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ANALYSIS OF ISENTROPIC TURBINE EFFICIENCY, PRESSURE AND TEMPERATURE ON A SUBCRITICAL ORGANIC RANKINE CYCLE (ORC) WITH SUPERHEAT USING WET AND DRY FLUIDS

Aline Jauch Antonio
Douglas Ricardo Fumes
Santiago del Rio Oliveira
São Paulo State University (UNESP), School of Engineering, Bauru
aline_jauch@hotmail.com
douglas.fumes@hotmail.com
santiago.oliveira@unesp.br

Abstract. *Through mathematical models, based on thermodynamic laws, working fluids can be selected to achieve greater thermal cycle efficiencies in organic Rankine cycles (ORC). The main objective of this study is to develop a theoretical mathematical model to obtain values of the cycle thermal efficiency and cycle specifc net work from the combination between some parameters, such as: turbine inlet temperature, saturation temperature and isentropic turbine efficiency for different fluids, classified as wet and dry. After the simulation, performing several variations in the cycle parameters, it was observed that the highest values of the cycle thermal efficiency and the cycle specific net work occurred for the combination between 90% of the critical pressure and 120% of the critical temperature for all fluids tested. It was analyzed the level of influence that each parameter presents in the cycle thermal efficiency, and the influence of combination of the parameters. It was concluded that the saturation temperature presents the highest influence in the cycle thermal efficiency. However, when a combination of all parameters is performed, a highest level of influence can be obtained. Therefore, in this study was developed a mathematical model for the cycle thermal efficiency as a function of the combination of studied parameters.*

Keywords: *cycle thermal efficiency, organic Rankine cycle, isentropic turbine efficiency, dry fluids, wet fluids.*

1. INTRODUCTION

Tchanche *et al.* (2009) mentioned the organic Rankine cycle (ORC) as a promising way to convert heat into electrical energy and the researches around this subject has increased over the last years. The authors also describes the several heat sources used in ORC energy plants, such as biomass combustion, solar radiation, industrial waste heat and so many others.

According to Tartière and Astolfi (2017), the ORC is a steam power cycle, which the main characteristic is to convert thermal energy in electrical energy from low temperatures. According to *Lecompte et al.* (2015) this cycle uses organics fluids instead of water as working fluid and the heat conversion into work occurs similarly to the conventional cycle, where the main difference regards to the amount of heat required for working fluid evaporation, once the water requires greater energy levels for evaporation than the organic fluids. The parameters of influence for these processes are cited by Dai, Wang and Gao (2009), being the saturation temperature, saturation pressure, turbine inlet temperature, isentropic turbine efficiency and the working fluids properties. Wei *et al*. (2007) describes the subcritical Organic Rankine cycle with superheating as a thermodynamic system where the main function regards to the conversion of thermal energy into electrical energy from low temperatures. This conversion occurs in a system composed by a pump, an evaporator, condenser and turbine, as illustrated in Fig. 1.

The pump is responsible for forwarding the working fluid at liquid state to the evaporator. At this point, there is no heat exchange, that is, the process is reversible and isentropic. The evaporator performs the transformation of the liquid compressed into superheated steam, using pressures and temperatures below the critical point. Due to this reason, the cycle is classified as subcritical. Still in the evaporator, the superheated steam undergoes through another heat operation, increasing its temperature, with the objective of reach higher cycle efficiencies. After that, the steam passes through an isentropic turbine, thus fulfilling the main purpose of the cycle, which is to generate electricity. After the turbine expansion, the fluid-steam resultant from the process meets the condenser, which the function is to convert this mixture into liquid, finishing all the processes of the first cycle and starting another one.
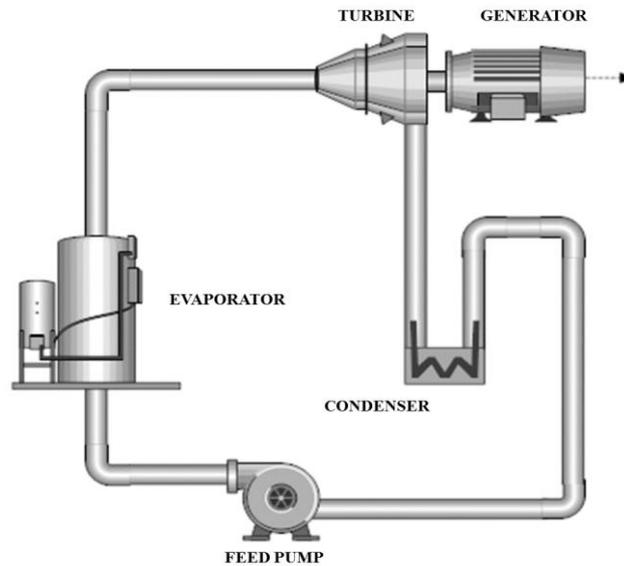
Figure 1. Architecture of a subcritical organic Rankine cycle. Adapted from: Lecompte *et al.* (2015).

This study shows the values of the cycle thermal efficiency and of the cycle specific net work, obtained from of variation and combination of the influence parameters. The influence parameters considered are the evaporation temperature, turbine inlet temperature and isentropic turbine. The working fluids considered for this study are classified as wet and dry. Through of simulations using the software EES (Engineering Equation Solver) it's possible to understand the influence level of each operating parameter on the cycle thermal performance and the influence level of parameters combinations. It was also developed a multiple regression equation that describes the behavior of the cycle thermal efficiency according to the influence parameters.

2. THERMODYNAMIC MODEL

Javanshir and Sarunac (2017) developed a theoretical mathematical model to calculate the thermal efficiency of organic Rankine cycles, based on the energy and entropy balances of the system, according to the first and seconds laws of thermodynamics. In the subcritical cycle with superheat, Fig. 2, both latent and sensible heat are considered in the input heat.
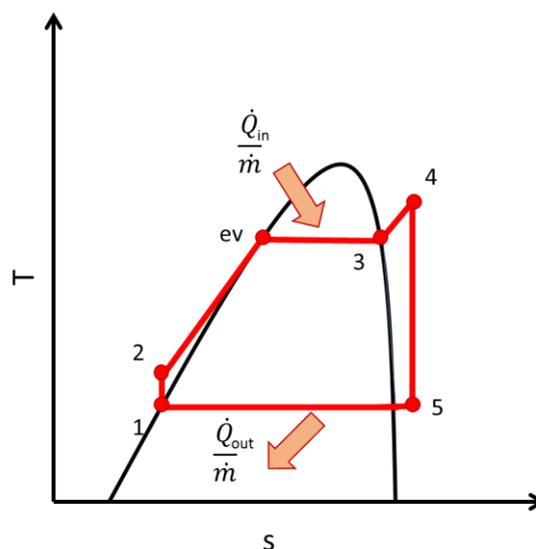

Figure 2. *T-s* diagram for a subcritical ORC with superheat. Adapted from: Javanshir and Sarunac (2017).

A mathematical expression for the cycle thermal efficiency was developed based on the model purposed by Javanshir and Sarunac (2017). To reach this model, a development of the thermodynamic first and second laws was performed, considering energy and entropy balances for the evaporator and for the condenser. The following assumptions for the thermodynamic model are: no heat loss due to friction in ducts and equipment, negligible kinetic and potential energies, steady state operation and no pressure drop in ducts and heat exchangers. Based on the assumptions above, from the first law of thermodynamics applied on the evaporator, we obtain:

$$\frac{\dot{Q}_{2,4}}{\dot{m}} = h_4 - h_2 \approx \frac{\dot{Q}_{1,4}}{\dot{m}} = h_4 - h_1 = (h_4 - h_g) + (h_f - h_1) \tag{1}$$

where q is the heat transfer per unit mass, h is the specific enthalpy, where the subscript f denotes the liquid saturation condition for the referred property and the subscript g denotes the steam saturation condition for the referred property and $\dot{m}$ is the working fluid mass flow rate. Using the latent heat, we can rewrite Eq. (1) as:

$$\frac{\dot{Q}_{in}}{\dot{m}} = c_{p1,ev}(T_{ev} - T_1) + h_{fg} + c_{p4,ev}(T_4 - T_{ev}) \tag{2}$$

where c_p is the specific heat at constant pressure and T is the temperature, where the subscript ev denotes the evaporation. From the second law of thermodynamics applied on the evaporator, we obtain:

$$s_4 - s_1 = c_{p1,ev} \ln\left(\frac{T_{ev}}{T_1}\right) + \frac{h_{fg}}{T_{ev}} + c_{p4,ev} \ln\left(\frac{T_4}{T_{ev}}\right) \tag{3}$$

where s is the specific entropy. From the first law of thermodynamics applied on the condenser, we obtain:

$$q_{1,5} = h_5 - h_1 \approx q_{1,4} = h_4 - h_1 \tag{4}$$

From the second law of thermodynamics applied on the condenser and using Eq. (3), we obtain:

$$\frac{\dot{Q}_{out}}{\dot{m}} = T_{eq,c}(s_4 - s_1) = T_{eq,c}\left[c_{p1,ev} \ln\left(\frac{T_{ev}}{T_1}\right) + \frac{h_{fg}}{T_{ev}} + c_{p4,ev} \ln\left(\frac{T_4}{T_{ev}}\right)\right] \tag{5}$$

where $T_{eq,c}$ is the average temperature between the turbine outlet and the condensation temperature. We can write an expression for the cycle net power as a function of the turbine isentropic efficiency in the following manner:

$$\dot{W}_{cycle} = \dot{m}\left(\frac{\dot{Q}_{in}}{\dot{m}} - \frac{\dot{Q}_{out}}{\dot{m}}\right)\eta_t \tag{6}$$

where $\dot{W}_{cycle}$ is the cycle net power and η_t is the turbine isentropic efficiency. Substituting Eqs. (2) and (5) in Eq. (6) we obtain:

$$\frac{\dot{W}_{net}}{\dot{m}} = \left\{c_{p1,ev}(T_{ev} - T_1) + h_{fg} + c_{p4,ev}(T_4 - T_{ev}) - T_{eq,c}\left[c_{p1,ev} \ln\left(\frac{T_{ev}}{T_1}\right) + \frac{h_{fg}}{T_{ev}} + c_{p4,ev} \ln\left(\frac{T_4}{T_{ev}}\right)\right]\right\}\eta_t \tag{7}$$

The cycle thermal efficiency is the ratio between the cycle net power and the cycle input heat rate, that is:

$$\eta = \frac{\dot{W}_{cycle}}{\dot{Q}_{in}} \tag{8}$$

Substituting Eqs. (2) and (7) in Eq. (8) and rearranging we obtain:

$$\eta = \eta_t \frac{\dfrac{c_{p1,ev}(T_{ev}-T_1)}{h_{fg}}+1+\dfrac{c_{p4,ev}(T_4-T_{ev})}{h_{fg}} - T_{eq,c}\left[\dfrac{c_{p1,ev}}{h_{fg}}\ln\left(\dfrac{T_{ev}}{T_1}\right)+\dfrac{1}{T_{ev}}+\dfrac{c_{p4,ev}}{h_{fg}}\ln\left(\dfrac{T_4}{T_{ev}}\right)\right]}{\dfrac{c_{p1,ev}(T_{ev}-T_1)}{h_{fg}}+1+\dfrac{c_{p4,ev}(T_4-T_{ev})}{h_{fg}}} \tag{9}$$

Wang *et al.* (2013) proposed to express the cycle thermal efficiency according to a variable that denotes the ratio between the sensible heat and the latent heat. This dimensionless parameter is called Jacob Number (Ja), that is:

$$\text{Ja} = \frac{\text{sensible heat}}{\text{latent heat}} = \frac{c_{p1,ev}(T_{ev}-T_1)}{h_{fg}} \tag{10}$$

For the superheated cycle, Javanshir and Sarunac (2017) proposed a similar variable called superheat Jacob number (Ja$_s$). In this case, the authors considered the sensible heat during the superheat process instead of the sensible heat between the beginning of the heating process and the evaporation point, that is:

$$\text{Ja}_s = \frac{\text{sensible heat (superheat)}}{\text{latent heat}} = \frac{c_{p4,ev}(T_4-T_{ev})}{h_{fg}} \tag{11}$$

For convenience, we can define T_{EC}, a ratio between the evaporation and condensation temperatures of the working fluid, that is:

$$T_{EC} = \frac{T_{evaporation}}{T_{condensation}} = \frac{T_{ev}}{T_1} \tag{12}$$

Using Eqs. (10-12), we can rewrite Eq. (9) in the following manner:

$$\eta = \eta_t \frac{\left\{\text{Ja}+1+\text{Ja}_s - \dfrac{T_{eq,c}}{T_1}\left[\dfrac{\text{Ja}}{T_{EC}-1}\ln(T_{EC})+\dfrac{1}{T_{EC}}+\left(\dfrac{\text{Ja}_s\ln\left(\dfrac{T_4}{T_{EC}T_1}\right)}{\dfrac{T_4}{T_1}T_{EC}}\right)\right]\right\}}{\text{Ja}+1+\text{Ja}_s} \tag{13}$$

3. RESULTS AND DISCUSSION

Javanshir and Sarunac (2017) proposed the implementation of two parameters for the results presentation, called dimensionless pressure P_r and dimensionless temperature (T_r). These two parameters can be easily obtained through the ratio between maximum value and the critical value, that is:

$$P_r = \frac{P_{max}}{P_{cr}} \tag{14}$$

$$T_r = \frac{T_{max}}{T_{cr}} \tag{15}$$

In this study, a range of P_r and T_r was predetermined for the calculations of the maximum values (temperature and pressure) for each fluid evaluated. For P_r a full range, from 0.1 to 0.9 was considered for calculations (it means a range from 10% to 90% of the critical pressure, for each fluid). The T_r values used to perform the analysis are located between 0.8 and 1.2 (80% to 120% of the critical temperature, for each fluid). Dry fluids (isobutane, R245FA and R236FA) and wet fluids (ammonia, R134A and R152A) were analyzed. Values of the critical temperatures and critical pressures were obtained through software Engineering Equation Solver (EES) and can be seen in Tab. 1:

Table 1. Values critical temperatures and critical pressure of each fluids.

	Fluids	Critical temperature (ºC)	Critical pressure (kPa)
Wet	Ammonia	132.30	11333
	R134A	101.00	4059
	R152A	113.30	4520
Dry	Isobutane	134.70	3640
	R236FA	124.90	3200
	R245FA	154.00	3651

Through Eq. (13), the behavior of cycle thermal efficiency and cycle specific network were calculated, considering the variation of the established parameters. The values variation during the simulations was performed for the evaporation temperature (dependent on the P_r) and turbine temperature (dependent on the T_r) for every fluid considered in this paper. The condenser temperature was set at 15 °C, according to the reference paper developed by Javanshir and Sarunac (2017). The cycle thermal efficiency and specific cycle network were analyzed according to P_r (% of critical pressure) and T_r (% of critical temperature) variations. These results can be seen in Figs. 3, 4, 5 and 6:
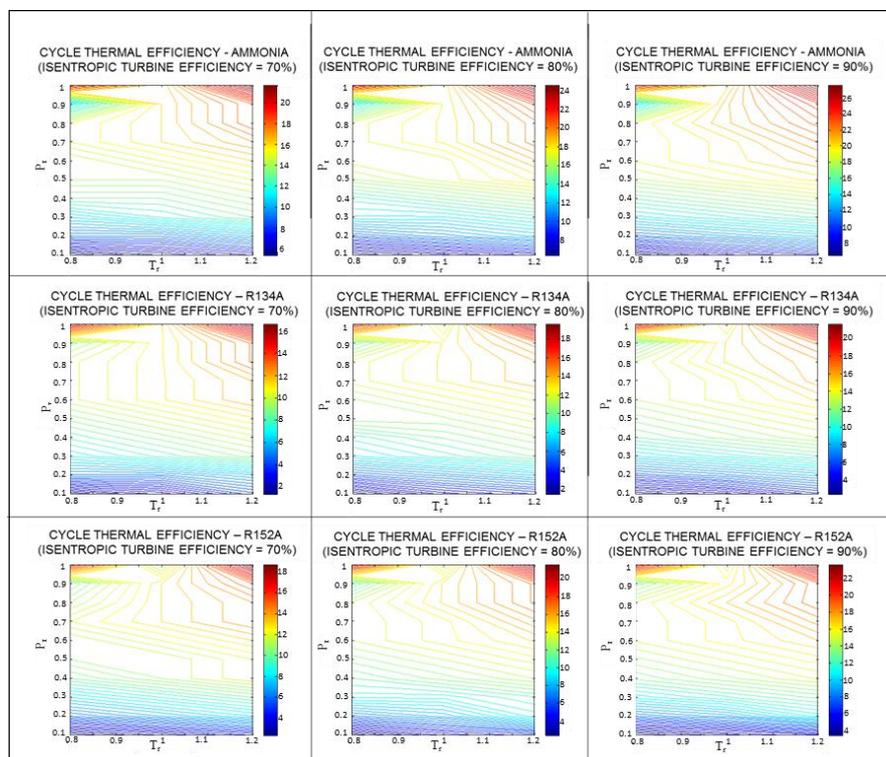

Figure 3. Cycle thermal efficiency for wet fluids.

Among the wet fluids, ammonia clearly presented the highest cycle thermal efficiency and specific cycle net work values. For the dry fluids, the R245FA presented the best performance regard to the cycle thermal efficiency, however, the isobutane presented the highest specific cycle net work value. All fluids presented its respectively highest values of cycle thermal efficiency and cycle specific net work for a combination between 90% of critical pressure and 120% of critical temperature (turbine inlet temperature). All the highest cycle thermal efficiency and cycle specific net work values were obtained considering an isentropic turbine efficiency equal to 90%, for all the fluids considered in this work. The resume of the results can be seen in Tab.2:
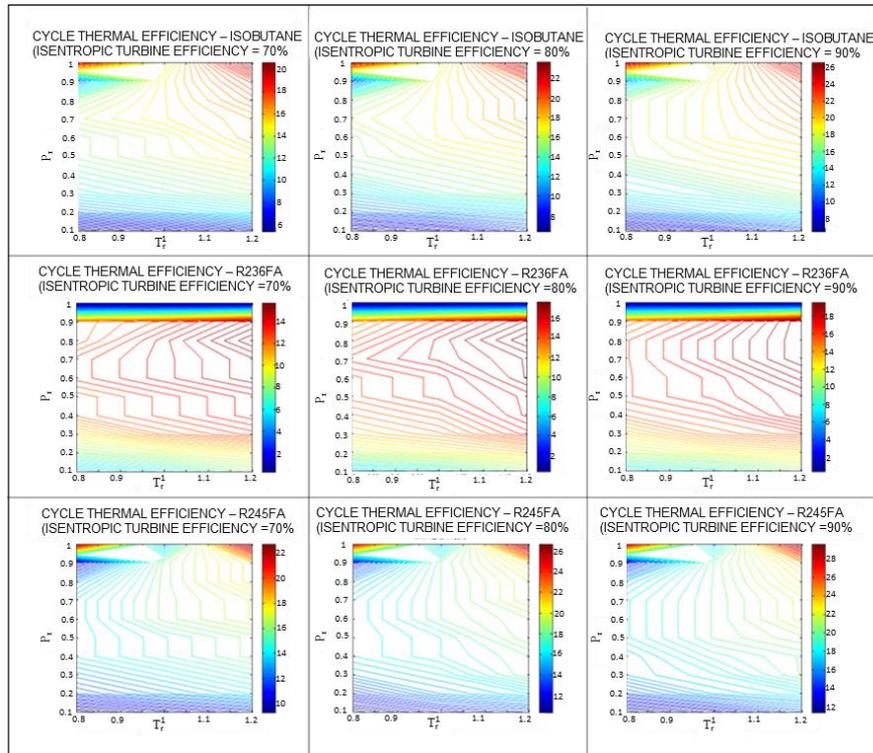
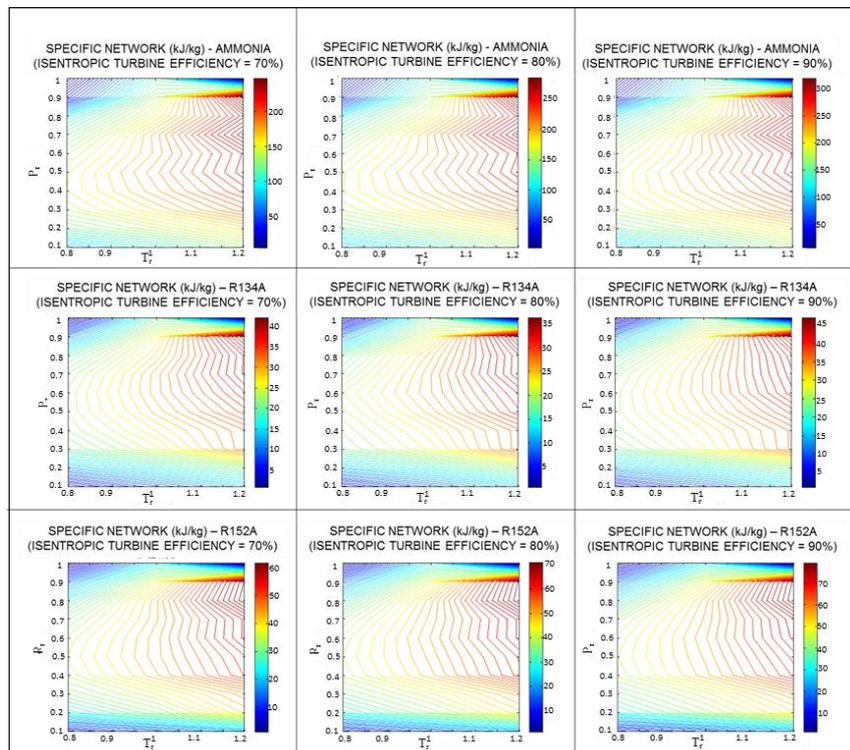Figure 4. Cycle thermal efficiency for dry fluids.

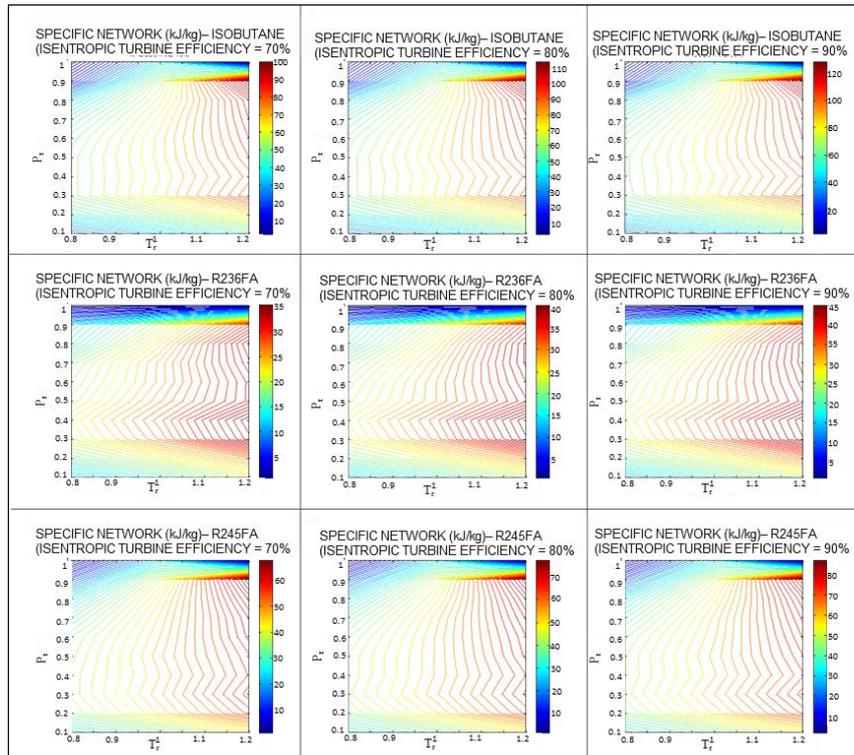Figure 5. Cycle specific net work for wet fluids.

Figure 6. Cycle specific net work for wet fluids.

Table 2. Cycle thermal efficiency and specific cycle net work for wet and dry fluids.

	Fluids	Cycle thermal efficiency (%)	Specific cycle net work (kJ/kg)
Wet	Ammonia	23.71	325.80
	R134A	16.97	47.79
	R152A	18.99	80.28
Dry	Isobutane	22.71	130.95
	R236FA	19.54	38.79
	R245FA	23.69	88.65

In order to obtain the influence level of each operating parameter on the cycle thermal performance, a statistical analysis was performed through the using of the Pearson correlation coefficient, denominated R. Thus, how much closer to unity is the value of the Pearson coefficient, the greater is the variable influence level on the cycle thermal efficiency result. The operating parameters of influence considered in this paper was T_{ev} (evaporation temperature), T_4 turbine inlet temperature) and η_t (isentropic turbine efficiency). According to results showed in Tab. 3, it was concluded that the evaporation temperature presents the highest level of influence on the cycle thermal efficiency, for both types of fluids (wet and dry), with an average correlation of 0.86 for wet fluids and 0.70 for dry fluids.

Table 3. Pearson correlation coefficients (R) for the cycle thermal efficiency and operating parameters.

Wet Fluids	R - Between T_{ev} and η	R - Between T_4 and η	R - Between η_t and η	Dry fuids	R - Between T_{ev} and η	R - Between T_4 and η	R - Between η_t and η
Ammonia	0.83	0.27	0.36	Isobutane	0.75	0.38	0.37
R134A	0.89	0.27	0.28	R236FA	0.79	0.34	0.42
R152A	0.87	0.28	0.31	R245FA	0.56	0.50	0.53
Average value	0.86	0.27	0.32	Average value	0.70	0.41	0.44

Through the Pearson correlation coefficients squared, the determination coefficients, denominated R^2 can be obtained. These determination coefficients correspond to the influence percentages that each parameter performs over the results of cycle thermal efficiency. This results can be seen in Tab. 4. For wet fluids, the evaporation temperature performs an

influence of 75% over the cycle thermal efficiency. The same parameter performs a lower influence (50%) over the cycle thermal efficiency for dry fluids. However, for the both type of fluids the evaporation temperature is the parameter with highest percentage of influence on the cycle thermal efficiency.

Table 4. Determination coefficients for the cycle thermal efficiency and operating parameters.

Wet Fluids	R² - Between T_{ev} and η	R² - Between T_4 and η	R² - Between η_t and η	Dry fuids	R² - Between T_{ev} and η	R² - Between T_4 and η	R² - Between η_t and η
Ammonia	69%	7%	13%	Isobutane	57%	14%	14%
R134A	79%	7%	8%	R236FA	62%	12%	18%
R152A	76%	8%	9%	R245FA	32%	25%	28%
Average Value	75%	7%	10%	Average Value	50%	17%	20%

The parameter of highest influence on cycle thermal efficiency is the evaporation temperature. However, the other parameters also performs certain influence over the cycle, in a lower scale. Thus, a correlation and determination coefficients were calculated between the cycle thermal efficiency and a combination of every influence parameter (evaporation temperature, turbine inlet temperature and isentropic turbine efficiency), as showed in Tab. 5:

Table 5. Determination (R²) and correlation (R) coefficients for the cycle thermal efficiency according to the suggested parameters.

Wet fluids	R - Between η, T_{ev}, T_4 and η_t	R² - Between η, T_{ev}, T_4 and η_t	Dry fluids	R - Between η, T_{ev}, T_4 and η_t	R² - Between η, T_{ev}, T_4 and η_t
Ammonia	0.95	90%	Isobutane	0.92	85%
R134A	0.97	94%	R236FA	0.95	90%
R152A	0.96	92%	R245FA	0.92	85%
Total	0.97	94%	Total	0.92	85%

Table 5 shows the influence level of the variables combination over the cycle thermal efficiency. Gathered, all the considered parameters performs an influence of 94% over the cycle thermal efficiency, considering wet fluids and 85% for the dry ones. Lastly, it is possible to conclude that the combination of the suggested parameters promotes a greater influence over the cycle thermal efficiency results than individually. Additionally, it was also developed a multiple linear regression equation for both types of fluid, in order to describe the cycle thermal efficiency behavior according to the influence parameters, that is, if we apply the parameters values in the regression equations, we will be able to obtain the cycle thermal efficiency results corresponding. The latter expressions presented a standard error of 1.14% for wet fluids and 1.44% for dry fluids. Additionally, it was developed a regression single equation to obtain the cycle thermal efficiency for both type of fluids, with a standard error of 1.49%. An expression for the cycle thermal efficiency for wet fluids selection can be written as:

$$\eta = -0.154 + 0.0012T_{ev} + 0.0006T_4 + 0.153\eta_t \tag{16}$$

Besides that, an expression for the cycle thermal efficiency for dry fluids selection can be written as:

$$\eta = -0.146 + 0.00076T_{ev} + 0.00053T_4 + 0.185\eta_t \tag{17}$$

Finally, a general expression for the cycle thermal efficiency for fluids selection can be written as:

$$\eta = -0.145 + 0.00094T_{ev} + 0.00051T_4 + 0.169\eta_t$$

$$\tag{18}$$

The expressions above are limited to the fluids considered this paper.

4. CONCLUSIONS

It can be concluded that the evaporation temperature is the parameter that performs the highest influence level on the cycle thermal efficiency. However, another important influence parameter for the cycle thermal performance is the fluid critical temperature. This effect assured to ammonia and R245fa the highest cycle thermal efficiencies, among its respectively categories wet and dry. The evaporation temperature express the highest influence level over the cycle thermal efficiency. This parameter denotes 75% of influence over the cycle performance for wet fluids and 50% for dry fluids. Considering the evaporation temperature influence level, it is also possible to conclude that there are others important parameters that can perform an intermediary influence over the cycle thermal efficiency. Thus, a combination of the influence parameters (evaporation temperature, turbine inlet temperature and isentropic turbine efficiency) were performed in order to verify the influence of the gathered on the cycle thermal efficiency. The gathered parameters resulted in an influence level of 94% over the cycle thermal efficiency for wet fluids and 85% for the dry ones. This result also allow us to understand that 19% (for wet fluids) and 35% (for dry fluids) of the cycle thermal efficiency can be explained according to the combination between the turbine inlet temperature and isentropic turbine efficiency influence. Through the statistical study, it was possible to develop a multiple linear regression equation, which can be used to predict the cycle thermal efficiency according to predetermined values of operating influence parameters.

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6. RESPONSIBILITY NOTICE

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