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DYNAMIC BEHAVIOR OF AXIALLY MOVING SYSTEMS WITH ELASTIC SUPPORTS

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Abstract. *Axially moving systems are present in a wide range of engineering devices such as transmission belts, variable transmission systems, band saw, cold drawing process, paper machines. However, above a certain critical speed, these systems can experience strong vibration and dynamic instabilities, leading to structural failure. In the literature, there are studies of such systems considering both the elastic and rigid supports as boundary conditions. It is known that this elastic foundation can change the behavior of axially moving structures. In this context, the stability and dynamic responses of these two types of systems have been widely studied to guarantee the success of the projects. Several methods were proposed for the structural analysis and among them, the spectral element method is very interesting in the dynamic analysis of structures. This work developed axially moving strings models using the spectral element method and analyzed the effects of the moving speed and the elastic foundation on the transversal vibrations. The moving speed considerably affects the natural frequency of the structures. The elastic foundation affects the natural frequencies and, together with the moving speed, attach a damping effect to the frequency response function.*

Keywords: *axially moving systems, elastic foundation, spectral element*

1. INTRODUCTION

Axially moving structures are of technological importance and widely present on engineering devices as transmission belts, CVT, band saws, aerial cable tramways, thread-lines in the textile industry and the like. Depending on the moving speed, these systems may experience strong vibrations and static and dynamics instability resulting in structural failure. Thus, the dynamic characteristics of such axially moving structures, focus on the transverse vibration, became an important subject to ensure stable work conditions. The literature presents a large number of papers on this topic.

(Lee *et al.*, 2004) formulated a spectral element model to analyze the transversal vibrations of an axially moving Timoshenko beam subjected to constant axial tension. The results were compared with analytical and finite element method solutions. (Lee and Oh, 2005) analyzed the viscoelasticity and the moving speed effects on an axially moving viscoelastic beam using the spectral element methods. (Xia *et al.*, 2015) performed an experimental study about nonlinear dynamics characteristics of an axially moving string. (Lee and Perkins, 1995) presented an experimental study of the nonlinear dynamic characteristics of taut steel cables using a 3-D motion analysis system.

(Oh *et al.*, 2004) studied the transversal vibration of an axially moving Euler-Bernoulli beam using the spectral element method. Also investigated the effects of the moving speed and axial tension at the vibrations and stability. The elastic foundation can severely affect the behavior of the axially moving structure. (Bhat *et al.*, 1982) studied axially moving belts supported on an elastic foundation and also considered nonlinearities. Assuming constant moving speed, they obtained the free vibration response of the belt in the presence of the elastic foundation for various velocities of the belt. (Banichuk *et al.*, 2014) studied the paper web machine considering the model of an infinite, homogeneous linearly elastic beam resting on a system of linearly elastic supports.

The purpose of this work is to develop axially moving strings models supported by an elastic foundation using the spectral element method to analyze the dynamic characteristics and the effects of the moving speed on the transversal vibrations. And also analyze the effects of periodicity.

2. SPECTRAL ELEMENT METHOD

The Spectral Element Method, SEM, is widely used on structural dynamics problems, wave propagation, and other related problems because it allows the accurate description of the dynamic behavior. According to (Lee, 2009), the SEM can be considered as a combination of the critical features of the Finite Element Method (FEM), the Dynamic Stiffness Method (DSM) and the Spectral Analysis Method (SAM).

2.1 Spectral Element Modeling

Figure 1 shows the first two models that are considered in this work. Model 1 is a simply supported uniform axially moving string and Model 2 is an axially moving string supported by an elastic foundation (a spring that can be linear or nonlinear). Both strings have the same properties, a uniform cross-section area A, traveling in its axial direction with a constant moving speed c. The supports are separated by a distance L, and the cables have mass density ρ.

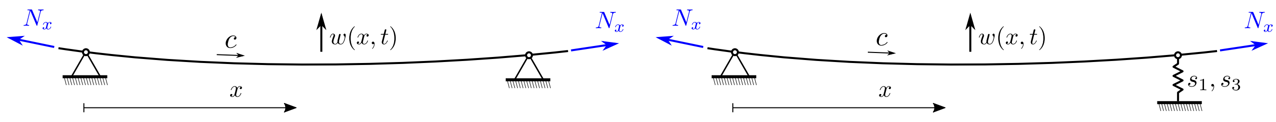


Figure 1. Axially moving string models, (a) Model 1 - simple support and (b) Model 2 - subject to elastic foundation

In both cases, the cables are subject to axial tension N_x. The transverse displacement $w(x,t)$ defines the motion of the cable as a function of time and position along its length. The equation of motion for the system, without considering any of the boundary conditions is given by the partial differential equation (Lee, 2009).

$$\rho A\ddot{w} + 2\rho Ac\dot{w}' + \rho Ac^2 w'' - N_x w'' = f(x,t) \tag{1}$$

The solution of Eq. 1 is obtained by the method of separation of variables, considering harmonic motion in the time domain as,

$$w(x,t) = \frac{1}{N}\sum_{n=0}^{N-1} W_n(x)\mathrm{e}^{i\omega t} \tag{2}$$

This way, the resulting equation of motion in the space domain is obtained as

$$-\omega^2\rho AW + i2\omega\rho AcW' - (N_x - \rho Ac^2)W'' = F(x) \tag{3}$$

Initially, considering the case of free motion ($F(x) = 0$) , a solution for the ordinary differential equation (Eq. 3) can be written as $W(x) = a\,\mathrm{e}^{-ikx}$, where a is a constant and k is the wavenumber given in rad/m. Applying this expression in Eq. 3, it is possible to obtain the dispersion equation

$$\left(N_x - \rho Ac^2\right)k^2 + 2\rho Ac\omega k - \rho A\omega^2 = 0 \tag{4}$$

The wavenumber is easily calculated from Eq. 4 as

$$k_{1,2} = -\omega\left(\frac{\rho Ac \pm \sqrt{N_x\rho A}}{N_x - \rho Ac^2}\right) \tag{5}$$

There is a critical value of cable speed, which vanishes the denominator of Eq. 5, $c_{cr} = \sqrt{N_x/\rho A}$. At this speed, the system becomes unstable, and the natural frequencies approach zero.

2.2 Harmonic Force Vibration

Considering the case of harmonic force vibration, the cable spectral element matrix can be calculate according the model developed in (Lee *et al.*, 2004), where for one element, the force vector f is related to the displacements $\mathbf{d}$ at the two ends of the cable, by the spectral element matrix $\mathbf{S}$ as

$$\mathbf{f} = \mathbf{S}(\omega)\mathbf{d}, \quad \text{where, } \mathbf{S}(\omega) = \mathbf{H}(\omega)^{-T}\mathbf{D}(\omega)\mathbf{H}(\omega)^{-1} \tag{6}$$

The spectral element matrix is calculated using

$$\mathbf{H} = \begin{bmatrix} 1 & 1 \\ \mathrm{e}^{ik_1L} & \mathrm{e}^{ik_2L} \end{bmatrix} \tag{7}$$

$$\mathbf{D}(\omega) = -(N_x - \rho A c^2)\mathbf{KEK} - \rho A\omega^2\mathbf{E} - \omega\rho Ac(\mathbf{KE} - \mathbf{EK}) + i\rho Ac(\omega\mathbf{I} - c\mathbf{K})\bar{\mathbf{E}} \tag{8}$$

the the matrix $\mathbf{K}$ is a diagonal matrix defined as $\mathbf{K} = \text{diag}(k_1, k_2)$, the elements of the matrix $\mathbf{E}$ are computed as

$$E_{rs} = \begin{cases} \frac{i}{k_r + k_s}\bar{E}_{rs}, & \text{if } k_r + k_s \neq 0 \\ L, & \text{if } k_r + k_s = 0 \end{cases} \tag{9}$$

where $\bar{E}_{rs}$ are the elements of the matrix $\bar{\mathbf{E}}$

$$\bar{\mathbf{E}} = \begin{bmatrix} \mathrm{e}^{-i2k_1L} - 1 & \mathrm{e}^{-i(k_1+k_2)L} - 1 \\ \mathrm{e}^{-i(k_1+k_2)L} - 1 & \mathrm{e}^{-i2k_2L} - 1 \end{bmatrix} \tag{10}$$

The spectral element matrices can be assembled in a completely analogous way to that used in FEM. Applying the boundary conditions after the assembly may provide a global system equation in the form as $\mathbf{S}_g(\omega)\mathbf{d}_g = \mathbf{f}_g$ where $\mathbf{S}_g(\omega)$ is the global dynamic stiffness matrix, $\mathbf{d}_g$ is the global spectral nodal DOFs vector, and f_g is the global spectral nodal forces vector. Considering the case of a cable simple supported at the left hand side and elastically supported on the right hand side by a spring with linear elastic constant s_1, the spectral element matrix can be written as

$$\begin{Bmatrix} f_1 \\ f_2 \end{Bmatrix} = \begin{bmatrix} S_{1,1} & S_{1,2} \\ S_{2,1} & S_{2,2} + s_1 \end{bmatrix} \begin{Bmatrix} d_1 \\ d_2 \end{Bmatrix} \tag{11}$$

where $S_{p,q}$ are the elements of spectral element matrix given by eq. (6). The application of the simple supported case, allows to write the displacement of the right hand side due to a harmonic force applied at the same location as

$$d_2 = (S_{2,2} + s_1)^{-1} f_2 \tag{12}$$

where, $S_{2,2}$, is calculated according to the procedure described previously

2.3 Periodic Model

Periodicity is an important issue for the vibro-isolation purposes because the location of stop bands may be tuned to match expected excitation frequencies, and the optimization issues.(Hvatov and Sorokin, 2015). In periodicity problems, impedance matrices are used, which are the dynamic stiffness, impedance and apparent mass matrices. The spectral matrix is a dynamic stiffnes matrix, therefore it can be used to build a periodic system.

Model 3, which contains periodicity and is shown in Fig. 2, is the another model also studied in this work. Each periodicity cell of the Model 3 corresponds to Model 2. The spectral matrix for Model 3 is made from the assembly of the Model 2 spectral matrix as a diagonal sum.

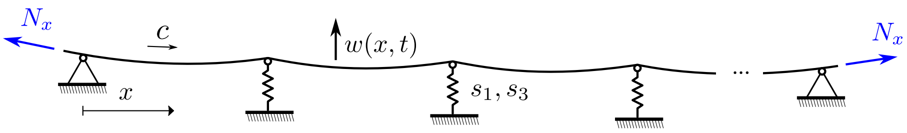


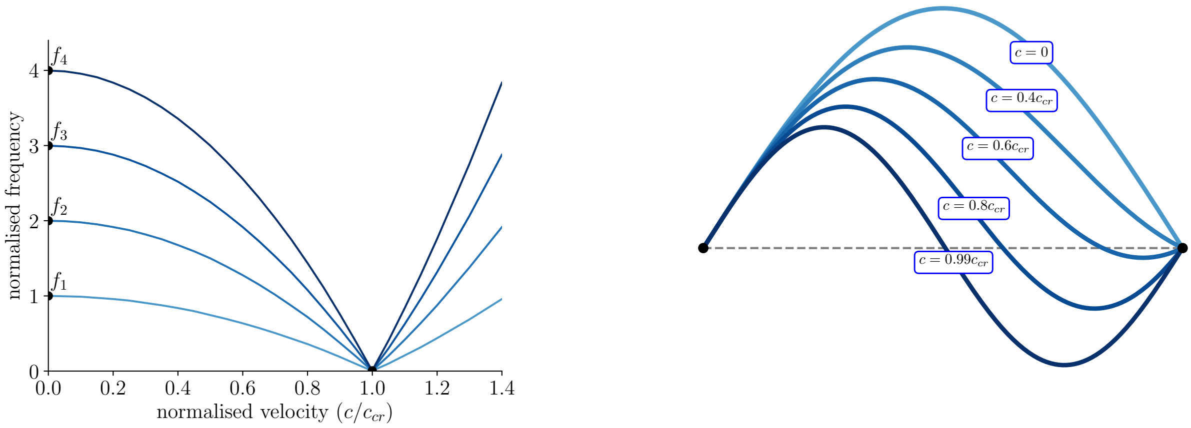
Figure 2. Periodic axially moving string model

3. RESULTS

For numerical illustration the three models previously introduced are considered. The geometric and material properties of the string are $L = 2$m, $\rho A = 1$[kg/m] and $N_x = 10$ [kN]. The results are presented in three sections for the Model 1, Model 2 and Model 3 respectively.

3.1 Model 1 - Simply supported model

The critical speed calculated for the simple supported case is the same presented in (Lee *et al.*, 2004), which is calculated using $c_{cr} = \sqrt{\frac{N_x}{\rho A}}$, using the numerical properties, the critical moving speed is $c_{cr} = 100$ [m/s]. The influence of the critical speed on the natural frequencies can be seen in Fig. 3(a).



(a) (b)

Figure 3. The influence of the cable speed on: (a) the natural frequencies and (b) first mode shape.

The mode shapes can be calculated considering the format given in Eq. 13 and the boundary conditions and the influence of the cable speed on the first mode is shown in Fig. 3(b).

$$W(x) = a_1 \sin(k_1 x) + a_2 \sin(k_2 x) \tag{13}$$

3.2 Model 2 - Elastic foundation model

For this model, it is important to find out what effect the elastic foundation has when compared to Model 1 and also whether the moving speed behavior at the natural frequencies of the system is the same for the simply supported case. Figure 4 is a frequence response function that shows the influence of the elastic foundation stiffness on natural frequencies for four cases with distinct moving speed. The Fig. 5 is similar to Fig. 4 but is the top view of the frequency response of each moving speed.

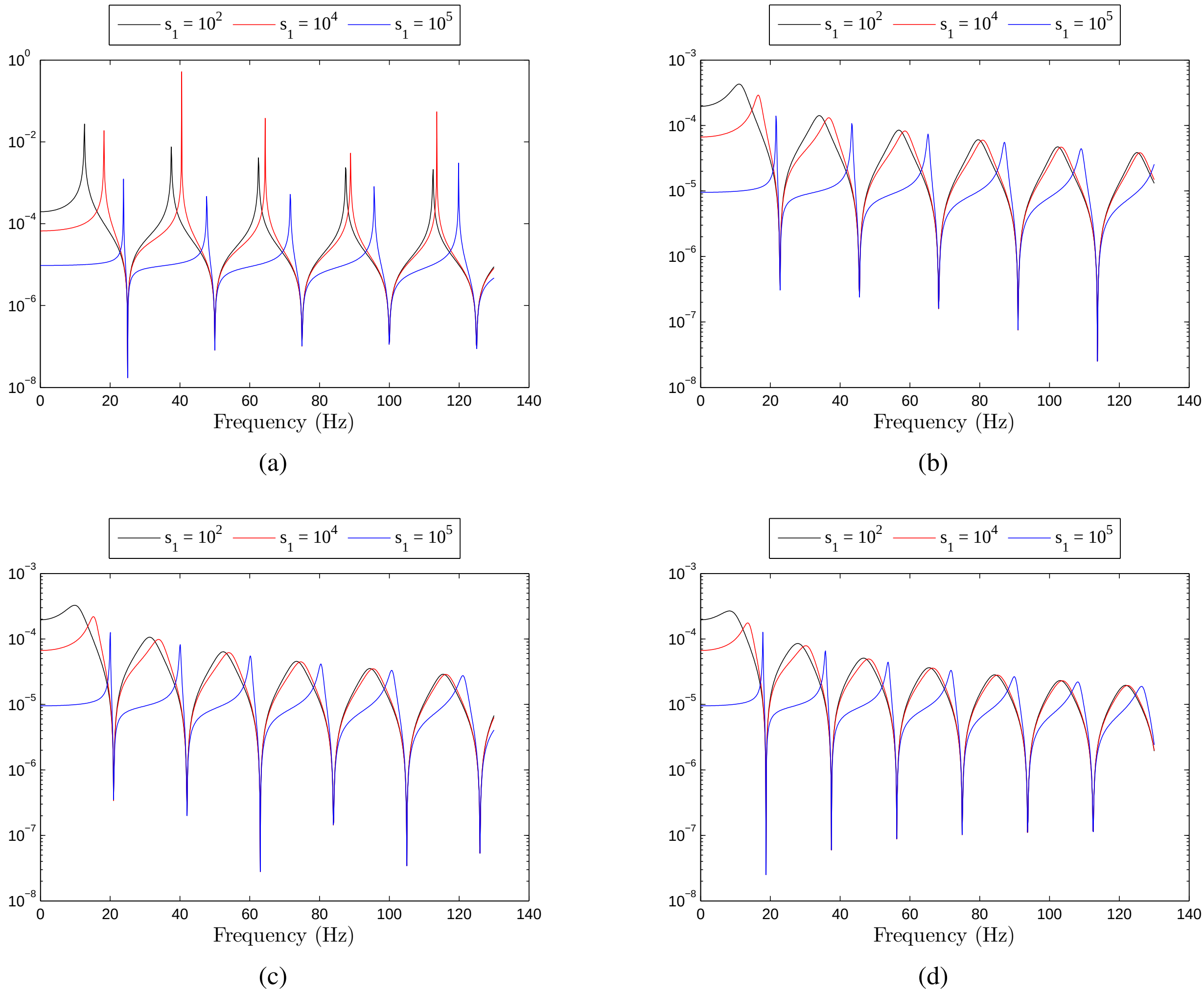


Figure 4. Frequency response for different elastic fundation stiffnes to show the influence the moving speed on the natural frequency. (a) $c = 0\,m/s$, (b) $c = 30\,m/s$, (c) $c = 40\,m/s$ e (d) $c = 50\,m/s$.

By establishing the elastic foundation stiffness, $s_1 = 10^5$, in Fig. 6, a frequency response, it is possible to see what the influence on the frequency response varying the moving speed.

Figure 7 is a top view of the frequency response that shows the influence on the frequency response varying the moving speed for Model 2 with four different elastic foundation stiffness values.

When the stiffness is sufficient high, the behavior of the model resemble the simply supported model. One may observe that the second mode natural frequency vanishes when the elastic foundation stiffness is increased to a certain high value. The behavior seems to be the simply supported model because the natural frequencies are decreased as the moving speed of the string is increased. Although, it seems to appear a damping effect as the moving speed is increased in the elastic foundation model.

3.3 Model 3 - Periodicity model

The periodic model considered has 20 cells. Figure 8 compares different values of the elastic foundation stiffness, s_1, for the same moving speed, $c = 0\,m/s$. It can be seen that for the higher the stiffness value, the larger the band-gap size, which are the regions where attenuation of the waves occurs.

The influence of the moving speed on the band-gap regions is shown in Fig. 9 and Fig. 10 , for this model the elastic

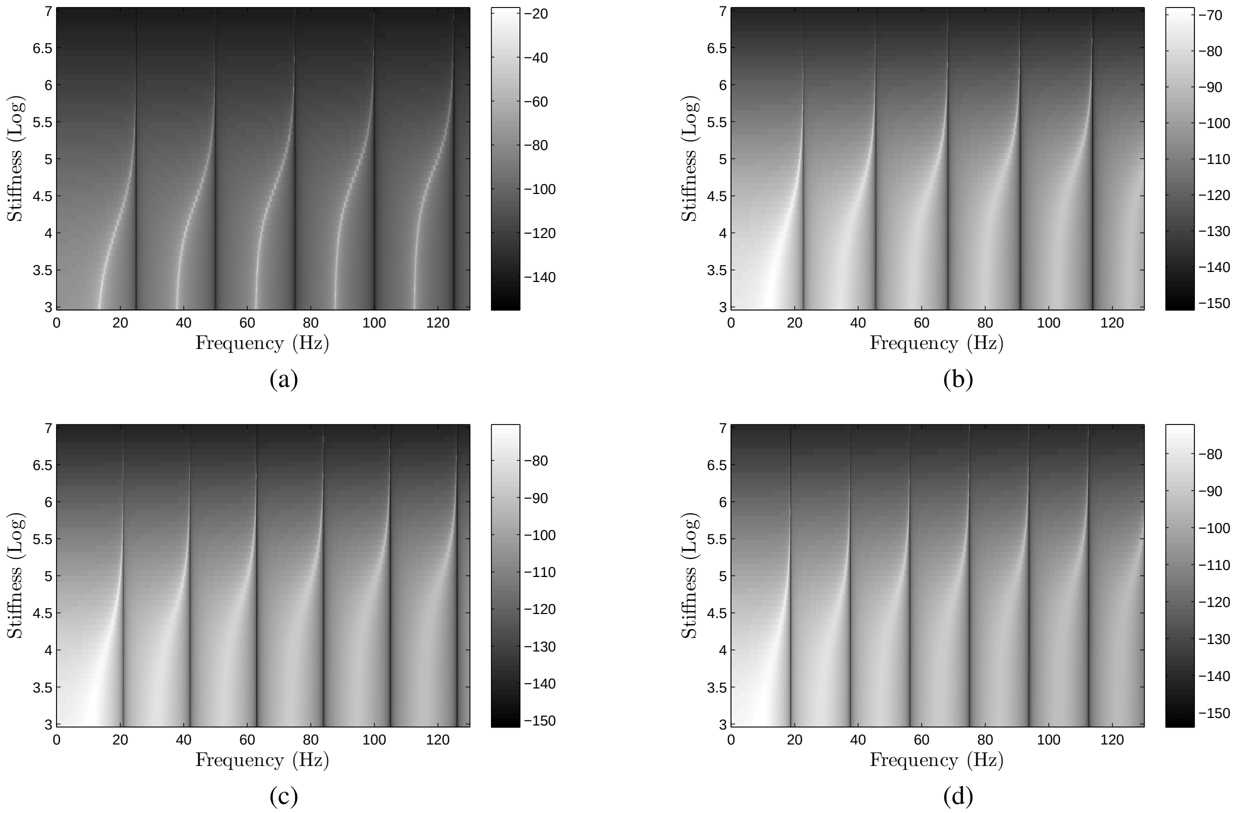


Figure 5. The influence on the frequency response varying the stiffness value of the elastic foundation to different moving speed. (a) $c = 0\,m/s$, (b) $c = 20\,m/s$, (c) $c = 30\,m/s$ e (d) $c = 50\,m/s$. These images are the top view of the frequency response and the colors indicate the amplitudes in dB.

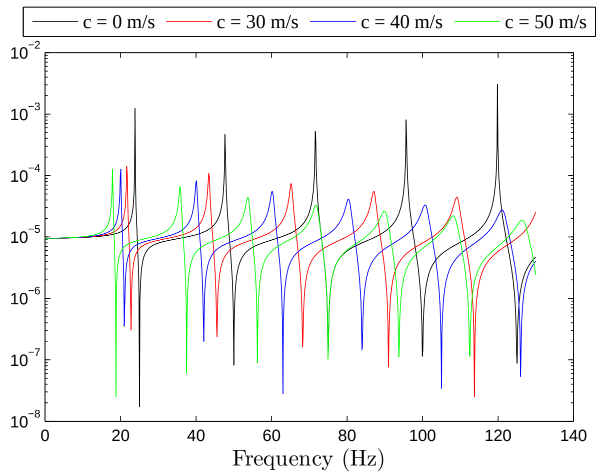


Figure 6. The influence on the frequency response varying the moving speed with elastic foundation $s_1 = 10^5$.

foundation stifness is $s_1 = 10^5$. It can be concluded that for higher moving speeds the natural frequencies get lower.

4. CONCLUSIONS

The present work is interested in the study of transverse vibration in axially moving strings. Axially moving structures are of technological importance and widely present on engineering devices. The moving speed considerably affects the natural frequency of the structures, and the frequencies decrease as the moving speed increases. The elastic foundation affects the values of the natural frequencies, the frequencies increase as the stiffness increases, and the moving speed

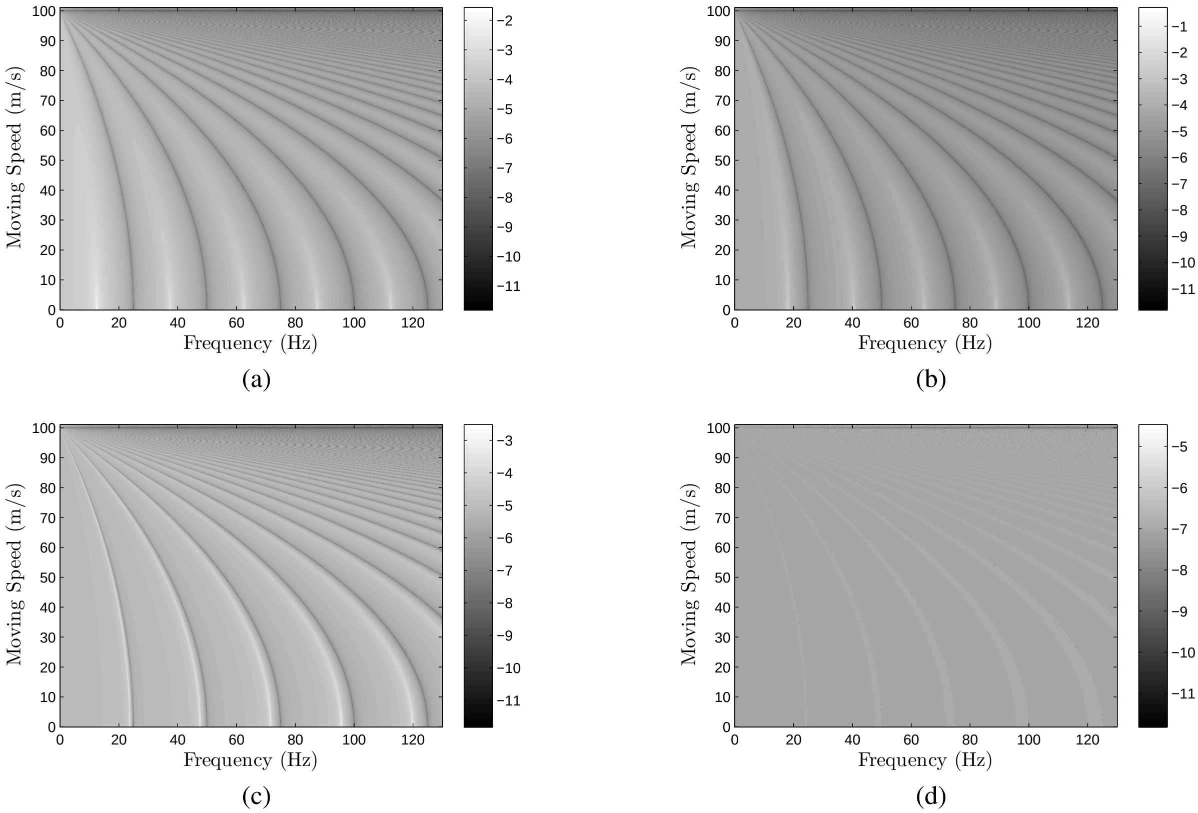


Figure 7. The influence on the frequency response varying the moving speed with elastic foundation (a) $s_1 = 10^3$, (b) $s_1 = 10^4$, (c) $s_1 = 10^6$, e (d) $s_1 = 10^7$. These images are the top view of the frequency response and the colors indicate the amplitudes in dB, with reference of 1 [m/N]

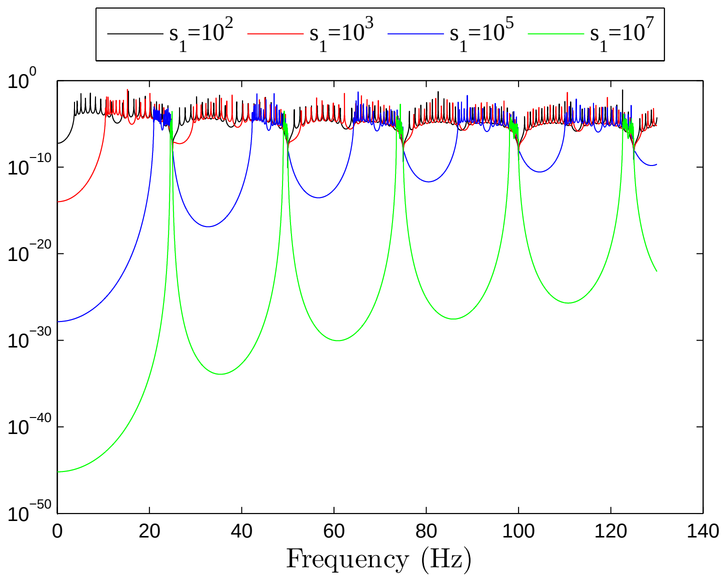


Figure 8. The influence of the elastic foundation stiffness on the band-gaps. Model with 20 periodic cells

creates a damping effect in the frequency response function.

This work is also interested in the study of periodic systems because the location of stop bands may be tuned to match expected excitation frequencies what is interesting for the vibro-isolation and optimization purposes. When there is the presence of elastic foundation there is a frequency range in which the amplitude starts to increase to the peak as the base stiffness value increases this frequency range becomes more extreme until it becomes a line when the stiffness is very

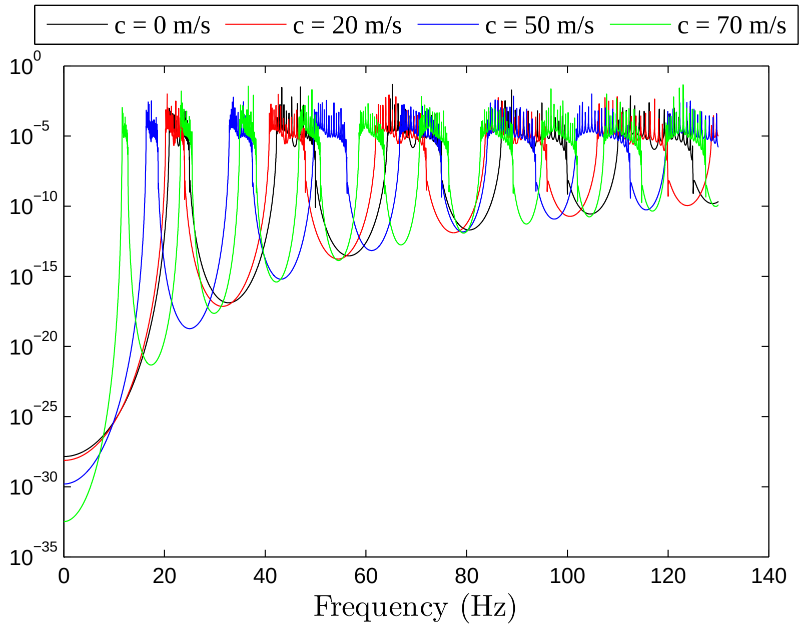


Figure 9. The influence on the frequency response varying the moving speed with elastic foundation $s_1 = 10^5$ for 20 cells.

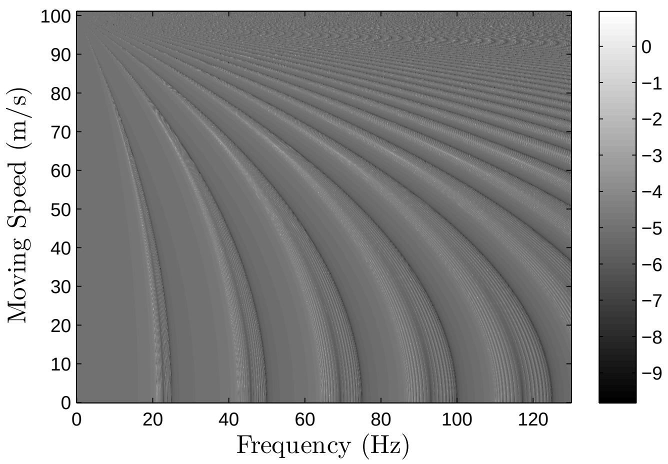


Figure 10. The influence on the frequency response varying the moving speed with elastic foundation $s_1 = 10^5$ for 20 cells. This figure is the top view of the frequency response and the colors indicate the amplitudes in dB, with reference of 1 [m/N]

high. When, besides the elastic foundation, the speed is inserted, what happens is that, as in the simply support model (Model 1), the speed decreases the natural frequency values of the system and also a damping effect on the amplitude peaks, as speed increases the peaks become softer. Model 3, with periodicity, behaves similarly to Model 2. The value of natural frequencies decreases with increasing speed. However, in model 3, there are several resonance peaks within the same frequency range, where in model 2 only one resonance peak appeared. The stiffness of the elastic foundation affects the band-gaps regions, the higher the stiffness value, the larger the band-gap size.

In conclusion, the three studied model exhibited similar behavior with respect to the moving speed, the natural frequencies decrease as the moving speed increases. In Model 2 the elastic foundation acts on the value of natural frequencies, the increase of stiffness produces increase in natural frequencies. For Model 3, the value of the elastic base stiffness interferes with the band-gap size, increasing the stiffness value and increasing the band-gap width. The values of the moving speed and elastic foundation stiffness can be set so that the frequency is within a suitable working range.

5. ACKNOWLEDGEMENTS

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6. REFERENCES

Banichuk, N., Ivanova, S., Jeronen, J. and Tuovinen, T., 2014. "Periodic spectral instability analysis of axially moving beam with elastic supports". *J. Struct. Mech*, Vol. 47, No. 1, pp. 1–16.

Bhat, R., Xistris, G. and Sankar, T., 1982. "Dynamic behavior of a moving belt supported on elastic foundation". *Journal of Mechanical Design*, Vol. 104, No. 1, pp. 143–147.

Hvatov, A. and Sorokin, S., 2015. "Free vibrations of finite periodic structures in pass-and stop-bands of the counterpart infinite waveguides". *Journal of Sound and Vibration*, Vol. 347, pp. 200–217.

Lee, C.L. and Perkins, N.C., 1995. "Experimental investigation of isolated and simultaneous internal resonances in suspended cables". *Journal of vibration and acoustics*, Vol. 117, No. 4, pp. 385–391.

Lee, U., 2009. *Spectral element method in structural dynamics*. John Wiley & Sons.

Lee, U., Kim, J. and Oh, H., 2004. "Spectral analysis for the transverse vibration of an axially moving timoshenko beam". *Journal of Sound and Vibration*, Vol. 271, No. 3-5, pp. 685–703.

Lee, U. and Oh, H., 2005. "Dynamics of an axially moving viscoelastic beam subject to axial tension". *International Journal of Solids and Structures*, Vol. 42, No. 8, pp. 2381–2398.

Oh, H., Lee, U. and Park, D.H., 2004. "Dynamics of an axially moving bernoulli-euler beam: Spectral element modeling and analysis". *KSME international journal*, Vol. 18, No. 3, pp. 395–406.

Xia, C., Wu, Y. and Lu, Q., 2015. "Experimental study of the nonlinear characteristics of an axially moving string". *Journal of Vibration and Control*, Vol. 21, No. 16, pp. 3239–3253.

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