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STUDY OF HEAT FLOW THROUGH MULTIPLE FORMS OF WAFER BAKING PLATES

Márcio César Carzino
Marcelo Risso Errera
Graduate Program in Environmental Engineering, PPGEA, Federal University of Paraná, Av. Cel. Francisco H. dos Santos, 100 - Jardim das Américas, Curitiba - PR, 81530-000
marcio.cesar.carzino@gmail.com
errera@ufpr.br

George Stanescu
Department of Mechanical Engineering, Federal University of Paraná, Av. Cel. Francisco H. dos Santos, 100 - Jardim das Américas, Curitiba - PR, 81530-000
stanescu@ufpr.br

Abstract. *There is a large worldwide demand to develop wafer baking equipments. The main goals are to increase production with quality and to reduce wastes and energy consumption. This work deals with thermal design in the heating side of baking plates of wafer gas ovens. The thermal interaction between the gas flames and the baking plates is considered. The flames are modelled as a radiant solid surface with fixed geometry and high-temperature. Heat flows mostly by thermal radiation to the back of the baking plates which design is under study. Steady, 2-d, heat conduction within the baking plate determines the superficial temperature of the baking side. All the physics is modelled mathematically and solved in a commercial finite-element software. The method of Constructal Design is employed in order to study designs of the back surface of the baking plates, for instance, the possibility of extended surfaces for a given volume of material. Variable rectangular, trapezoidal and parabolic fins were considered. They varied in number and geometry accordingly. Results showed the baking plate with 17 fins provides a better heat distribution and increase of near 20ºC from the plate with 3 fins. This means an overall gain of approximately 10%. In sum the new designs provide better heat transfer that in turn decreases the baking time, and it also improves heat distribution thus warranting product quality. Furthermore, the new designs provide the same mechanical resistance with less 17% material.*

Keywords: *wafer biscuit oven, baking plate, heat conduction, radiation heat transfer, constructal design*

1. INTRODUCTION

Biscuits, otherwise called as wafers, are known in Europe for centuries. They were initially baked between two cast iron plates (Moor, 1994). Production started in simple baking plates then evolved to large and complex automated ovens in order to meet the large production demand and quality standards. Today the main goals for improvements are the process itself, the quality, productivity, automation and ultimately reduction of wastes and energy. In view of the large-scale production, even incremental gains lead to substantial benefits year-round. The starting point to achieve those goals is the thermal interaction between the batter and the heat source. Heat is provided by arrays of gas burners to the baking plate which in turn bakes the dough. Today's ovens work with a set of lower and upper baking plates that are mounted on a baking trolley (Fig. 1). The baking trolley moves through the oven in a loop over rails fixed in the structure (Haas, *et al.* 1984). Heat is transferred to the baking trolley through the tubular burners located under the cart, approximately 50 mm below for combustion requirements. The burners are located along the longitudinal direction of the oven. The baking trolley moves along the oven with the plate fins in the direction transverse to the burners.

The flames are modelled as a fixed geometry high temperature slender rectangle radiating solid body. Heat flows mostly by thermal radiation to the back of the baking plates, which design is under study. Steady, 2-d, heat conduction within the baking plate determines the superficial temperature of the opposite side where baking takes place. All the physics is modelled mathematically and solved in a commercial finite-element software (COMSOL, 2019).

The method of Constructal Design (Bejan and Lorente, 2008) is employed in order to study designs of the back surface of the baking plates. According to the method, the better designs promote greater access of the heat flow across the system (baking plate). For instance, we study the possibility of extended surfaces for a given volume of material. A variable number of triangular, rectangular, trapezoidal and parabolic fins were considered. It was tested from 3 to 17 fins in a total of 68 computer simulations. The method has been successfully applied earlier (Bejan and Almogbel, 2000) for purely convective fins in a different application.

The thermal impact of different designs is reflected in the temperature level and its distribution on the baking surface: the greater the access, the higher the temperature, and lower the nonuniformity. Fin designs have been addressed widely in the literature, in particular Bejan and Almogbel, 1999, Torabi, *et al.* 2013, and Lane and Heggs, 2005.

This paper deals with the influence of the shape of the fins on the heat flow through the baking plates, new fin shapes, homogeneous baking temperature and possible weight reduction of the baking plate is the object of this study. The target is therefore to provide better access to the heat from the burners for the surface baking plate. Results showed that indeed the design of heating side of the plates can provide significant improvement for instance, near 10% between two distinct designs.

The new designs provide better heat transfer that in turn decreases the baking time, and it also improves heat distribution thus warranting product quality. Furthermore, the new designs provide the same mechanical resistance with less material.

2. BRIEF DESCRIPTION OF WAFER BAKING OVENS

Wafer ovens can produce several kinds of different biscuits, namely, flat wafers, hollow wafer, waffles, pancakes, ice cream cones among others. Baking ovens are designed for the industrial production of wafer in large scale, and they are composed of a front cabinet where the liquid batter is deposited on the baking plates (Fig. 2). Afterwards, the batter is deposited in the baking form then the plates are shut off thus creating a pressure zone where the final product is baked. The baking plates are supported on baking trolleys, which have rolling wheels that slide along rails (Fig. 1). A set of trolleys forming a chain run through a circuit inside the oven passing through the firing chamber where gas burners heat the plates and transfer heat by providing a quick bake. In about 2 minutes the cycle is complete and the plates open. The baked wafer sheets are then removed in the beginning of cabinet and the baking sheets are ready to receive a new product injection and start a new cycle (Haas, *et al.* 1984).

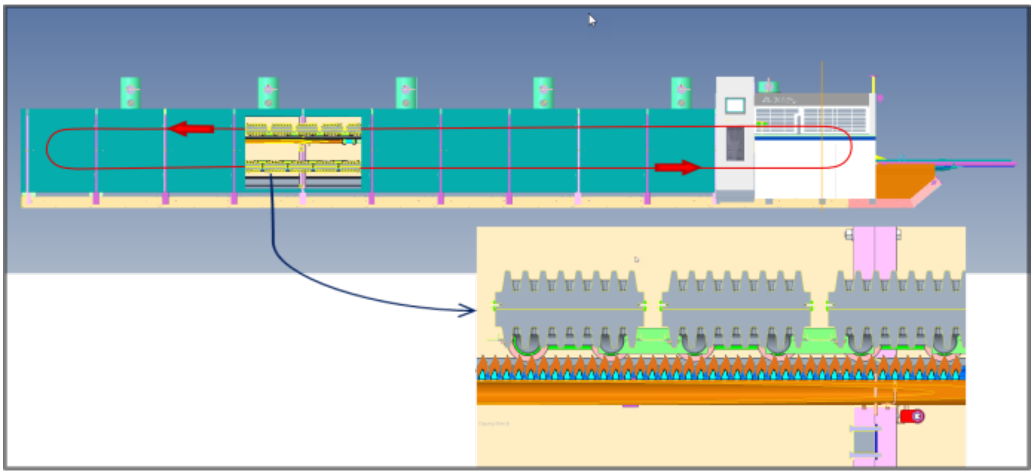

Figure 1. Schematic drawing of an industrial flat wafer oven.

The heat is provided by the flame of the gas burner (NG or LPG). The baking plates move across to the burner nozzles. The heat is absorbed by the fins on the back of the cooking plate and it flows by conduction in the cast iron plates to reach the baking surface, where the wafer batter is baked at a temperature of approximately 160 °C and 180 °C depending of the wafer recipe and ingredients (Tiefenbacher, 2014).

3. PHYSICAL AND MATHEMATICAL MODEL

The bottom side of a single plate is considered in order to capture the main features of the heating process (Fig. 2). The flames are considered to form a continuous flat plate that produces a heat flux of 20 kWm^{-2}. The only fixed temperature is the baking. (average phase change) temperature of the dough of 180 °C, read far from the plate. The oven chamber ambient temperature is assumed to be 150 °C, considering the combustion gases removed by forced exhaustion. The view factor between the virtual flame and the finned surface is calculated for each point along the surface coupled with the finite element method. The heat transfer by thermal radiation prevails. Surfaces are opaque. The heat balance will be described later. The model is two dimensional. Even though the baking process is transient, the process is considered steady-state at average conditions (cooking heat and temperature). For the first step of the design process, the heat flux demanded by the baking process is due a highly convective regime at the batter side, namely, 100 $Wm^{-2}K$ this value is applied for convective heat flux in biscuit ovens (Sakin *et al.*, 2009).

For sake of brevity, readers are directed to classic heat transfer textbooks (e.g. Incropera, 2003 and Bejan, 1993) for the details of the thermal radiation model for every point lying along the finned surface (Fig. 2). The surface temperature field is coupled with 2-D, steady-state conduction with homogenous and isotropic thermal conductivity in the plate as Eq. (1).

$$\frac{\partial^2 T_p}{\partial x^2} + \frac{\partial^2 T_p}{\partial y^2} = 0 \qquad (1)$$

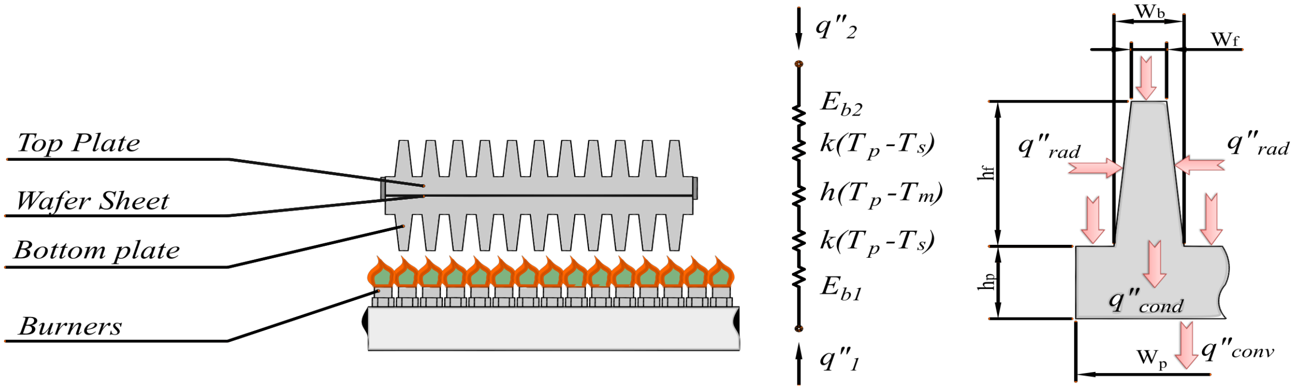

Figure 2. Skematic drawing of a baking plate with burners with heat transfer circuit and a slice of the baking plate and main dimensions.

The temperature of the plate $T_p(x, y)$ lies in a domain with x ranging from 0 to W_p, where W_p is the width of the base of the plate, and y ranging from 0 to h_t, where h_t is the total height of the baking plate, $h_p + h_f$, the height of the base baking plate plus the height of the fins.

The numerical solutions in the commercial software COMSOL were tested by the usual procedures of meshing and validity. This procedure is omitted of the sake of conciseness as well.

The design of the baking plate must be within some constructive dimensions necessary for project and some degrees of freedom were defined by constructal design. The material used for baking plates was nodular cast iron (3,7C-3Si). The heating was designed like a flat flame along the baking plates irradiating to the fins of the back of baking plates, the flames are from propane gas at a high temperature irradiating.

The internal ambient temperature of the oven is considered constant and uniform as 150 ºC. The batter is fed at the initial temperature of 20 ºC (Tiefenbacher, 2014). The mean temperature of the flame is considered to be 1,400 ºC.

The properties of the baking plate are, the heat capacity of C_{pff} = 460.55 $J.kg^{-1}.K^{-1}$, the mass density of 7,200 $kg.m^{-3}$, thermal conductivity of 52 $W.m^{-1}.K^{-1}$ and the surface emissivity of 0.67 (Bejan, 1993).

An oven yields around 10.5 kW per baking plate, this means 60,000 W/m^2 installed but part of it is lost. It was then considered a heat transfer flux of 20,000 W/m^2 for each plate by radiation surface-to-surface, with controlled opacity.

The overall emissivity of the flame was set as 0.23 considering mainly CO_2 e O_2 as combustion products (Andersson and Johnsson, 2016)

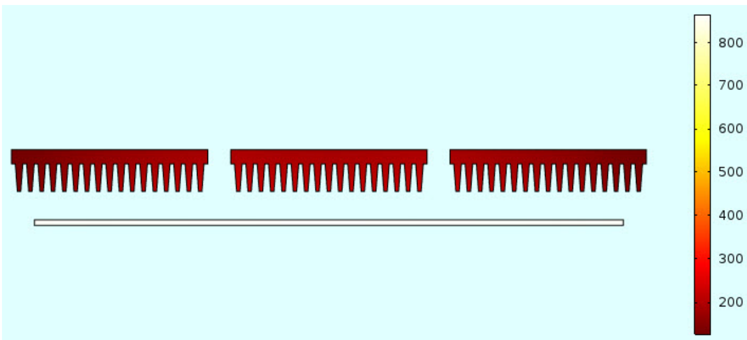

Figure 3. Temperature field of the middle plate, with 17 trapezoidal fins and the flat radiant burner.

The software computes the total radiative flux leaving a flame (radiosity). The flame condition was simplified as a solid body with a constant emissivity during an average baking time. Irradiation of the flame is diffuse, as well as the baking plates surfaces. The radiation spectrum solely depends on the temperature of the baking plates (Eck *et al.*, 2016).

In short, the mathematical formulation consists of computing the heating emitted by the burners, the heating transferred to baking plates and the heat transferred from baking plates to wafer biscuit undergoing baking. The physics is considered bi-dimensional (2-D). The remaining hypotheses are stated below:

1. steady-state operation of the baking plates running by the oven, because of great mass of cast iron from the baking plates;
2. heat losses for surroundings are negligible because of thermal insulation of the oven;
3. diffuse radiation is the predominant mechanism of heat transfer inside the oven;
4. all flows throughout the oven are considered constant pressure flows;
5. the liquid batter deposited on baking plate is scaled by the boiling water heating transfer coefficient;
6. the ambient temperature of the oven is considered stable around 150 ºC, by the medium temperature of the literature (Mukherjee *et al.*, 2017).

Energy balance considers the heat transfer modes by conduction, convection and radiation in each surface. The energy balance at the rear surface of the baking plate is given by:

$$\dot{Q}_{cond} + \dot{Q}_{conv} + \dot{Q}_{rad} = 0 \tag{2}$$

Where Q_{rad} [W] is the energy transmitted by the burner by radiation, and it can be defined accounted for as follows:

$$\dot{Q}_{rad} = A_s \ F_{bp} \left(J_b - J_p\right) \tag{3}$$

Where A_s [m^2] is the rear surface area from the baking plate, F_{bp} [-] is the view factor from the burner to the baking plate. The total radiative flux leaving the burners is J_b [W/m^2] and total radiative flux leaving the rear baking surface of the plate is J_p [W/m^2]. Therefore,

$$\dot{Q}_{rad} = \dot{Q}_{bp} = A_s F_{bp} \sigma \left(T_b^4 - T_p^4\right) \tag{4}$$

Where Q_{bp} [W] is the radiation net rate that leaves the burners and goes to the baking plate, σ is the Stephan-Boltzmann constant with the value 5,670373x10^{-8} [W.m^{-2}.K^{-4}]. T_b [K] is the flame temperature, T_p [K] is the local temperature of the rear of the baking plate. The view factor depends of the geometry of the radiation of the burner and the baking plates, and it can be seen as the radiation from burner that is intercepted by baking plate, which means:

$$F_{bp} = \frac{radiation\ leaving\ burner\ A_b\ and\ hitting\ the\ plate\ A_p}{total\ radiation\ leaving\ the\ burner\ A_b} \tag{5}$$

In simulation software the surfaces are divided into facets in the meshing geometry, the parts were design in a CAD software and imported into the simulator. The view factor is computed internally by COMSOL, using a hemicube model.

The radiant net exchange Q_{rad} [W] in which the radiation leaves the burner and arrives in the baking plate can be calculated also by the radiative interactions absorbed and reflected.

$$q_{rad} = A_s \left(J_p - G_p\right) \tag{6}$$

Equation (7) provides the radiosity. E_p [W/m^2] is the emissive power of the surface of the plate and ρ_p [-] is the baking surface reflectivity and G_p [W/m^2] is the baking plate irradiation.

$$J_p = E_p + \rho_p G_p \tag{7}$$

which in turn yields:

$$J_p = \varepsilon_{ff} \ E_{bp} + (1 - \varepsilon_p) G_p \tag{8}$$

where: ε_{ff} [-] is the emissivity of the cast iron set as 0.67, E_{bp} is the power emissivity of baking plate surface as defined by law of Stefan-Boltzmann:

$$E_{bp} = \sigma \left(T_b^4 - T_p^4\right) \tag{9}$$

Then the heat transfer net rate from radiation over the rear baking plate is:

$$q_{rad} = \varepsilon_p A_s \frac{E_{bp}-J_p}{(1-\varepsilon_p)} \quad (10)$$

More details of the finite element of heat conduction and other details can be found in COMSOL, 2019.

4. CONSTRUCTAL DESIGN OF THE FINNED SURFACE

The method of Constructal Design is employed in order to explore design trends. The model consists of identifying the essential flows of a system, the constraints, the source of imperfections and the degrees of freedom. It is based on the Constructal Law, which is detailed in Bejan and Lorente, 2008 and further discussed by Errera, 2018.

Briefly, there are 5 degrees of freedom for a trapezoidal fin that can be varied in order to generate designs that are tested. The Constructal Law states that the configurations that facilitate the flow the most tend to prevail. In this work, this is translated into the average temperature at the baking side of the plate and its non uniformity. From limit of 2D design were simulated only the number of fins, *N*, for four different fin geometry will be shown.

The main constraint is the amount of material of the heated surface of the baking plate. For instance, if the material volume is turned into trapezoidal fins, the constraint is given by Eq. (11), based on the indications of Fig. 2.

$$V_{cte} = \left\{ N \left[\frac{(W_b+W_f)}{2} \right] h_f + h_p W_p \right\} \mathrm{L} \quad (11)$$

Where, V_{cte} [m^3] is the plate fixed volume, W_b [m] is the width of the fins base, W_f [m] is the width of the tip of the fin, h_f [m] is the height of the fin, h_p [m] is the thickness of the base of the plate, W_p [m] is the width of the base of the plate, and *L* [m] é a length of the plate in the third dimension. In rectangular fins, $W_b = W_f$. In parabolic fins, the shape is fixed for initial simulations. The geometric parameters (Eq. (11) and Fig. 2) of each configuration (design) are determined in a non-linear algebraic solver implemented in electronic spreadsheet. W_b, W_f and h_f were determined for a fixed condition for h_p while the number of fins, *N,* varied between 3 e 17. Physical and constructive constraints were taken into account.

The baking plates are supported by tongs, and on them are hinges that allow a precise closure of the plates against each other. When the plates are closed, a pressure chamber is formed, the plates are loaded by the steam pressure from the batter. The load results in an elastic deformation of the baking plates, which is similar to deflection of a beam. A relation between small deformation of the plates and the better heat transfer could be achieved by calculation and graphics and relating to fin numbers. The deformation is calculated by the formula:

$$\delta = \frac{5\, F\, L^4}{384\, E\, J_x} \quad (12)$$

Where, δ [m] is the deformation in the baking plate, F [N] is the force load in the baking plates due the steam pressure, E [N.m^{-2}] is the elasticity module of the cast iron, J_x [m^4] is the inertia moment of the cross section of the baking plates. The baking pressure is around 700 [kPa] due the saturation of the water in a temperature of 170 ºC, the elasticity module for cast iron was considered to be E = 137×10^9 [N.m^{-2}], and the inertia moment in a cross section was found through the design of each shape and fin number by computer aided design using toll mass properties.

5. RESULTS AND DISCUSSION

Initially, we compared a finned surface with a plain rectangular plate with the same volume. That showed indeed it is worthwhile a finned surface. In Fig. 4 we show the temperature profile along the width of the baking surface of the plate for trapezoidal fins for different numbers of fins, *N*. Higher temperatures of the baking surface are reached with larger number of fins. Temperature non-uniformity seems however unavoidable since the finned surfaces are not irradiated uniformly neither the thermal resistance of the plate is uniform.

Commercial baking plates are done with 6 to 11 fins. The objective is to show that using more fins could get a better heat flow transfer from burners to the baking plate surface, with a more homogeneous temperature. The most important heat transfer mechanism is the radiation from the flame to the back of the baking plates. And the view factor from the burner to the fins are higher when we increase the number of the fins. On the other hand, when we add material to plate, we increase its weight and consequently increase the production costs. The new goal is then to increase the amount of fins in the plate and keep the plate weight constant, so there should be a situation where the plates has as many fins as possible, does not lose mechanical strength and transfers as much heat as possible.

Figures 4 to 7 shows the baking surface temperature increasing with the number of the fins, that is, the more fins are arranged on the plate the higher the surface baking temperature for trapezoidal, triangular, rectangular and parabolic arrangements. Figure 8 shows that trapezoidal shape reaches higher temperature than other shapes.

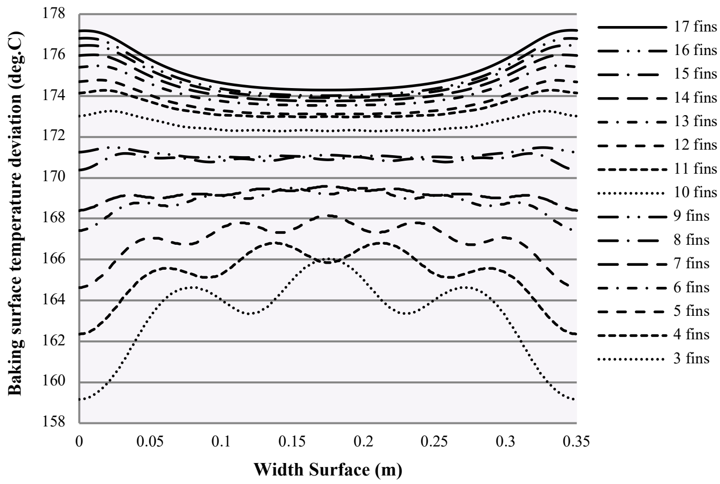

Figure 4. Temperature profile along the width of the baking surface of the plate for trapezoidal shape for different numbers of fins, N.

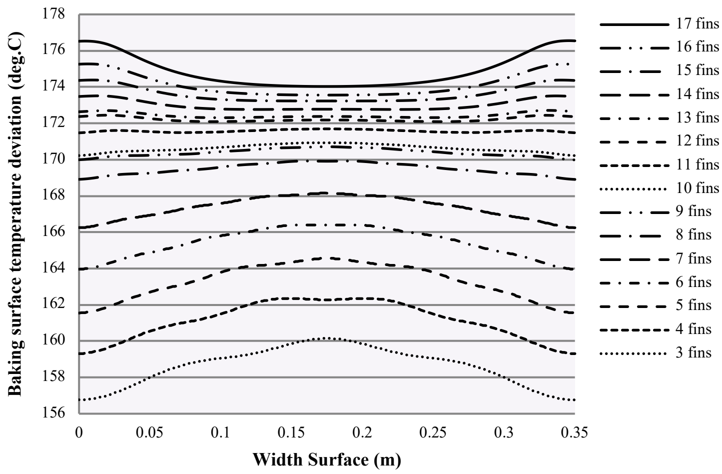

Figure 5. Temperature profile along the width of the baking surface of the plate for triangular shape for different numbers of fins, N.

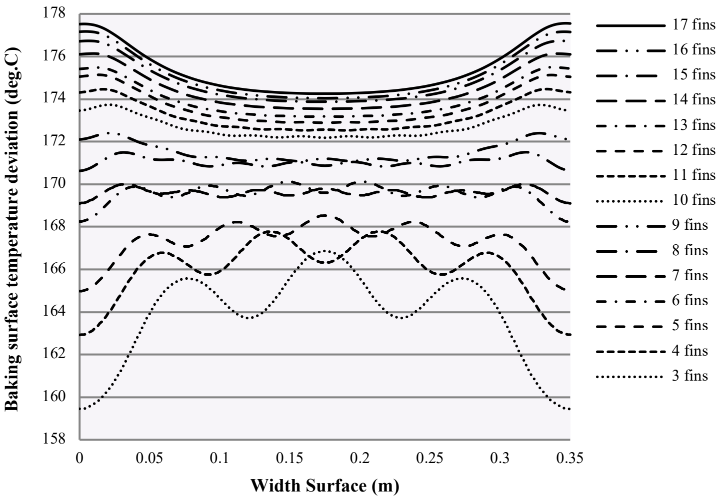

Figure 6. Temperature profile along the width of the baking surface of the plate for rectangular shape for different numbers of fins, N.

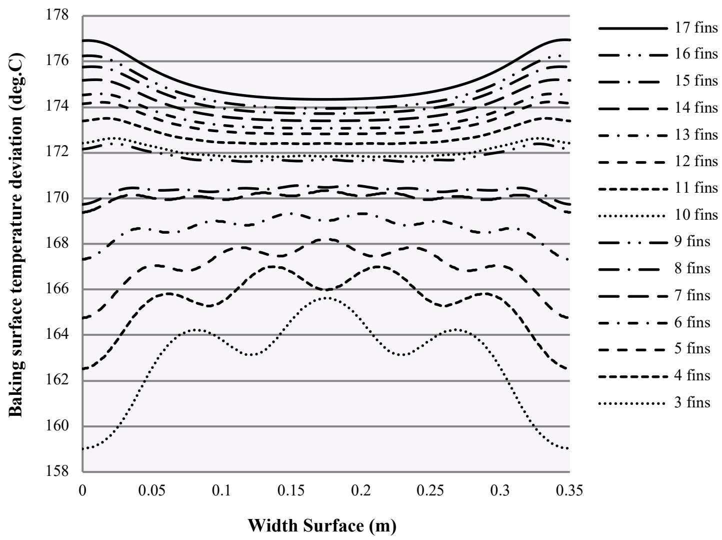

Figure 7. Temperature profile along the width of the baking surface of the plate for parabolic shape for different numbers of fins, N.

Figure 8 shows the average surface temperature of the baking side of the plate finned with variable number of trapezoidal, parabolic, triangular and rectangular fins. The curves are ragged due to Eq. (11) and constructive constraints. Larger number of fins facilitates the heat flow from the burners to the batter dough. Triangular fins have a lower heat flow compared to other shapes, despite the volume of the plate is the same, the view factor is affected by the reduced of polygon area.

Figure 9 shows the difference between maximum and minimum temperature for each shape and fin number, the uniformity of the temperature in the baking plate is important to reach a good baking of wafer and avoid moisture areas. The triangular shape reaches a minimum difference between 11 and 13 fins. While trapezoidal and parabolic are between 8 and 10. Rectangular is at 8 fins the minimum difference. When the baking plates have so much fins the uniformity of the baking surface becomes more difficulty.

Figure 10 shows the calculation of the deformation of the plate due the baking pressure, when the inertia moment increase the deformation decrease, the minimum deformation was found for rectangular shapes, the triangular shape have the worst performance in this case. By the practical experience a deformation bigger than 250 µm would be not acceptable.

One commercial baking plate with six fins has a weight of 74 kg and was calculated a deformation of 220 µm, the average temperature of baking surface was simulated and reach 163.02 ºC and the difference between maximum and minimum temperature in the baking surface was 3.87 ºC. It was compared with the new design proposal of baking plate of fourteen fins, was calculated the weight of 61.2 kg, this means a possible weight reduction of cast iron of 17.3 %. For sake the deformation of the plate is a same 220 µm to remains the same quality. The average temperature of baking surface was simulated with the same conditions and reaches 174.2 ºC a substantial increasing of temperature of 11.2 ºC, with this higher temperature can be converted into a reduced fuel consumption. The difference between maximum and minimum temperature in the baking surface was 2.26 ºC, it was reduced 1.61 ºC compare to commercial baking plate and this means a better uniformity of baking surface temperature, in practical means a better moisture control of the wafer sheet.

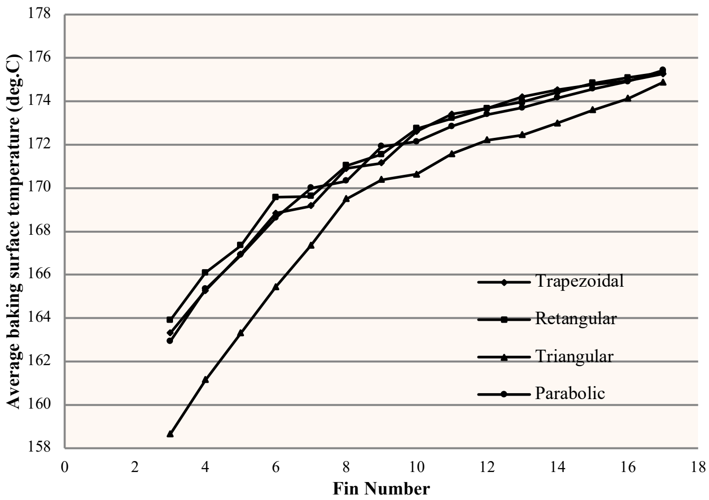

Figure 8. Average baking surface temperature of the plate finned with variable number of trapezoidal, triangular parabolic and rectangular fins.

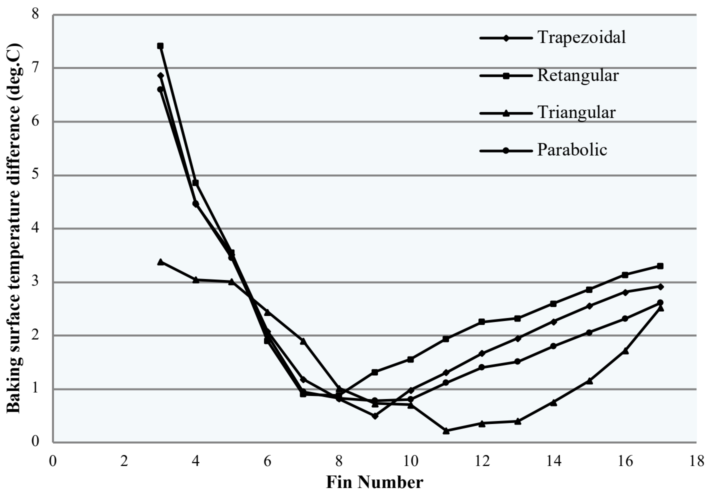

Figure 9. Difference between maximum and minimum temperature at baking surface of the plate finned with variable number of trapezoidal, triangular parabolic and rectangular fins.

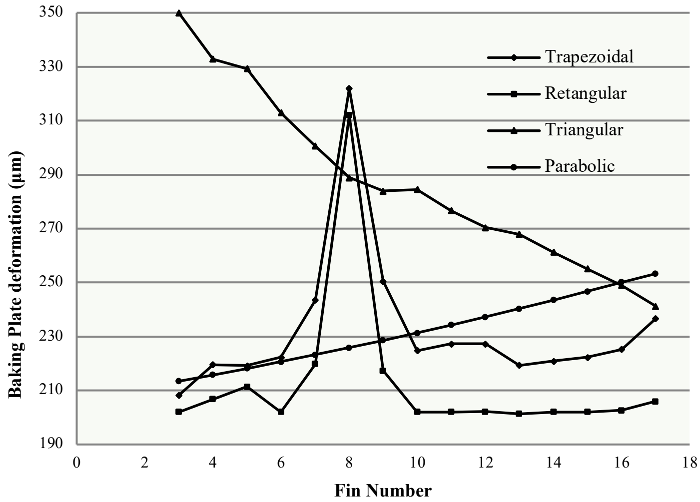

Figure 10. Deformation of the plate due baking pressure

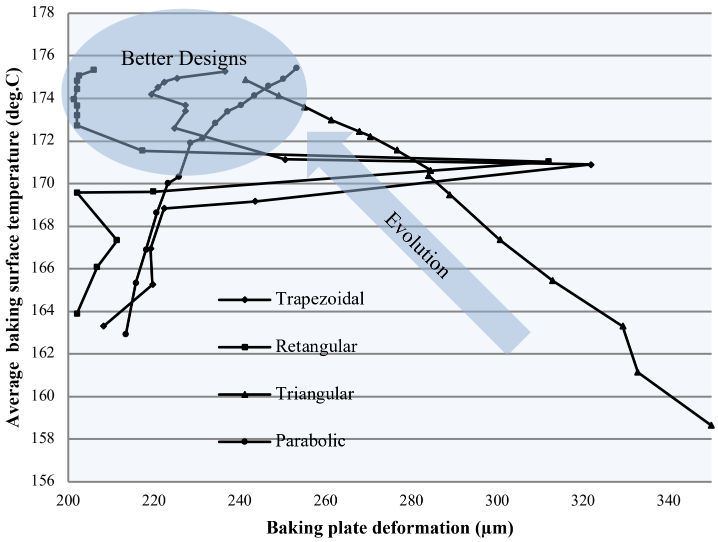

Figure 11. Trade-off baking plate deformation versus average baking surface temperature

Figure 11 shows the deformation behavior of each plate with fins numbers from 3 to 17 related to the average baking surface temperature. In this graph it is possible to observe that higher temperatures are reached by the larger fin plates and the deformation of the rectangular and trapezoidal plates are within an acceptable value below 250um except for variations due to the freedom of the constructal design.

6. CONCLUSIONS

This paper explores the design possibilities of baking plates for wafer. The design of those plates must consider a trade-off of mechanical resistance (low deformation), performance (higher temperature for a given heat flux) and temperature uniformity for quality purposes.

The numerical model was simple enough to sense how the main features of the design affects the project criteria set forth.

The method of constructal design provided a landscape of possibilities that will aid project engineers to choose the configurations that meet the goals when manufacturing, operational and economic issues are later addressed.

In sum the new designs provide better heat transfer that in turn decreases the baking time, while it also improves heat distribution thus warranting product quality. Furthermore, the new designs provide the same mechanical resistance with less 17% material.

7. ACKNOWLEDGEMENTS

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9. RESPONSIBILITY NOTICE