

AUXETIC STRUCTURE DESIGN USING STRUCTURAL OPTIMIZATION TECHNIQUES

Paula Fregnani

Cícero Ribeiro de Lima

Universidade Federal do ABC - Campus SBC, Alameda da Universidade, Anchieta, s/n, SP, 09606-045

paula.fregnani@ufabc.edu.br; cicero.lima@ufabc.edu.br

Abstract. The material with a negative Poisson's ratio is known as auxetic and has not convectional behavior, since they expand laterally when tensioned and contract all sides when compressed. In the last decades, the importance of this metamaterial has gained notorious prominence in the scientific community, aiming at engineering applications. The main motivation of this work is to contribute with a more systematic methodology to design of the auxetic structures, make it independent of the designer expertise. For this, some computational simulation tools have been adopted, such as CAD for geometry modeling and CAE software for the finite element analyses. With the parametric optimization results were possible to create an optimized structure, considering an auxetic behavior using a conventional material. This example demonstrated in this study an improvement of 107% in the auxetic behavior (negative Poisson's ratio).

Keywords: optimization, auxetic structure, computational simulation, negative Poisson's ratio.

1. INTRODUCTION

Conventional material reduces its transversal section in the opposition direction when tensioned in one direction and for compression these materials expanded in the transversal direction. The Poisson's ratio (ν) is defined by Eq. (1), where ϵ_{load} is the deformation of load direction, while ϵ_{transv} is the deformation perpendicular or transversal to load direction (Lim, Teik-Cheng, 2015).

$$\nu = - \frac{\epsilon_{transv}}{\epsilon_{load}} \quad (1)$$

The deformation occurs in the opposite direction of load, for that a negative signal is inserted on equation to obtain a positive Poisson's ratio, regarding conventional material. Structures with auxetic behavior have negative Poisson's ratio, that is, they expand laterally when tensioned and contract all sides when compressed.

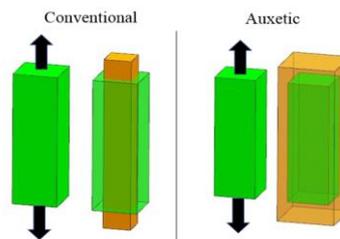


Fig. 1 – Conventional and Auxetic material

The auxetic material is divided into two groups, natural and man-made. The natural auxetic material is subdivided into molecular (arsenic, cadmium, etc) and biomaterial (salamander skin), according to examples presented by Jayanty (2010). A structure is man-made when conventional material is applied to generate a re-entrant structure, which has the auxetic behavior. Structures are known as re-entrant, according shown by the comparative study carried out by Álvarez Elipe et al (2012) and as exemplified in figure 2. An auxetic structure can be designed by the means of structural optimization method, in order to generate the geometry that gives the best response to the desired behavior. The focus of this work is the man-made structures and how to obtain this auxetic behavior, considering geometry of the structure as the major factor to be optimized.

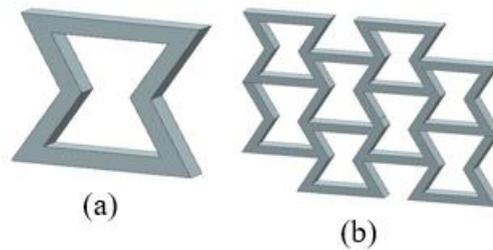


Fig 2 – (a) Unitary Cell – (b) macrostructure

Jayanty (2010) shows the relation between the mechanical properties and concludes that the auxetic effect can provide to the materials many benefits, such as increase in stiffness, improvement of tenacity to fracture and improvement in shear stiffness.

Due to improved absorption impact properties, the auxetic materials have great potential for use in the automotive, textile, military and health industries, as shown in some examples described by Prawoto (2012) and Evan et al (2000). For example, in Biomedicine, auxetic materials can be applied to surgical implants, prosthetic material, and dental floss; in Footwear industry, they can be seen in tennis soles that expand during walking or running, increasing flexibility; and in Military/Defense industry, these metamaterials are very suited to be employed to helmet, bulletproof vest, knee guard, glove, and protective equipment.

As shown by Imbalzano (2015) and Mir et al (2015), the auxetic structures can be manufactured with different types of materials (polymers, metals, ceramics and composites), which are chosen according to the application, although it is not widely disseminated.

The objective of this work is to develop a man-made auxetic microstructure, with the aid of the optimization tool that can be reached through a systematic design methodology. In Section 1, auxetic microstructures are introduced. Section 2 presents an analysis of the parameters employed to the auxetic structure, which will be the basis for the development of the parametric structural optimization. Section 3 shows how software works to applied optimization procedure, Section 4 shows the results and Section 5 gives the conclusion.

2. EXPERIMENT PLANNING

This first analysis aims to identify the parameters of the geometry that most influence the auxetic behavior of the structure. That is, the main goal of this analysis is to know the behavior of auxetic structure as some parameters (width, height, thickness wall and inclination angle) are changed individually. These analyses are useful for us to find a preliminary parameters combination that improve auxetic behavior of the structure, and to be used as guidance in a structural optimization process, in which an optimized set of parameters can be reached through a systematic design methodology.

The following parameters (see figure 3) are considered: angle (α_1), length (L1), height (H1) and thickness (all structure has same thickness -T1).

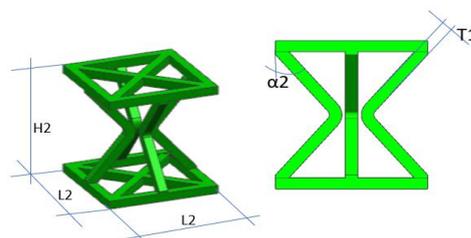


Fig. 3 – Geometry and parameters

A well-known 3D re-entrant auxetic structure of the literature (Ren et al., 2018) is adopted as initial design domain, in which the parameters (figure 3) have been evaluated. Boundary conditions are applied to evaluate the geometry of these structures, according to figure 4.

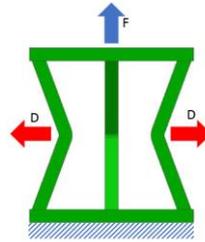


Fig.4 – Application of the boundary conditions (input load and desired output displacement)

As shown in figure 4, F and D indicate the displacement in the direction of the applied force and the desired output displacement, respectively. A large combination of the parameters and amount of time was carried out to simulate these experiment performed here. Chart 1 shows a summary of the results obtained from these analyses.

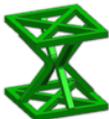
|  | ângulo $\alpha 1$ | | | esp (mm) | | | L1(mm) | | | H1(mm) | | | Poisson | % |
|---|-------------------|-----|-----|----------|---|---|--------|----|----|--------|----|----|---------|------|
| | 20° | 30° | 40° | 2 | 3 | 4 | 25 | 30 | 35 | 25 | 30 | 35 | | |
| 1° | x | - | - | x | - | - | x | - | - | x | - | - | -0.62 | -10% |
| 2° | - | x | - | x | - | - | x | - | - | x | - | - | -0.64 | -6% |
| 3°* | - | - | x | x | - | - | x | - | - | x | - | - | -0.69 | - |
| 4° | - | - | x | - | x | - | x | - | - | x | - | - | -0.55 | -20% |
| 5° | - | - | x | - | - | x | x | - | - | x | - | - | -0.38 | -44% |
| 6° | - | - | x | x | - | - | - | x | - | x | - | - | -0.63 | -8% |
| 7° | - | - | x | x | - | - | - | x | - | - | x | - | -0.85 | 24% |
| 8° | - | - | x | x | - | - | - | x | - | - | - | x | -0.63 | -8% |
| 9° | - | - | x | x | - | - | - | - | x | x | - | - | -0.62 | -9% |
| 10° | - | - | x | x | - | - | - | - | x | - | x | - | -0.85 | 24% |
| 11° | - | - | x | x | - | - | - | - | x | - | - | x | -0.69 | 0% |
| 12° | - | - | x | x | - | - | x | - | - | - | x | - | -0.74 | 7% |
| 13° | - | - | x | x | - | - | x | - | - | - | - | x | -0.68 | -1% |

Chart 1 – Results obtained from the experiment planning

Initially, the analyses were set to specific parameters and started varying only the angle $\alpha 1$, as can be seen in the first three lines of the chart 1. After that, the angle $\alpha 1 = 40^\circ$ (3°* model) has been chosen as baseline for the other models shown in chart1. In addition, the other parameters (L1, H1, T1) are changed, considering predetermined values to analyze auxetic behavior. The structure models are analyzed by a finite element (FE) mesh 94,800 tetrahedral elements. Each FE analysis has been solved in approximately 2 minutes by using notebook Core i7 7th generation and RAM 8GB.

These analyses were considered to understand the impact of each parameters in these investigated models (chart 1), and it is noticed that the angle $\alpha 1$ has great influence in re-entrant structure. Therefore, the arrangement of $\alpha 1$ with others different parameters (shown in chart 1) should improve the auxetic behavior (negative Poisson's ratio) of the results.

3. OPTIMIZATION

Techniques of structural optimization can support this auxetic structure design considering a large range of parameters, combining them simultaneously at the same computational simulation. In this work, OPTISTRUC (Altair) is adopted as the optimization tool. OPTISTRUC is one of commercial software used in industry for modern structural analysis and optimization designs, and is also available in the university for academic purposes.

This work is focused on the parametric optimization, where one or more dimensional parameters (design variables) of the auxetic structure can be considered in a large range with no predetermined values. The objective to be achieved in this optimization is to maximize the desired output displacement and, thus, to obtain an auxetic geometry with better response. The objective function is represented by expression of the Poisson's Ratio, Eq. (1), written in terms of the displacements shown in figura 4.

$$v = -\frac{\boldsymbol{\varepsilon}_{\text{transv}}}{\boldsymbol{\varepsilon}_{\text{load}}} = -\frac{D}{F} \quad (2)$$

Optimization is defined as the automatic process to make a component as good as possible based on an objective function and subject to certain design constraints. Using OPTISTRUC, same boundary condition as indicated on figure 4 were defined to the geometry, as an input to linear analyze and evaluate behavior of the re-entrant this structure.

There are many different methods that can be used to optimize a structure, in OPTISTRUC size optimization (parametric) is based on gradient methods (Rao, 1996). Gradient methods depend on the sensitivity of the structure responses, considering changes in design variables in order to understand the effect of the design changes and optimization. The gradient method can be ascendant direction or descendent, to find the maximum or the minimum respectively. The gradient method can be generalized by a function of n variables:

$$\text{Ascent Method} \quad x^{(k+1)} = x^{(k)} + \lambda * \nabla f |_{x^{(k)}} \quad (3)$$

$$\text{Descent Method} \quad x^{(k+1)} = x^{(k)} - \lambda * \nabla f |_{x^{(k)}} \quad (4)$$

The gradient method is a first order iterative optimization algorithm, from which the negative gradient points the direction of the function at the current point. To maximize a function the positive gradient will indicate the direction for verification and λ will indicate the step that varies in each iteration.

In the optimization process, sensitivity analysis (gradients of the objective function) is evaluated at each step of the algorithm. For this problem the answers of interest are of displacements. Thus, the optimization problem is given as follows.

$$\begin{aligned} &\text{maximize} \quad D \\ &\quad \quad \quad L1, H1, T1, \alpha \\ &\text{subject to} \quad \mathbf{K}\mathbf{u} = \mathbf{f} \quad (\text{equilibrium equations}) \end{aligned} \quad (5)$$

The values maximum and minimum considered for these analyses:

$$\begin{aligned} L1 &= 25 - 35 \text{ mm}; \\ H1 &= 25 - 35 \text{ mm}; \\ T1 &= 2 - 4 \text{ mm}; \\ \alpha &= 15^\circ - 40^\circ. \end{aligned}$$

The direct analytical method is obtained by the derivative of the equilibrium equation (4):

$$\frac{dK}{dx} u(x) + K \frac{du}{dx} = \frac{dF}{dx} \quad (6)$$

Where du/dx represents the sensitivity of the displacements with respect to the design variables, which can be calculated by using the adjoint method (Rao, 1996).

The stiffness matrix K is factored by Cholesky decomposition, that is, a non-singular lower triangular matrix. The equation (5) needs to be solved n times, to calculate the sensitivity for all the design variables. The stress constrains was considered to evaluate this structure.

4. RESULTS

Initially the parameters of geometry on Fig. 5a are: angle 40° , width 25mm, height 25, thickness 2mm. Figure 5b shows the structure optimized after 7 iterations, in which final geometry parameters are angle 19° , width 26, height 35 and thickness 2mm.

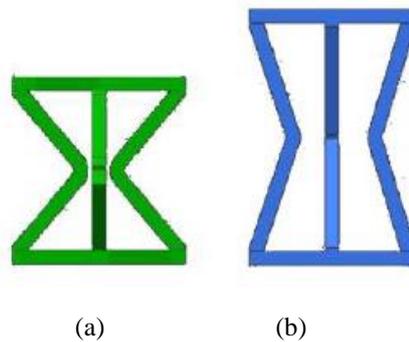


Fig.5 (a) Initial structure; (b) Structure optimized

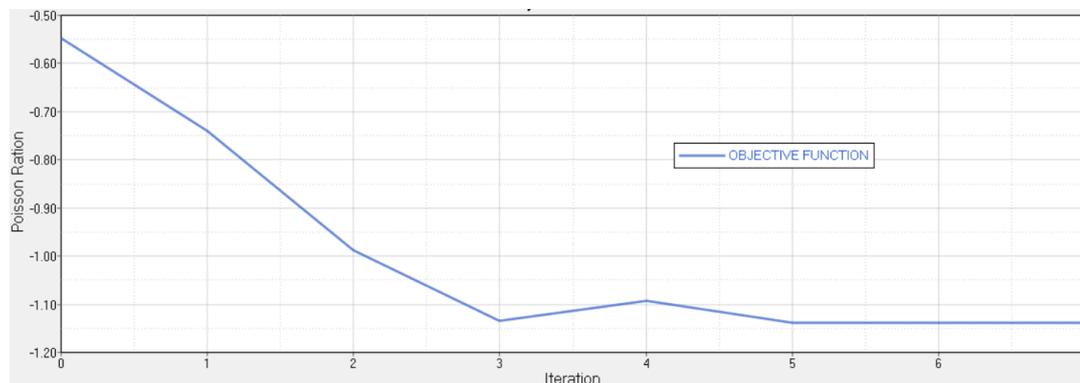


Fig. 6 Convergence of objective function

The plot of Fig 6 illustrates the convergence of iterative optimization process, in which the objective function starts from Poisson's ratio of -0.55 and achieves a value close to -1.14 at the convergence. That is, it represents an improvement of 107% in the auxetic behavior (negative Poisson's ratio). This work demonstrates the auxetic structure combining the parameters that bring the maximization behavior and a way to construct in a systematic form.

Comparing the results of experimental analyses, which was conducted using predetermined values, optimization shows results that was not considered previously. This demonstrates the importance of this tool to improve the design of the auxetic structures.

5. -CONCLUSIONS

This work shows the capability within the commercial software Altair OPTISTRUC during the process of extending it into a general structural optimization tool that provides parametric optimization as an option. Emphasis has been placed on integrating state-of-the-art techniques that enhance the overall efficiency of the optimization process.

With the combination of variables obtained in the optimization process, there was an improvement in geometry performance when compared to the initial one. This way with this project it was possible through a systematic methodology to create a microstructure with optimized auxetic behavior using a conventional material. Moreover, auxetic structures (macrostructures) can be designed by applying a periodic repetition of the optimized microstructure cell over a desired domain.

This method of performing parametric optimization has resulted in knowledge in how to implement in the design process and understand how software works to run this kind of analysis. Experimenting with different combinations of variables has helped to understand the behavior of the structure.

6. REFERENCES

- Evans, Ken E. and A. Alderson. Auxetic materials: functional materials and structures from lateral thinking!, *Advanced Materials*, 12 (9), 617 - 628, 2000.
- Imbalzano, Gabriele et al, Three-dimensional modeling of auxetic sandwich panels for localized impact. *Journal of Sandwich Structures and Materials*, December 2015
- Jayanty, Sharmila. Negative Poisson's ratio composite: finite element modeling and experiments, Dissertation 2010, The University of Toledo.
- Lim, Teik-Cheng. *Auxetic materials and structures*, Springer Singapore, 2015.
- Prawoto, Yunan. Seeing auxetic materials from the mechanics point of view: A structural review on the negative Poisson's ratio, *Computational Materials Science*, Elsevier – 2012.
- Ren, Xin et al. Auxetic material and structure review, *Smart material and structure*, 2018.
- Rao, S. S. *Engineering optimization: theory and practice*. John Wiley & Sons, 1996.

7. RESPONSIBILITY NOTICE

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