x 51 x 9.5 | | 0.842 | |
| 1 | 2L 76 x 76 x 4.8 | 6 | 0.838 | 0.59 |
| 3 | 2L 76 x 76 x 9.5 | | 0.843 | |

Furthermore, a comparison of the numerical results with the results obtained through standards procedures is presented in Tab. 4. It was verified that the design proposed by the NBR 8800:2008 and ANSI/AISC 360-16 displays similar values to the results obtained through the numerical modeling and experimental tests. However, the EN 1993-1-1:2005 presents a superior limitation for the shear lag phenomenon. In this sense, the European code assumes a way more conservative approach regarding the nonuniform tension distribution in steel angles. In this last case, for failure analyses in which the tensile rupture in the net section governs the design, the numerical analysis can be used as a mean to obtain less conservative design for steel angles bolted at one leg, in order to avoid structural oversizing, and guarantee structural safety.

Table 4. Comparison between numerical models, tests of Kulak and Wu (1997) and standards results

| Specimen | $C_{t,ABAQUS}$ | $C_{t,Tests}$ | $C_{t,NBR}$ | $C_{t,AISC}$ | $C_{t,EN}$ |
|---|---|---|---|---|---|
| 1 | 0.83 | 0.89 | 0.90 | 0.95 | 0.55 |
| 2 | 0.82 | 0.82 | 0.90 | 0.92 | 0.55 |
| 3 | 0.79 | 0.79 | 0.90 | 0.93 | 0.55 |
| 4 | 0.83 | 0.82 | 0.90 | 0.95 | 0.55 |
| 5 | 0.85 | 0.90 | 0.90 | 0.96 | 0.55 |
| 6 | 0.84 | 0.86 | 0.90 | 0.97 | 0.55 |
| 7 | 0.84 | 0.90 | 0.90 | 0.95 | 0.55 |
| 8 | 0.78 | 0.72 | 0.84 | 0.84 | 0.48 |

## 4. CONCLUSION

In this paper, the influence of shear lag on the behavior of steel angles connected by one leg was analyzed through finite element analysis developed with the software ABAQUS. For this, a consistent numerical model was generated and validated against experimental results proposed by Kulak and Wu (1997), in order to obtain accurate values for the reduction coefficient of the net area. Then, a study was accomplished diversifying the following parameters: connection length, steel angle thickness and angle disposition for the connection.

Through the study it was verified that the connection length has a direct relationship with net section efficiency. However, the effectiveness of the cross-section tends to converge to a specific value with the increment of the length of connection. The two-bolt connection presented a substantial reduction in efficiency when compared to the four-bolt connection. However, the difference between the four-bolt and six-bolt connections was, for most cases, non-expressive.

Regarding the angle disposition for the connection, it was concluded that steel angles connected by the long leg display a better efficiency of the net area. This result is due to the lower value of eccentricity obtained through the connection in the long leg, which provides an increment of the reduction factor for the same connection length. From the study, it was also possible to conclude that thicker steel angles result in a better efficiency of the cross-section.

From the comparison of the numerical results with the analytical procedures proposed by standards, it is possible to conclude that the EN 1993-1-1:2005 presents a superior limitation for the shear lag phenomenon. In this sense, the European code assumes a way more conservative approach regarding the nonuniform tension distribution in steel angles.

In addition, the present work displays the importance of the numerical modelling for structural purposes, allowing a more in-depth study of a phenomenon and the consideration of different parameters. Thus, numerical analysis can be used as a mean to obtain a more realistic design, in order to guarantee structural safety and efficiency.

## 5. ACKNOWLEDGEMENTS

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