25th ABCM International Congress of Mechanical Engineering
October 20-25, 2019, Uberlândia, MG, Brazil

COB-2019-1745
COMPOSITE PANEL OPTIMIZATION USING LAMINATION PARAMETERS AND INVERSE DISTANCE WEIGHTING INTERPOLATION

Victor N. Capacia
Jose A. Hernandes
Solid Mechanics and Structures, ITA - Av. Heitor Villa Lobos, 1850
victorncapacia@hotmail.com; hernades@ita.br

Saullo G. P. Castro
Delft University of Technology, Department of Aerospace Structures and Materials, Kluyverweg 1, 2629 HS Delft, The Netherlands
S.G.P.Castro@tudelft.nl

Abstract. *Since the middle of last century, the use of composite structures has increased in the aerospace industry. The process of defining efficient structure laminate is still a challenge even with the processing power of modern computers. At the same time, the competitive aeronautical industry demands an efficient way for defining the ideal structure stiffness associated with specific requirements.*
Throughout the past decades, the scientific academy evolved its knowledge and strategy to design orthotropic laminates. The formulation and use of lamination parameters in optimizations has shown to be a good alternative to local minimum problems.
In this work, the use of lamination parameters is used to define the stiffness matrix of specially orthotropic laminates. Equally, an interpolation strategy is implemented as a manner to reduce the number of design variables within the process of an structural optimization.
Keywords: *Structural analysis, laminates, optimization*

1. INTRODUCTION

The application of composites in civilian or military aircraft followed the typical stages that every new technology goes through during its implementation. At the beginning, limited application on secondary structure minimized risk and improved understanding by collecting data from tests and experience. This limited usage was followed by wider applications, first in smaller aircraft, capitalizing on the experience gained earlier.

The structural optimization of composite material is one of the main themes of this growing technology. However, the optimization task of such complex material is no easy task. The most common parametrization when optimizing composite material is the use of lamination angles gradients in order to design the laminate. However, the problem becomes poorly conditioned, ending up in local optimum instead of the final solution of the problem. In the decade of 1960, Tsai and Pagano (1968) first developed an elegant way to express the stiffness transformation of an orthotropic material from one coordinate system to a generic one. By use of various trigonometric identities between sin and cos, the transformed reduced stiffness could be written as a function of some constants, called the invariant properties of an orthotropic lamina.

The lamination parameters were initially introduced by Hahn and Tsai (1980) and represent a non-dimensional through the thickness integration of the layer angles. For layup optimization problems, using lamination parameters, reduces significantly the number of design variables in comparison to using ply orientation angles and thickness, which may result in non-convex optimization problem related to periodic functions and discrete number of plies or constrained design space (HAMMER et al., 1997a). In the recent past, the academic community noticed its potential to be used as design variables in laminates optimization problems. Moreover, only eight lamination parameters are necessary to describe a general laminate, independently of its total thickness, and its domain is always within the range of [-1,1]. Fukunaga and Vanderplaats (1991) optimized the buckling load of a cylindrical shell making use of a symmetric and balanced laminate. Also considered the coupling effects of bend-twisting negligible, making the laminate especially orthotropic and reducing the necessary lamination parameters from eight to four. In their work, the optimization behaved well and converged always for the same optimum point.

Despite the fact of being a good alternative, the use of lamination parameters also has its difficulties. For example, not all combination of lamination parameters can be associated with a real laminate. To transform the orientations of a real laminate into lamination parameters is an easy task, however to start from lamination parameters and end up with the orientations of a laminate is a non-linear problem and it can have one, many or no solutions. There is a relation between the lamination parameters in order for then to represent a real laminate. This relation is called feasible region and many authors along the last years tried to develop a simple manner of representing this relation. Grenestedt and Gudmundson (1993) used a variational approach to implicitly determine the feasible region of orthotropic symmetric laminates. Furthermore, Grenestedt and Gudmundson (1993) derived explicit expressions between certain sets of the in-plane and out-of-plane lamination parameters and additionally proved that the feasible region was necessarily convex (Bloomfield et al., 2009).

In this work, the design variables will be interpolated using the inverse distance weighting (IDW) Method, as a manner of reducing the total number of design variables an interpolation approach is investigated in order to find a satisfactory balance between computational cost and weight reduction. The parametrization method is called inverse distance weighted interpolation and it can be applied in any type of geometry. The optimization is performed in a two-step approach as a manner to, first, maximize the loads multiplier of a linear buckling problem and, afterwards, minimize the wing total weight. The case study is performed for two different numbers and distributions of design variables. A commercial software called MSC NASTRAN is used in order to model and analyze the wing structure.

2. OPTIMIZATION FORMULATION

Lamination parameters were first introduced only as a simpler and elegant manner of expressing the stiffness transformation of a general laminate. However, its potential to be used as design variables was identified by the scientific academy and it has been used in many recent studies in the area.

2.1 Lamination parameters

Tsai and Pagano (1968), formulated an elegant way of representing laminate stiffness, in terms of orthotropic layer invariants and trigonometric functions:

$$\begin{aligned}
\overline{Q}_{11} &= U_1 + U_2 cos2\theta + U_3 cos4\theta \\
\overline{Q}_{12} &= U_4 - U_3 cos4\theta \\
\overline{Q}_{22} &= U_1 - U_2 cos2\theta + U_3 cos4\theta \\
\overline{Q}_{16} &= \frac{1}{2} U_2 sin2\theta + U_3 sin4\theta \\
\overline{Q}_{26} &= \frac{1}{2} U_2 sin2\theta - U_3 sin4\theta \\
\overline{Q}_{66} &= U_5 - U_3 cos4\theta
\end{aligned} \tag{1}$$

Where $\overline{Q}_{ij}$ is the reduced transformed stiffness matrix of the laminate and U_i are the invariants properties of an orthotropic lamina. The laminate invariants are the following:

$$\begin{aligned}
U_1 &= \frac{3Q_{11} + 3Q_{22} + 2Q_{12} + 4Q_{66}}{8} \\
U_2 &= \frac{Q_{11} - Q_{22}}{2} \\
U_3 &= \frac{Q_{11} + Q_{22} - 2Q_{12} - 4Q_{66}}{8} \\
U_4 &= \frac{Q_{11} + Q_{22} + 6Q_{12} - 4Q_{66}}{8} \\
U_5 &= \frac{Q_{11} + Q_{22} - 2Q_{12} + 4Q_{66}}{8}
\end{aligned} \tag{2}$$

Where, Q_{ij} are the reduced stiffnesses of the lamina.

In the integration through the thickness of a laminate with only one material type, the terms Ui for i=1,2...5, become constant and the following so called lamination parameters are defined:

$$V_1(_{A},_{B}\ ,_{D}\ ) = \int_{-\frac{1}{2}}^{\frac{1}{2}} cos(2\theta)[1, \overline{z}, \overline{z}^2] d\overline{z} \tag{3a}$$

$$V_2(_{A},_{B},_{D}) = \int_{-\frac{1}{2}}^{\frac{1}{2}} sin(2\theta)[1, \overline{z}, \overline{z}^2]d\overline{z} \tag{3b}$$

$$V_3(_{A},_{B},_{D}) = \int_{-\frac{1}{2}}^{\frac{1}{2}} cos(4\theta)[1, \overline{z}, \overline{z}^2]d\overline{z} \tag{3c}$$

$$V_4(_{A},_{B},_{D}) = \int_{-\frac{1}{2}}^{\frac{1}{2}} sin(4\theta)[1, \overline{z}, \overline{z}^2]d\overline{z} \tag{3d}$$

Where, $\overline{z}$ is the normalized through the thickness coordinate of the layers. Therefore, matrices A, B, D can be rewritten as

$$A = t(\Gamma_0 + \Gamma_1 V_{1A} + \Gamma_2 V_{2A} + \Gamma_3 V_{3A} + \Gamma_4 V_{4A}) \tag{4a}$$

$$B = \frac{t^2}{4}(\Gamma_1 V_{1B} + \Gamma_2 V_{2B} + \Gamma_3 V_{3B} + \Gamma_4 V_{4B}) \tag{4b}$$

$$D = \frac{t^3}{12}(\Gamma_0 + \Gamma_1 V_{1D} + \Gamma_2 V_{2D} + \Gamma_3 V_{3D} + \Gamma_4 V_{4D}) \tag{4c}$$

Where, t is the laminate thickness. The following matrices of invariants are used:

$$\Gamma_0 = \begin{bmatrix} U_1 & U_4 & 0 \\ U_4 & U_1 & 0 \\ 0 & 0 & U_5 \end{bmatrix} \tag{5a}$$

$$\Gamma_1 = \begin{bmatrix} U_2 & 0 & 0 \\ 0 & -U_2 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{5b}$$

$$\Gamma_2 = \begin{bmatrix} 0 & 0 & \frac{U_2}{2} \\ 0 & 0 & \frac{U_2}{2} \\ \frac{U_2}{2} & \frac{U_2}{2} & 0 \end{bmatrix} \tag{5c}$$

$$\Gamma_3 = \begin{bmatrix} U_3 & -U_3 & 0 \\ -U_3 & U_3 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{5d}$$

$$\Gamma_4 = \begin{bmatrix} 0 & 0 & U_3 \\ 0 & 0 & -U_3 \\ U_3 & -U_3 & 0 \end{bmatrix} \tag{5e}$$

Lamination parameters are interesting option of design variables for optimization, since a generic laminate with any number of layers can be fully described with only 12 lamination parameters. This number can be reduced to 8, in case of a symmetric laminate constituted of the same material. In this case, because of the symmetry of the stiffness matrix and thickness, all the bending-extension coupling (B matrix) is reduced to 0. Likewise, when defining a balanced laminate all the coupling between shear and extension cease to exist, eliminating terms A16 and A26. Thereafter, parameters V2A and V4A also become zero. Finally, for simplicity, the coupling terms of the bending-twisting, D16 and D26 are going to be neglected in the current work; then the symmetric and balanced laminate can be regarded as an especially orthotropic laminate (FUKUNAGA et al., 1991). Therefore, the laminate stiffness can be represented by only 4 lamination parameters.

Lamination parameters are defined to be nondimensional quantities that represent the stiffness of a laminate and, as trigonometric equations, are restricted in the interval between -1 and 1. However, not all combinations among lamination parameters results in physically viable laminates. The formulation of the relationship between lamination parameters has been developed by several authors in an incremental fashion to define a feasible region for special laminates. In the especially orthotropic laminated plates and shells that eliminate all coupling terms, the stiffness characteristics are governed by two in-plane and two out-of-plane lamination parameters (DIACONU et al., 2002). The first to define a fundamental relationship between two in-plane or two out-of-plane lamination parameters was Miki (1982) and Miki (1985). Yet, he did not described a relationship between in-plane and out-of-plane parameters. Later, Fukunaga and Sekine (1992) described a feasible region for a symmetric laminate with extension-shear coupling or bending-twisting coupling. This type of laminate is governed by four in-plane and four out-of-plane lamination parameters. However, a relationship between in-plane and out-of-plane parameters was not described.

The lamination parameters have been used successfully as design variables in the optimization problems for vibration by Fukunaga et al. (1994), buckling by Fukunaga et al. (1995) and topological design by Hammer et al. (1997) (DIACONU et al., 2002). Next, Grenestedt and Gudmundson (1993) used a variational approach to implicitly determine the feasible region of orthotropic symmetric laminates. Furthermore, Grenestedt and Gudmundson (1993) derived explicit expressions between certain sets of the in-plane and out-of-plane lamination parameters and additionally proved that the feasible region was necessarily convex (Bloomfield et al., 2009). For an especially orthotropic laminate, Grenestedt and Gudmundson (1993) formula for the coupling of in-plane region with out-of-plane region becomes:

$$\frac{1}{4}(V_{iA}+1)^3-1 \leq V_{iD} \leq \frac{1}{4}(V_{iA}-1)^3+1 \tag{6}$$

2.2 Inverse Distance Weighting

In the inverse distance weighting method, each sample point is pondered during interpolation according with its distance from the unknown point. The farther a sample point gets from the interpolated one, less influence it has over it.

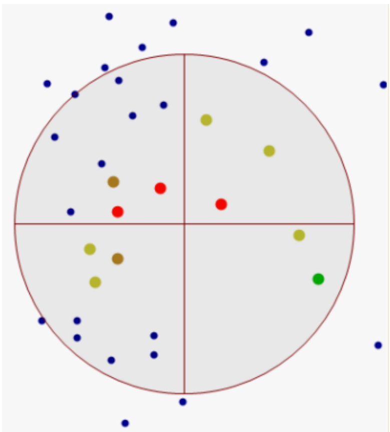

Figure 1. Influence zone

Figure 1 shows an example of an interpolated point in the center of the circle. In a scale from blue to red it is shown qualitatively the measured point influence over the interpolated one. A weighting coefficient is applied to sample points values in order to account for its distance and consequently its influence over the interpolated point. The closer the sample point is to the interpolated one, the greater the coefficient will be, and closer in value both points will be. As the coefficient decreases, the value of the unknown point deviates the value of the nearest observational point. The weighting function can be seen in Eq. (7).

$$w_i = \frac{1}{d(x,x_i)^p} \tag{7}$$

Where d is the distance between x (interpolated point) and xi (known point). At the same time, the inverse distance weighting interpolation method has some disadvantages. The quality of the interpolation result can decrease, if the distribution of sample data points is uneven. Besides that, maximum and minimum values in the interpolated surface can only occur at sample data points. This often results in small peaks around the sample data points and it is the main reason to strategically choose the sample points location. The interpolating function for a value u at a point x on samples xi = u(xi) for i=1,2,..n can be seen in Eq. (8).

$$u(x) = \frac{\sum_{i=1}^{n} w_i(x)u_i}{\sum_{i=1}^{n} w_i(x)} \tag{8}$$

In the case of having a big interpolating area, it is possible to restrict the area that is going to be included in the summation of Eq. (8). The shape of the neighborhood restricts how far and where to look for the measured values to be used in the prediction. Other neighborhood parameters restrict the locations that will be used within that shape. As the structure of the investigated wing presents no directional influence over the thickness and lamination parameters, all data points are going to be considered equally in all interpolations. In this work, the search neighborhood is going to be considered a circle with half wing span radius. It is also possible to divide the search neighborhood into sectors and consider only the maximum value of each sector. No sectioning restriction is going to be applied in the current optimization.

3. Wing structure

The conceptual aircraft used by Castro (2009) is called PRIME 900 and was a preliminary project of the 10th EMBRAER's specialization program. The current composite wing holds twenty five ribs and three spars. In Tab. 1 it is possible to visualize the main dimensions of the wing.

Table 1. Prime 900 Wing Dimensions

Variable	Value
Root chord	7.164 m
Kink chord	4.229 m
Tip chord	1.480 m
Dihedral Angle	5.0^o
Sweep Angle	38.5^o
Kink Span	3.684 m
Half Span	12.874 m
Front wing box height	0.727 m
Rear wing box height	0.650 m

The current wing holds three spars and twenty five ribs. No stringers were used in the current wing. Castro (2010) also optimized wing rib geometry and its position. In the current work his optimized geometry is used in order to optimize its composite structure stiffness. The wing can be visualized in Fig. 3. The maximum takeoff weight of this aircraft is 100,000lbs. Consequently 50,000 lbs will be supported by each wing. In the current model two load cases will be used, a 3G positive (667 kN) and a 1G negative (222 kN). Also, for simplification a trapezoidal load distribution will be used instead of an elliptic. The pressure applied is a result of the total load divided by the upper or lower skin area.

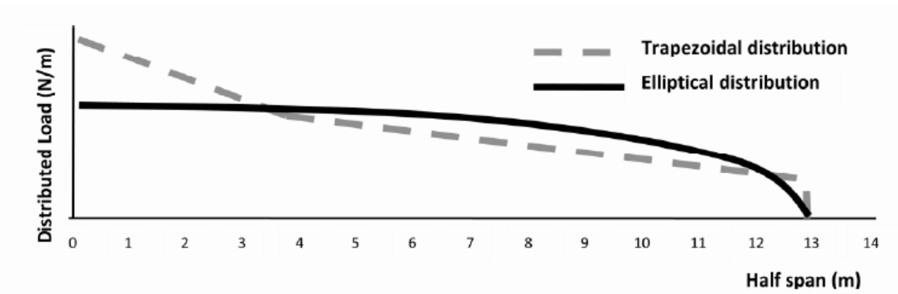

Figure 2. Representation of wing loading simplification, Lemos (2013)

Each color depicted in Fig. 3 corresponds to an elastic region of the finite element model, where thickness and material properties will be optimized. As discussed previously, the elastic region of the current wing needs four lamination parameters to be defined. In this work, using the idea of the IDW, a few property regions on upper and lower skin will be chosen to have its lamination parameters used as design variables. All the other regions will have their lamination parameters interpolated by the IDW equations.

Also, the thicknesses will be optimized independently for all elastic regions. Having in mind the need of maximizing the buckling efficiency at the same time as reducing the total weight of the wing, the optimization will be divided into two steps. Each step will have its own design variables and constraints as well as, its specific design objective.

4. Two-step approach

At the first optimization step the stiffness matrix of the wing constitutive material will be enhanced in order to maximize its buckling eigenvalue. Using lamination parameters as design variables in a few property regions it is possible to interpolate all the other regions stiffness matrices using a few design variables to optimize the entire wing structure. In

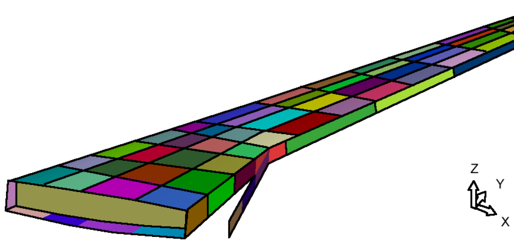

Figure 3. Property regions

this manner, the focus is perform a computationally cheap optimization step maximizing the wing buckling eigenvalue while constraining the laminate feasible region. Two different model configuration with two different number of lamination parameter design variables will be optimized in this first step. The main idea of testing two different models is to understand the influence of the number of design variables and its distribution through each skin on a big structure linear buckling load multiplier, using a reduced number of design variables.

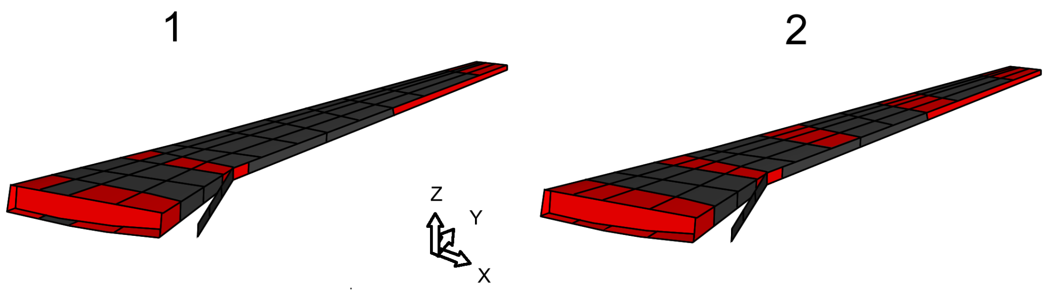

Figure 4. Lamination parameters design variables distribution

Spars 1 and 2 will also have its structure optimized in the same manner. A few property regions will be chosen to have its lamination parameters optimized and the remaining regions will have lamination parameters interpolated. Spar 3 will not be optimized. All the ribs structures will also have its lamination parameters optimized, however, having in mind its small dimensions, nothing will be interpolated. In Fig. 4 it is possible to visualize two models that will be verified. In the figure, regions with red color represent design variable region. While grey color illustrates interpolating regions. Case 1 represents only three independent property regions on each skin chordwise and also three span-wise, with a total of 9 independent regions on each skin. Three regions on spar 1 and spar 2 and one region per rib. Resulting in a sum of 49 regions. In case 2, four independent property regions are used chordwise on each skin. No modification on ribs and spars design regions were made from case 1. Also, case 2 adds two rolls of independent regions span wise, summing a total of 71 design regions on the entire wing, of which, 20 of them in each skin.

In a summarized way, the first step optimization problem statement holds Eq. (9) form.

$$
\begin{aligned}
& Max \quad \lambda \\
s.a:& \\
& 1 - V_{1D} + \frac{1}{4}(V_{1A}+1)^3 \geq 0 \\
& V_{1D} - \frac{1}{4}(V_{1A}+1)^3 + 1 \geq 0 \\
& 1 - V_{3D} + \frac{1}{4}(V_{3A}+1)^3 \geq 0 \\
& V_{3D} - \frac{1}{4}(V_{3A}+1)^3 + 1 \geq 0 \\
var.:& -1 \leq V_{1A} \leq +1 \\
& -1 \leq V_{3A} \leq +1 \\
& -1 \leq V_{1D} \leq +1 \\
& -1 \leq V_{3D} \leq +1
\end{aligned}
\tag{9}
$$

As a manner of translating the step 1 optimization result in the maximization of the buckling load multiplier, a second step optimization is going to be performed. The focus is to take advantage of the first optimization and reduce the wing weight through a sizing step. The design objective is the minimization of weight while restraining wing elements from buckling. Plate elements thicknesses are going to be used as design variables for this step. Differently than the previous approach no interpolation is going to be used and all property regions within the wing are going to receive its own design variable. The main objective at this stage is to investigate if the step 1 approach is sufficiently capable of defining feasible laminates for a wide structure while reducing the numbers of design variables to maintain a computationally cheap and efficient optimization. The final weight is also going to be a good manner of comparing the current work results with previous works performed by several authors with the same wing geometry and loads. A total of 143 thicknesses design variables are implemented at this stage. They are divided as:

- 52 design variables for the upper skin;
- 52 design variables for the lower skin;
- 25 design variables for the ribs (1 for each rib);
- 14 design variables for the spars (7 for each spar);

The problem statement of the second step optimization is as follows:

$$\begin{aligned} & Min \quad Weight \\ s.a: & \\ & \frac{1}{\lambda} < 1 \\ var.: & \quad 3mm \quad \leq t \leq \quad 20mm \end{aligned} \tag{10}$$

5. Runs and analysis

This section aims to expose all results obtained in a resumed manner and debate some conclusions that were taken from it. After this preview a best solution case is going to be chosen in order to deeply expose its own results in the next subsections. For both model configuration a total of 4 optimization runs were performed. For each design variable configuration, the optimization was investigated for the power function owning the values of p = 1 and p = 2. Table 2 holds the resume of results.

Table 2. Step 1 results summary

	Model 1		Model 2	
Power function	1	2	1	2
Design Cycles	36	42	26	38
$\lambda_{initial}$	2.16	2.16	2.16	2.16
λ_{final}	3.44	3.40	3.38	3.41

According to Tab. 2, it is easy to see that Model 1, using p value of 1 was the one with the best final value. These results are counter intuitive, having in mind that it is common to think that the more design variables a problem have, the best the result. However, what recent optimization publications shows, is that, typically, the optimization performance in finding the optimum solution increases until an optimum point of number of design variables, after which, it only decreases with the increase of design variables. Quadros (2017), with his Lagrange interpolation faced the same conclusion. More variables trends to add more non-linearities, which, in turn, can add more local minimum to the objective function when the problem is not convex by its nature. And even if the increase of number of design variable does not affects convexity, it can change the objective function curvature, also decreasing convergence capabilities of the solver. Furthermore, a problem with more non-linearities is hard to converge, either in gradient-based algorithm or not. The results also shows that, in this case, Model 1, with less design variables was capable of adapting better to the current buckling problem and, despite converging faster, Model 2 was not able to hold the best results. Besides that, it was noticed a small difference between the results for the interpolation power functions, p = 1 and p = 2. Further analysis on this fact is going to be drawn in the next subsection.

Focusing on performing a deeper investigation of the influence of the power function in the interpolation method, a Python algorithm was wrote in order to be able to see the interpolation function behavior. Figure 5 shows the interpolation function acting on the upper panel, upon the result for lamination parameter V1A for Model 1 with p = 1.

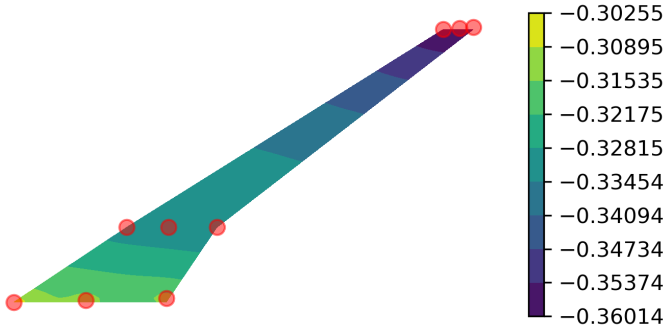

Figure 5. Contour of the interpolation function of V_{1A} results at upper panel of Model 1 with $p = 1$

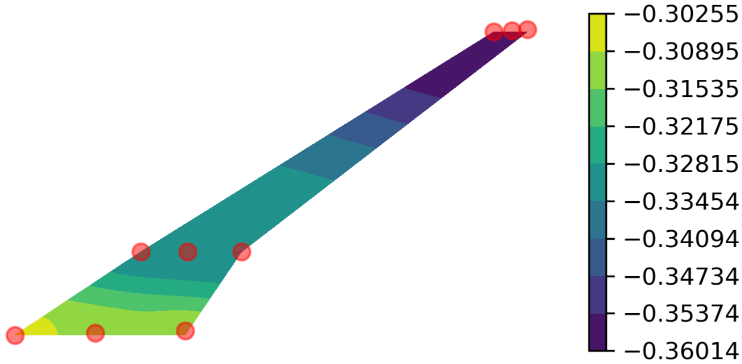

Figure 6. Contour of the interpolation function of V_{1A} results at upper panel of Model 1 with $p = 2$

Yet, in the current model, properties are interpolated based on the location of a center element. As a consequence, small variations of the power function do not have a big influence on the model interpolating results. As an example of that, Fig.7 shows how Fig. 5 interpolating function really appears in the model results.

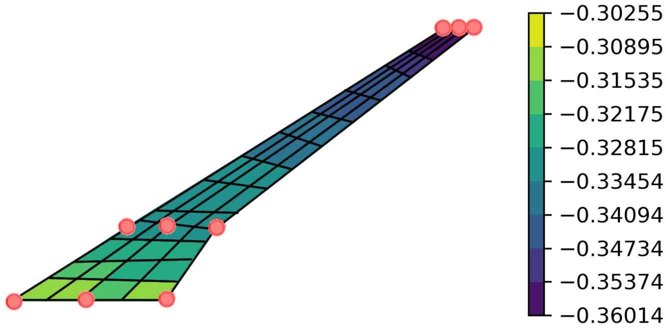

Figure 7. Contour of real V_{1A} results for the upper panel of Model 1 with $p = 1$

The above figure shows the exact result that Model 1 holds for p = 1. With the change of p = 1 to p = 2 would have a tiny influence on the final result for the same values. Yet, Model 1 with power function of 1 was able to find a load multiplier 1% bigger than when using a power function of 2. It is easy to conclude that an equilibrium between number and location of design variables has to be found, as well as, its combination with the power function value for the algorithm to find the optimum solution. At this stage, in Step 2, a sizing optimization is performed in order to quantify how much the found lamination parameters could reduce the total weight of the wing. Table 3 resumes the final weight obtained for each group of structure, as well as, the total wing weight for Model 1 with interpolating power function of 1.

Table 4 resumes the current optimization results. It is easy to conclude that Step 2 is much more computationally expensive than Step 1. With a total of 1127 design cycles with an elapsed time of 6 hours.

Table 5 resumes the final weight obtained for each group of structure, as well as, the total wing weight for Model 2 with interpolating power function of 1.

Is possible to perceive a reduction in both upper and lower panel weights. That can be explained by the increase in the total number of design variables in each panel from 9 to 20. However, the optimizer seemed not to found such good

Table 3. Components weight

Component	Weight [kg]
Upper skin	319.73
Lower skin	203.17
Ribs	133.94
Spar 1	23.97
Spar 2	17.11
Spar3	4.46
Total	702.38

Table 4. Optimization summary for Model 1, Step 2, with p=1

	Model 1
Power function	1
Design Cycles	1127
Initial weight	1801.1kg
Final weight	702.38

Table 5. Component weight

Component	**Weight [kg]**
Upper skin	315.10
Lower skin	198.34
Ribs	148.76
Spar 1	26.22
Spar 2	19.12
Spar3	4.46
Total	**712.0**

results for the ribs and spars. In comparison to configuration 1 both total spar and ribs weight increased.

6. Conclusion

In this work, an interpolation parametrization was presented for a linear buckling design optimization of a composite wing structure through lamination parameters theory. The stated approach focused on investigating the potential of reducing the number of design variables while maximizing the critical buckling load and, at the same time, reducing the total weight of a complex aeronautical structure. By the use of the proposed method, it was possible to increase the buckling load of a composite wing up to 159.26%, starting with a quasi-isotropic laminate. It was also possible to reduce its weight to almost 38.54% of its initial total weight. The analysis was extended to a second configuration case where more panel design variables were included. It was concluded that, contrary to what was expected, more design variables added more complexity to the optimization model. Typically, the optimization performance in finding the optimum solution increases until an optimum number of design variables, after which, it only decreases with the increase of design variables. More variables tends to add more non-linearities, which, in turn, can change the objective function curvature, or create local minima, thereby, decreasing convergence capabilities of the solver.

Also, for each panel the behavior of the interpolating function was evaluated. Two types of power function values were used on the inverse distance weighting interpolation method, p = 1 and p = 2. The interpolating surface became smoother with the increase of p value up to a point where local values generates steps in its surface. With higher values of p, the interpolated points get to be more influenced by local values and suffer less influence of the further points. However, the current model interpolates properties, not elements. The consequence is that a few elements gets the interpolated results based on the location of a center element. Hence, small variations of the power function does not have a big influence on the model interpolating results. Yet, the configuration holding a p value of 1 showed the best results. The strategy to use lamination parameters as design variables, had the purpose of obtaining an objective function with better behavior and more convex. Yet, local minima were identified such that different starting points led to distinct optimum solution. Having in mind the focus of understanding the behavior of the optimization algorithm, it was decided to start always from the same point in which V1A = V3A = V1D = V3D = 0. This combination represents a quasi-isotropic laminate and, at this manner, a fair comparison between the model could take place. All design variables respected the Grenested and Gudmunson (2013) feasible region imposed. Also, the fact that the feasible region equation results in a continuous domain, all the interpolated lamination parameters also necessarily respected the constraint.

7. REFERENCES

Bloomfield, M., Diaconu, C. and Weaver, P., 2009. “On feasible regions of lamination parameters for lay-up optimization of laminated composites”. *Proceedings of the Royal Society of London Series A*, Vol. 465, pp. 1123–1143.

Campen, J.M.J.F., Seresta, O., Abdalla, M.M. and Gurdal, Z., 2008. “General blending definitions for stacking sequence design of composite laminate structures”. *4th AIAA Multidisciplinary Design Optimization Specialist Conference*.

Castro, S.G.P., 2009. *Otimização eficiente de asa em material compósito via algoritmo genético*. Master’s thesis, ITA, São José dos Campos, SP - Brasil.

Diaconu, C.G., Sato, M. and Sekine, H., 2002. “Feasible region in general design space of lamination parameters for laminated composites”. *AIAA jounal*, Vol. 40, n.3, pp. 559–565.

Dutra, T.A., 2015. *Otimização de rigidez de compósitos laminados utilizando parametros de laminação*. Mestrado em ciência em engenharia aeronáutica e mecânica, Instituto Tecnológico da Aeronáutica, São José dos Campos.

Engelstad, S.P., Barker, D.K., Ellsworth, C.S. and Proctor, L., 2003. “Optimization strategies for the f/a 22 horizontal stabilator”. *44th AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conference*.

Foldager, J., Hansen, J.S. and Olhoff, N., 1998. “A general approach forcing convexity of ply angle optimization in composite laminates”. *Structural Optimization*, Vol. 16, pp. 201–211.

Fukunaga, H. and Sekine, H., 1992. “Stifness design method of symmetric laminates using lamination parameters”. *AIAA journal*, Vol. 30, n. 11, pp. 2791–2793.

Fukunaga, H., Sekine, H. and Sato, M., 1994. “Optimal design of symmetric laminated plates for fundamental frequency”. *Journal of sound and vibration*, Vol. 171, n. 2, pp. 219–229.

Fukunaga, H., Sekine, H., Sato, M. and Iino, A., 1995. “Buckling design of symmetrically laminated plates using lamination parameters”. *Computers & Structures*, Vol. 57, n. 4, pp. 643–649.

Fukunaga, H. and Vanderplaats, G.N., 1991. “Stiffness optimization of orthotropic laminated composites using lamination parameters”. *AIAA journal*, Vol. 29, pp. 641–646.

Grenestedt, J.L. and Gudmundson, P., 1993. “Layup optimization of composite material structures”. *Optimal design with advanced materials, Elsevier Science Amsterdam*, pp. 311–336.

Hahn, H.T. and Tsai, S.W., 1980. *Introduction to composite materials*. CRC press, Boca Raton, Flórida - USA.

Hammer, V.B., Bendsoe, M., Lipton, R. and Pedersen, P., 1997. “Parametrization in laminate design for optimal compliance”. *International Journal of Solids and Structures*, Vol. 34, pp. 415–434.

I. M. Daniel, O.I., 1994. *Engineering mechanics of composite materials*. Oxford University Press, Oxford, Reino Unido.

Kassapoglou, C., 2013. *Design and analysis of composite structures: with application to aerospace structures*. John Wiley & Sons, New Jersey, USA.

Kaw, A.K., 2005. *Mechanics of composite materials*. CRC press, Boca Raton, Flórida - USA.

Lemos, V.L., 2013. *A Two step Approach for Structural Optimization Of Blended Composite Wing Box*. Mestrado em ciência em engenharia aeronáutica e mecânica, Instituto Tecnológico da Aeronáutica, São José dos Campos.

Miki, M., 1985. “Design of laminated fibrous composite plates with required flexural stiffness”. *Recent Advances in Composites in the United States and Japan*, pp. 387–400.

Quadros, H.B., 2017. *Buckling Optimization of Variable Stiffness Composite Structures Using Lamination Parameters*. Mestrado em ciência em engenharia aeronáutica e mecânica, Instituto Tecnológico da Aeronáutica, São José dos Campos.

Setoodeh, S., Abdalla, M.M. and Gurdal, Z., 2006. “Design of variable stiffness laminates using lamination parameters”. *Composites Part B: Engineering*, Vol. 37, n. 4, pp. 301–309.

Shepard, D., 1968. “A two-dimensional interpolation function for irregularly-spaced data”. *Proceedings of the 1968 ACM National Conference*, pp. 517–524.

Tsai, S.W. and Pagano, N.J., 1968. “Invariant properties of composite materials”. *Composite Materials Workshop*, pp. 233–253.

8. RESPONSIBILITY NOTICE

The authors are the only responsible for the printed material included in this paper.