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STUDY OF TURBULENT FORCED CONVECTION IN THERMALLY DEVELOPING

Dhiego Luiz de Andrade Veloso
Carlos Antônio Cabral dos Santos
Instituto Federal da Paraíba - IFPB, Jaguaribe - João Pessoa - PB - Brasil - CEP: 58015-430
Universidade Federal da Paraíba - UFPB, Cidade Universitária - João Pessoa - PB - Brasil - CEP: 58051-900
dhiego.veloso@ifpb.edu.br
carloscabralsantos@yahoo.com.br

Pedro Granville Gonçalves
Márcio Andrade Rocha
Universidade Federal da Paraíba - UFPB, Cidade Universitária - João Pessoa - PB - Brasil - CEP: 58051-900
pedrogranville@gmail.com
rochandrademarcio@hotmail.com

Abstract. *This work describes a solution for heat transfer by forced convection in the thermal input region of a rectangular or circular duct, for the turbulent flow dynamically developed and thermally developing of Newtonian fluids. The energy equation is solved using the Generalized Integral Transformation Technique (GITT). The temperature field and the local Nusselt number are evaluated for several values of the Reynolds and Prandtl numbers, where the length of thermal development was also investigated. The results are presented in the form of tables and graphs, comparing with existing literature values observing a good agreement between them.*

Keywords: *forced convection, turbulent flow, GITT, temperature field, Nusselt.*

1. INTRODUCTION

The fluids mechanics and heat transfer always aroused curiosities between physicists and mathematicians, due to its variety of applications and complexity at the same time. Presently, with the great technological advance, an in-depth knowledge about the real processes of heat transference becomes essential, because it allows a better dimensioning of the thermal equipment, a better choice of materials suitable for pipes, that prevent corrosion and extend the useful life of the equipment, besides optimizing the heat transfer in processes, allowing to increase the efficiency of these equipments.

The turbulent forced convection in the thermal entrance region of circular and rectangular ducts has been extensively studied, considering several models of turbulence and several boundary conditions. In the works of (Sleicher *et* al., 1970; Notter and Sleicher, 1971; Notter and Sleicher, 1972; Shibani and Özisik, 1977; Özisik *et* al., 1989; Santos *et* al., 1995; Brown *et* al., 1997; Quaresma *et* al., 2001) is possible to make a comprehensive review of the turbulent forced convection in thermal developing and hydrodynamically developed, object of study of the present work.

There is no univocal form, or a fundamental theory, for the treatment of turbulence, however, there are empirical or semi-empirical relations that allow this study through of turbulence models. The calculations of practical interest parameters, such as friction factors and heat transfer coefficients, are totally dependent on the adopted turbulence model (Santos *et al.,* 2001). Each model of turbulence presents a margin of precision in relation to the obtained results. The present work numerically solves the turbulent flow of Newtonian fluids, using as reference the turbulence model based on the works of (Prandtl, 1910 and Taylor, 1916).

2. MATHEMATICAL MODELING

The following simplifying assumptions are considered in the problem analysis:

- Steady forced convection in thermally developing, hydrodynamically developed ;
- Viscous dissipation, free convection and axial conduction effects are neglected ;
- Physical properties are taken as constant ;
- The duct wall is subjected to a uniform temperature (T_w) ;

- The fluid enters the duct with a constant temperature (T_i).

The mathematical formulation for this forced convection problem is dimensionless form is written as:

Energy equation

$$U(R)\frac{\partial \Theta(X,R)}{\partial X} = \frac{1}{R^m}\frac{\partial}{\partial R}\left[R^m E_h(R)\frac{\partial \Theta(X,R)}{\partial R}\right] \; ; \; X>0 \; and \; 0<R<1 \quad (1)$$

where, the constant *m* is related to the geometry of the duct. If *m=0* the duct is rectangular (parallel-plates channel), if *m=1* the duct is circular (circular tube).

Boundary Conditions

$$\frac{\partial \Theta(X,R)}{\partial R} = 0 \; ; \; X>0 \; and \; R=0 \quad (2)$$

$$\Theta(X,R) = 0 ; \; X>0 \; and \; R=1 \quad (3)$$

Inlet condition

$$\Theta(X,R) = 1 ; \; X=0 \; and \; 0 \le R \le 1 \quad (4)$$

For the analysis of the problem were defined the following dimensionless parameters:

$$X = \frac{2^{(4-2m)}\left(x/D_h\right)}{C.\mathrm{Re}.\mathrm{Pr}} \; ; \; R = \frac{r}{r_0} \; ; \; U(R) = \frac{u(r)}{u_{max}} = \frac{u(r)}{Cu_m} \; ; \; C = \frac{u_{max}}{u_m} \; ; \; \mathrm{Pr} = \frac{\nu}{\alpha} \quad (5a\text{-}e)$$

$$\mathrm{Pr}_t = \frac{\varepsilon_m}{\varepsilon_h} \; ; \; \mathrm{Re} = \frac{D_h.u_m}{\nu} \; ; \; \Theta(X,R) = \frac{T(x,r)-T_w}{T_i - T_w} \; ; \; E_h(R) = 1 + \frac{\varepsilon_h}{\alpha} = 1 + \frac{\mathrm{Pr}}{\mathrm{Pr}_t}\frac{\varepsilon_m}{\nu} \quad (5f\text{-}i)$$

where ν represents the kinematic viscosity, α is the thermal diffusivity of the fluid, r_0 is the characteristic length, $D_h = 2^{(2-m)}.r_0$ is the hydraulic diameter and *Pr*, *Re* and Pr_t are, respectively, the numbers of Prandtl, Reynolds and turbulent Prandtl.

The turbulence model is based on analytical expressions proposed for the universal velocity profile - Law of the Wall (Kestin and Richardson,1963; Kakaç *et. al*, 2014 and Santos *et al.*,2001). The turbulence model is the fully-developed two-layer model for velocity distribution together with two-layer model for the momentum eddy diffusivity, based on the works of Prandtl, 1910 and Taylor, 1916 (Schlichting, 1960).

The two-layer turbulent velocity distribution is taken as:

$$\mathrm{u}^+ = \mathrm{y}^+ \; ; \; for \;\; 0 \le y^+ \le 11.5 \; , \textit{laminar sublayer} \quad (6)$$

$$\mathrm{u}^+ = 5.5 + 2.5\ln\left(\mathrm{y}^+\right) \; ; \; for \;\; y^+ > 11.5 \; , \textit{turbulent core} \quad (7)$$

The two-layer model for the momentum eddy diffusivity is taken as:

$$E_h(R) = 1 + \frac{\mathrm{Pr}}{\mathrm{Pr}_t}\left(\frac{\varepsilon_m}{\nu}\right) = 1 \; ; \; for \;\; 0 \le y^+ \le 11.5 \quad (8)$$

$$E_h(R) = 1 + \frac{\mathrm{Pr}}{\mathrm{Pr}_t}\left(\frac{\varepsilon_m}{\nu}\right) = 1 + \frac{\mathrm{Pr}}{\mathrm{Pr}_t}\left(0.4y^+ - 1\right) \; ; \; for \;\; y^+ > 11.5 \quad (9)$$

Several dimensionless groups were included in equations (6-9), where are defined in the following form:

$$y^{+}=(1-R)R^{+}\ ;\qquad R^{+}=\frac{\mathrm{Re}}{2^{(2-m)}}\sqrt{\frac{f}{8}}\ ;\qquad u^{+}=\frac{u(r)}{u_m\sqrt{\frac{f}{8}}}\ ;\qquad f=\frac{4\tau_w}{\frac{\rho u_m{}^2}{2}} \tag{10a-d}$$

where f is the friction factor. In the present work, empirical correlations are used for the friction factor, based on the works of (Filonenko, 1954; Dean, 1978; Bhatti and Shah, 1987).

Filonenko correlation for circular tube or parallel-plates channel:

$$\frac{f}{4}=\left[1.58\ln(\mathrm{Re})-3.28\right]^{-2}\ ;\ 1\times10^{4}\le\mathrm{Re}\le1\times10^{7} \tag{11}$$

Dean correlation for parallel-plates channel:

$$\frac{f}{4}=0.073\left(\frac{\mathrm{Re}}{2}\right)^{-0.25}\ ;\ 1.2\times10^{4}\le\mathrm{Re}\le1.2\times10^{6} \tag{12}$$

Bhatti and Shah correlation for circular tube or parallel-plates channel:

$$\frac{f}{4}=0.00128+\frac{0.1143}{\mathrm{Re}^{1/3.2154}}\ ;\ 4\times10^{3}\le\mathrm{Re}\le1\times10^{7} \tag{13}$$

3. APPLICATION OF THE GENERALIZED INTEGRAL TRANSFORMING TECHNIQUE

3.1 Auxiliary problem of eigenvalue

The auxiliary problem for determining the temperature field is taken as:

$$\frac{1}{R^m}\frac{\partial}{\partial R}\left(R^m E_h(R)\frac{\partial\Psi_i(\mu_i,R)}{\partial R}\right)+\mu_i^2U(R)\Psi_i(\mu_i,R)=0,\qquad 0<R<1 \tag{14}$$

$$\frac{\partial\Psi_i(\mu_i,R)}{\partial R}=0\ ;\ R=0 \text{ and } \mu_i>0 \tag{15}$$

$$\Psi_i(\mu_i,R)=0\ ;\quad R=1 \text{ and } \mu_i>0 \tag{16}$$

3.2 Integral transformation of the temperature field

The pair transformed integral and inverse, defined for this problem is given by:

$$\overline{\Theta}_i(X)=\frac{1}{N_i^{1/2}}\int_0^1 R^mU(R)\Psi_i(\mu_i,R)\Theta(X,R)dR \qquad \textit{Transform} \tag{17}$$

$$\Theta(X,R)=\sum_{i=1}^{\infty}\frac{\Psi_i(\mu_i,R)\overline{\Theta}_i(X)}{N_i^{1/2}} \qquad \textit{Inverse} \tag{18}$$

Applying integral operators in equation (1), with the aid of the auxiliary problem and the transformed-inverse pair, it is possible to transform this partial differential equation into a system of ordinary differential equations with solution given by:

$$\overline{\Theta}_i(X)=\overline{\Theta}_i(0).e^{-\mu_i^2X} \tag{19}$$

With transformed inlet condition given by:

$$\overline{\Theta}_i(0) = \frac{1}{N_i^{1/2}} \int_0^1 R^m U(R).\Psi_i(\mu_i, R) dR = \bar{f}_i \tag{20}$$

3. 3 Temperature field solution

Using the inverse formula it is possible find the general solution of the temperature field for the proposed physical problem. Through from this solution it is possible to calculate the average temperature and the number of local Nusselt through the expressions:

$$\Theta(X)_{average} = \frac{\int_0^1 R^m U(R) \Theta(X, R) dR}{\int_0^1 R^m U(R) dR} \tag{21}$$

$$Nu(X) = -\frac{2.(2-m)}{\Theta(X)_{average} - \Theta(X,1)} \left.\frac{d\Theta(X,R)}{dR}\right|_{R=1} \tag{22}$$

For the evaluation of the asymptotic Nusselt number and validation of the results obtained in the present work, it is used the expression obtained by (Gnielinski, 1976) given by:

$$Nu_\infty = \frac{\left(f/2\right)(\mathrm{Re}-1000)\mathrm{Pr}}{1+12.7\left(f/2\right)^{\frac{1}{2}}\left(\mathrm{Pr}^{\frac{2}{3}}-1\right)}, \quad for \quad \begin{matrix} 2.3\times10^3 \le \mathrm{Re} \le 5\times10^6 \\ 0.5 \le \mathrm{Pr} \le 2000 \end{matrix} \tag{23}$$

Also it is used the expression obtained by (Prandtl, 1910 and Taylor,1916) given by:

$$Nu_\infty = \frac{\left(f/2\right)\mathrm{Re}\,\mathrm{Pr}}{1+5\left(f/2\right)^{\frac{1}{2}}(\mathrm{Pr}-1)}, \quad for \quad \begin{matrix} 5\times10^3 \le \mathrm{Re} \le 5\times10^6 \\ \mathrm{Pr} \le 10 \end{matrix} \tag{24}$$

4. RESULTS

For the purposes of *benchmarking* the results of the present study were confronted with results found in the specialized literature, particularly in Gnielinski (1976) and Taylor(1916), showing the robustness and effectiveness of GITT in solving of proposed physical problem. In this paper is analyzed the situation of a flow inside a channel of parallel flat plates and inside a circular tube, as can be seen in presented results in tables 1-4 and figures (1-6), considering differents values of Reynolds and Prandtl.

The tables 1 and 2 show the remarkable influence that the Reynolds and Prandtl numbers exert on the prediction of the asymptotic convective Nusselt number. For a specific number of Reynolds, an increase in the number of Prandtl leads to an increase in the asymptotic Nusselt number. This behavior also occurs with the increase of Reynolds number, keeping the same number of Prandtl, as we can also observe in figures 1 and 4. The above mentioned facts make it possible to conclude that the increase in Reynolds and Prandtl numbers produces an increase in the heat transfer rates, causing the axial length of thermal development to be reduced, as can be seen in Figures 2, 3, 5 and 6.

As predicted, the results obtained in the present study, shown in tables 1 and 2, do not show perfect compliance with the results predicted by the correlations of Gnielinski (1976) and Taylor (1916), because it is the comparison of experimental studies with numerical simulation studies. However, it is possible to observe in tables 1 and 2 a very similar variation between the results, which validates the study developed in the present work.

Table 1. Asymptotic Nusselt number considering different numbers of Reynolds and Prandtl for the turbulent flow between flat plates.

Flat Plates - $Pr_t = 1$								
$Pr = 0.72$			$Pr = 1$			$Pr = 2$		
$Re = 1.10^4$	$Re = 5.10^4$	$Re = 1.10^5$	$Re = 1.10^4$	$Re = 5.10^4$	$Re = 1.10^5$	$Re = 1.10^4$	$Re = 5.10^4$	$Re = 1.10^5$
38.31[a]	123.91[a]	211.26[a]	41.69[a]	141.75[a]	247.16[a]	47.58[a]	177.16[a]	326.20[a]
38.17[b]	123.82[b]	210.54[b]	41.52[b]	141.64[b]	246.27[b]	47.37[b]	177.03[b]	324.86[b]
39.87[c]	129.41[c]	219.26[c]	43.46[c]	148.23[c]	257.14[c]	49.77[c]	185.74[c]	341.14[c]
30.24[d]	105.97[d]	181.85[d]	35.41[d]	128.37[d]	222.65[d]	48.25[d]	185.79[d]	328.93[d]
29.93[e]	105.78[e]	180.40[e]	35.09[e]	128.15[e]	220.99[e]	47.87[e]	185.52[e]	326.80[e]
33.67[f]	118.33[f]	198.48[f]	39.07[f]	142.23[f]	241.65[f]	52.38[f]	202.91[f]	353.14[f]
31.06[g]	101.59[g]	173.44[g]	39.35[g]	130.99[g]	224.90[g]	59.91[g]	208.59[g]	363.59[g]
30.76[h]	101.41[h]	172.10[h]	38.98[h]	130.76[h]	223.22[h]	59.42[h]	208.27[h]	361.13[h]
34.43[i]	113.02[i]	188.80[i]	43.41[i]	145.14[i]	244.09[i]	65.30[i]	228.67[i]	391.48[i]

[a] – Present work: Nusselt with friction factor of Filonenko (Model of Prandtl and Taylor)
[b] – Present work: Nusselt with friction factor of Shah and Bhatti (Model of Prandtl and Taylor)
[c] – Present work: Nusselt with friction factor of Dean (Model of Prandtl and Taylor)
[d] – Gnielinsk's empirical correlation with Filonenko's friction factor
[e] – Gnielinsk's empirical correlation with Shah and Bhatti's friction factor
[f] – Gnielinsk's empirical correlation with Dean's friction factor
[g] – Taylor's empirical correlation with Filonenko's friction factor
[h] – Taylor's empirical correlation with Shah and Bhatti's friction factor
[i] – Taylor's empirical correlation with Dean's friction factor

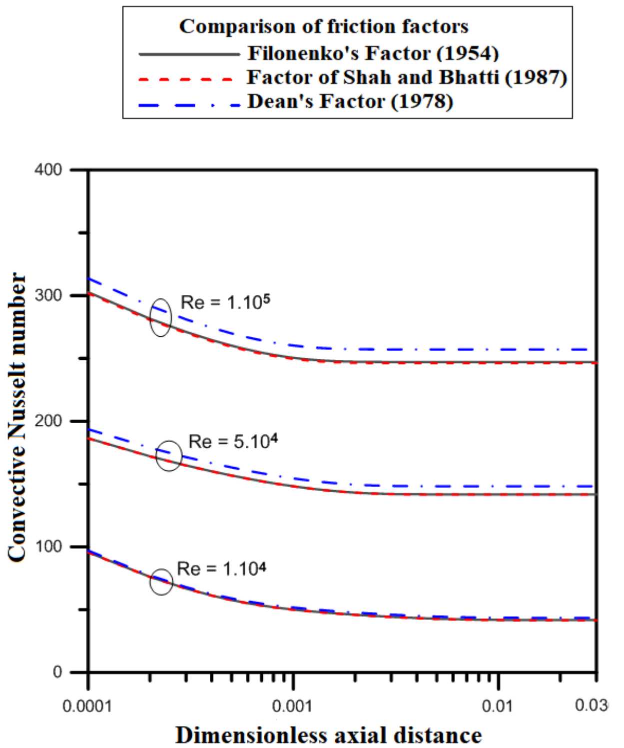

Figure 1. Local Nusselt number for turbulent flow between flat plates, considering Pr = 1, Pr_t = 1 and turbulence model of Prandtl and Taylor (Flat plates).

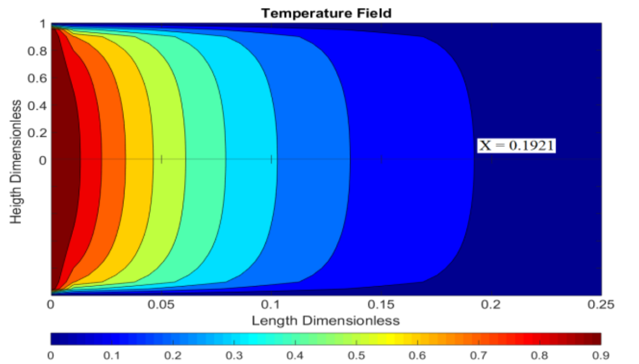

Figure 2. Temperature field for turbulent flow between flat plates, considering Pr = 1, Pr_t = 1, Re = 1.10^4 and friction factor of Filonenko.

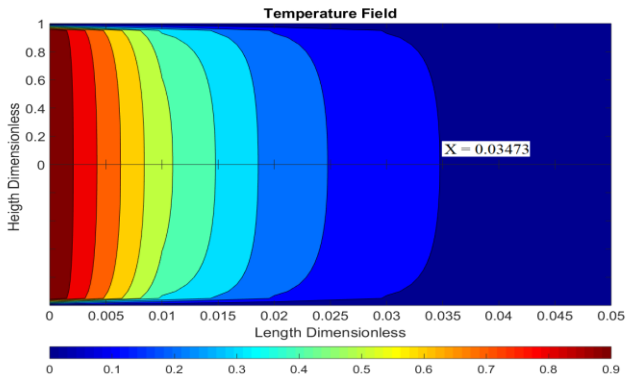

Figure 3. Temperature field for turbulent flow between flat plates, considering Pr = 1, Pr_t = 1, Re = 1.10^5 and friction factor of Filonenko.

Table 2. Asymptotic Nusselt number considering different numbers of Reynolds and Prandtl for the turbulent flow inside a circular tube.

Circular Tube - $Pr_t = 1$								
$Pr = 0.72$			$Pr = 1$			$Pr = 2$		
$Re = 1.10^4$	$Re = 5.10^4$	$Re = 1.10^5$	$Re = 1.10^4$	$Re = 5.10^4$	$Re = 1.10^5$	$Re = 1.10^4$	$Re = 5.10^4$	$Re = 1.10^5$
34.68[a]	116.63[a]	201.56[a]	38.48[a]	136.46[a]	241.81[a]	45.41[a]	180.15[a]	346.03[a]
34.55[b]	116.54[b]	200.85[b]	38.33[b]	136.35[b]	240.87[b]	45.21[b]	179.99[b]	344.22[b]
30.24[c]	105.97[c]	181.85[c]	35.41[c]	128.37[c]	222.65[c]	48.25[c]	185.79[c]	328.93[c]
29.93[d]	105.78[d]	180.40[d]	35.09[d]	128.15[d]	220.99[d]	47.87[d]	185.52[d]	326.80[d]
31.06[e]	101.59[e]	173.44[e]	39.35[e]	130.99[e]	224.90[e]	59.91[e]	208.59[e]	363.59[e]
30.76[f]	101.41[f]	172.10[f]	38.98[f]	130.76[f]	223.22[f]	59.42[f]	208.27[f]	361.13[f]

[a] – Present work: Nusselt with friction factor of Filonenko (Model of Prandtl and Taylor)
[b] – Present work: Nusselt with friction factor of Shah and Bhatti (Model of Prandtl and Taylor)
[c] – Gnielinsk's empirical correlation with Filonenko's friction factor
[d] – Gnielinsk's empirical correlation with Shah and Bhatti's friction factor
[e] – Taylor's empirical correlation with Filonenko's friction factor
[f] – Taylor's empirical correlation with Shah and Bhatti's friction factor

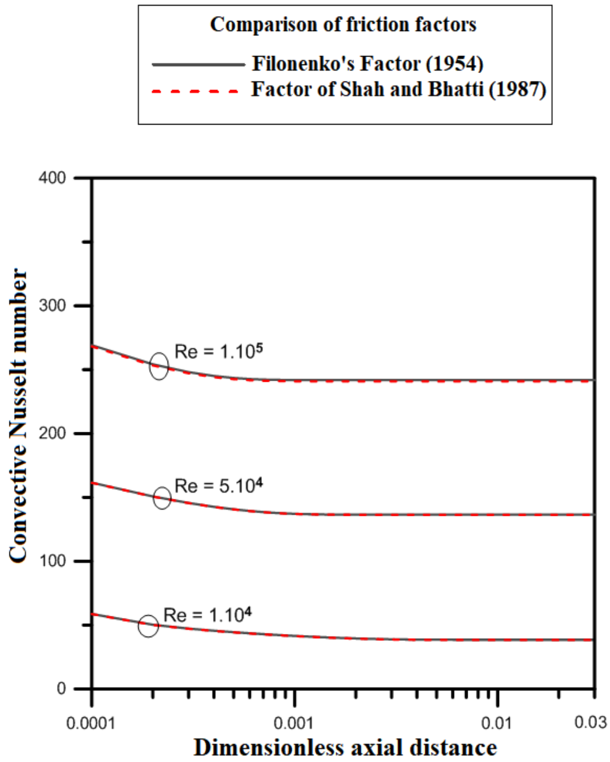

Figure 4. Local Nusselt number for turbulent flow inside a circular tube, considering Pr = 1, Pr_t = 1 and turbulence model of Prandtl and Taylor (Circular tube).

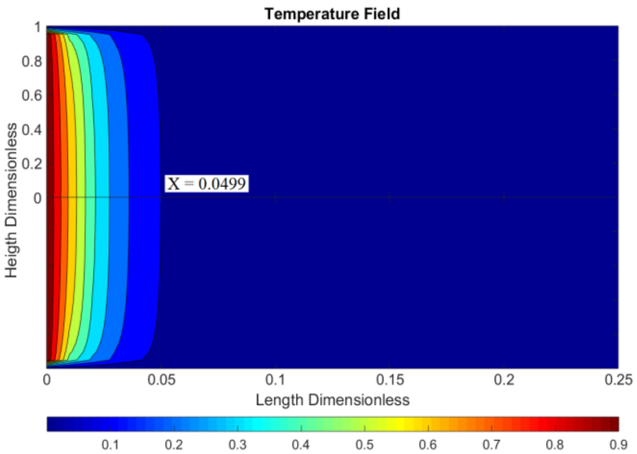

Figure 5. Temperature field for turbulent flow inside a circular tube, considering Pr = 1, Pr_t = 1, Re = 1.10^4 and friction factor of Filonenko.

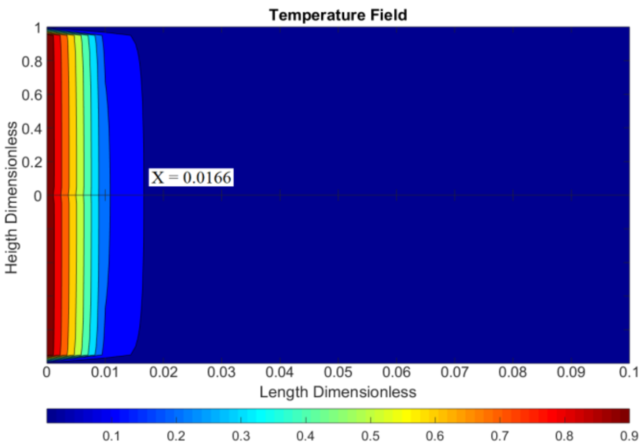

Figure 6. Temperature field for turbulent flow inside a circular tube, considering Pr = 1, Pr_t = 1, Re = 5.10^4 and friction factor of Filonenko.

The tables 3 and 4 show the influence that the friction factor and the Reynolds and Prandtl numbers exert on the prediction of the dimensionless thermal input length. In the table 3 analyzes the case of flow between flat plates and in the table 4 explores the situation of flow inside a circular tube. In both tables it is considered the combination of different numbers of Reynolds (*1.10⁴*, *5.10⁴* and *1.10⁵*), different numbers of Prandtl (*0.72*, *1* and *2*) and different correlations for the friction factor (Dean, Filonenko and Bhatti-Shah). The results presented allow to estimate, for each investigated situation, a trend value for the dimensionless length of thermal development, which can represent extremely important information in practical situations.

Table 3. Dimensionless length of thermal development for the turbulent flow between flat plates.

Flat Plates - $Pr_t = 1$								
$Pr = 0.72$			$Pr = 1$			$Pr = 2$		
$Re = 1.10^4$	$Re = 5.10^4$	$Re = 1.10^5$	$Re = 1.10^4$	$Re = 5.10^4$	$Re = 1.10^5$	$Re = 1.10^4$	$Re = 5.10^4$	$Re = 1.10^5$
0.2100[a]	0.0676[a]	0.0399[a]	0.1921[a]	0.0588[c]	0.0347[a]	0.1670[a]	0.0471[a]	0.0266[a]
0.2108[b]	0.0676[b]	0.0400[b]	0.1929[b]	0.0588[b]	0.0349[b]	0.1676[b]	0.0471[b]	0.0267[b]
0.2022[c]	0.0649[c]	0.0388[c]	0.1847[c]	0.0565[c]	0.0333[c]	0.1598[c]	0.0451[c]	0.0256[c]

[a] – Present work: Dimensionless length of thermal development with friction factor of Filonenko

[b] – Present work: Dimensionless length of thermal development with friction factor of Shah and Bhatti

[c] – Present work: Dimensionless length of thermal development with friction factor of Dean

Table 4. Dimensionless length of thermal development for the turbulent flow inside a circular tube.

Circular Tube - $Pr_t = 1$								
$Pr = 0.72$			$Pr = 1$			$Pr = 2$		
$Re = 1.10^4$	$Re = 5.10^4$	$Re = 1.10^5$	$Re = 1.10^4$	$Re = 5.10^4$	$Re = 1.10^5$	$Re = 1.10^4$	$Re = 5.10^4$	$Re = 1.10^5$
0.0563[a]	0.0185[a]	0.0103[a]	0.0499[a]	0.0166[a]	0.0096[a]	0.0420[a]	0.0122[a]	0.0091[a]
0.0565[b]	0.0185[b]	0.0104[b]	0.0501[b]	0.0167[b]	0.0096[b]	0.0423[b]	0.0122[b]	0.0092[b]

[a] – Present work: Dimensionless length of thermal development with friction factor of Filonenko

[b] – Present work: Dimensionless length of thermal development with friction factor of Shah and Bhatti

In the present study the thermal input length is defined as the maximum axial length required for the fluid to reach its final temperature with a margin of 10% relative difference. In practical situations this information may be relevant in the dimensioning process of the thermal equipment.

As predicted, the results differ from each other, but not with a very significant difference. Through this table it is possible to estimate, for a specified situation, the dimensional length of thermal development through equation 5a, that is, it is possible to find how many "meters" of duct are necessary to reach the thermal development.

5. CONCLUSIONS

It is concluded from the analysis of the results obtained that the application of GITT is effective in solving the problem proposed, since the presented formulation was validated with the results found in the specialized literature. In this way, the objectives were reached satisfactorily, where the influence of the Reynolds and Prandtl numbers on the development of the thermal field and the local Nusselt number was shown. The analysis made in the present work is of extreme relevance, since the study of turbulence assumes great importance in engineering since it has seen the great number of practical applications in which it is present. In heat transfer, the involvement of a fluid in turbulent motion appears in most processes involving the transport of energy. In applied areas such as heat exchanger design, reactor engineering and power engineering, laminar flow is an exception rather than a rule.

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