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TOTAL VARIATION PRIOR AND INFRARED THERMOGRAPHY APPLIED TO THE INVERSE PROBLEM OF TWO-DIMENSIONAL AND TIME DEPENDENT HEAT FLUX ESTIMATION

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Abstract. *In this work the problem of heat flux estimation with two-dimensional and temporal variations applied to one of the surfaces of a thermally thin aluminium plate is addressed. The direct problem consists in the identification of the temperature field on a thermally thin plate, which was solved by using the finite difference method and a lumped formulation. The solution of the inverse problem is obtained using the Markov chain Monte Carlo Method, in a Bayesian approach. The experimental data was collected through the use of infrared thermography. For the regularization of the inverse problem it is proposed the application of the Total Variation prior information, a Markov Random Fields formulation. The results, using real measurements, show that the method is able to accurately estimate the space and time dependent heat flux.*

Keywords: *Heat Flux Estimate, Bayesian Inference, Inverse Problems, Markov Random Fields, Infrared Termography*

1. INTRODUCTION

The phenomenon of miniaturization of electronic components, meeting increasingly rigorous demands, such as the development of microchips and nanochips, justifies the investigation of several issues that emerges in its production. Problems of overheating, and consequently heat dissipation, are examples of obstacles constantly faced in industry (Hetsroni *et al.*, 2001). Thus, solutions to accurately and quickly identify heat fluxes in small electronic components play a prominent role in both academy and industry.

The estimation of possible heat fluxes from temperature field measurements is characterized as an inverse problem, and attracts the attention of researchers from different fields of knowledge, initially through deterministic approaches such as the iterated regularization method of Alifanov (Silva Neto and Özisik, 1993; Su and Silva Neto, 2001), among others. Stochastic optimization methods have also been used in such inverse heat transfer problems (Blaudt *et al.*, 2018). In recent years, however, the popularization of Bayesian approaches, such as the Monte Carlo with Markov Chains method (MCMC), allowed the study of new problems, equally relevant (Kaipio and Fox, 2011; Watanabe *et al.*, 2017).

Inverse heat transfer problems can be divided into two types: parameter estimation and function estimation (Kaipio and Somersalo, 2008). The second case is the one of interest in the present work, and it has the characteristic of involving a high computational cost and the obtained solutions may not be enough regularized. In fact we consider a discretized version of the unknown function, with a relatively high set of discrete unknowns. In the Bayesian framework, finding adaptive methods and prior information to help overcome these problems has been a central topic in many studies. Several approaches, with different aspects, have been presented over the years, such as Markov Random Fields priors (Abreu *et al.*, 2017; Neves *et al.*, 2018).

This work addresses the case of the heat flux estimation problem that abruptly varies spatially, in two dimensions, and over time. It is proposed the use of infrared thermography images to provide the experimental data and the Total Variation (TV) prior, a Markov Random Fields (MRV) technique, to regularize the inverse problem. The proposed approach allowed to obtain regularized results for the presented case of study. The expected heat flux was satisfactorily reconstructed and regularized.

2. METHODOLOGY

2.1 Forward Problem

Consider the problem of two-dimensional transient heat conduction, generated by the application of a heat flux that varies in the directions x and y, as well as over time (t), at the lower surface (z = 0) of a thermally thin aluminium plate of dimensions $L_x \times L_y \times L_z$, subject to the initial condition $T(x, y, z, 0) = T_\infty$, the ambient temperature, while its opposite surface $(z = L_z)$ is subjected to the convective heat exchange with the environment at a temperature T_∞, with a heat transfer coefficient h (as shown in Fig. 1). It is assumed that the thickness, L_z, of the plate is much smaller than its width, L_x, and its length, L_y, so that the heat losses at the edges are neglected (Orlande *et al.*, 2013).

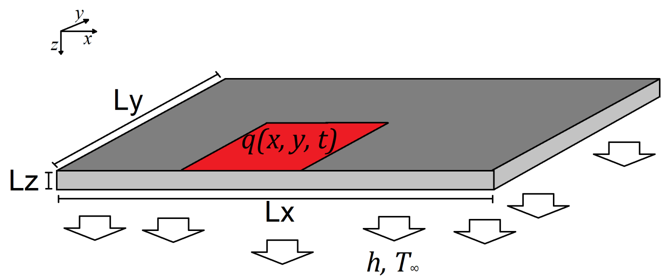


Figure 1. Schematic representation of the physical problem.

Considering a lumped formularion in the z direction and that there is no volumetric heat generation inside the plate, this physical problem is modeled with the transient heat conduction equation (Neves, 2019):

$$\rho c_p \frac{\partial T}{\partial t} = k\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) + \frac{q}{L_z} + \frac{h}{L_z}(T - T_\infty) \tag{1}$$

where ρ is the specific mass (kg/m^3), c_p is the specific heat $(J/(kg/K))$, and k is the thermal conductivity $(W/(m \cdot K))$, which are considered constant.

The boundary and initial conditions imposed on the problem are given by

$$\frac{\partial T(\mathbf{x}, t)}{\partial \mathbf{n}} = 0,\ x = 0,\ x = L_x,\ y = 0,\ y = L_y \tag{2}$$

and

$$T(\mathbf{x}, 0) = T_\infty \tag{3}$$

where $0 \leq x \leq L_x,\ 0 \leq y \leq L_y,\ 0 \leq z \leq L_z,\ t \geq 0$ and $\mathbf{x} = (x, y)$. With the lumped formulation in the z direction, the corresponding boundary conditions are incorporated in Eq. (1).

2.2 Experimental Data and Setup

For the validation of the proposed methodology it was built an experimental setup in the LEMA (Patrícia Olivia Soares Laboratory for Experimentation and Numerical Simulation in Heat and Mass Transfer), Polytechnic Institute (IPRJ/UERJ).

The experimental data was acquired by an infrared thermography camera that relates the emitted infrared radiation with the temperature throughout the surface of a thermally thin aluminium plate with dimensions $0.08 \times 0.04 \times 0.002\,m$ painted with a high emissivity carbon based black paint ($\varepsilon = 0.96$, according to its manufacturers). The material, aluminium, was chosen based on the relevance of applications and already developed research works in the laboratory. Material properties were considered known, accordingly with Bergman *et al.* (2011), as shown in Tab. 1.

Table 1. Prescribe known problems for the inverse problem solution.

Material	Aluminium
$\rho\ [kg/m^3]$	2754
$c_p\ [J/kg \cdot K]$	887
$k\ [W/m \cdot K]$	194

A resistance, with dimensions $0.02 \times 0.02\,m$, was positioned in the center of the plate, as shown in Fig. 2, in order to create a heat flux throughout the aluminium plate, simulating a $q(x, y, t)$ heat flux aimed to be estimated.

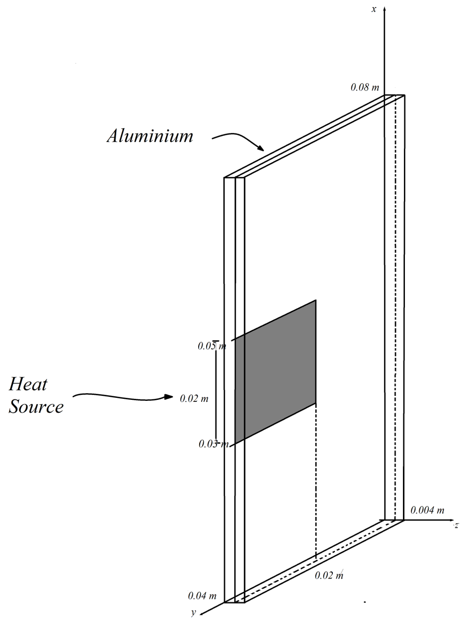
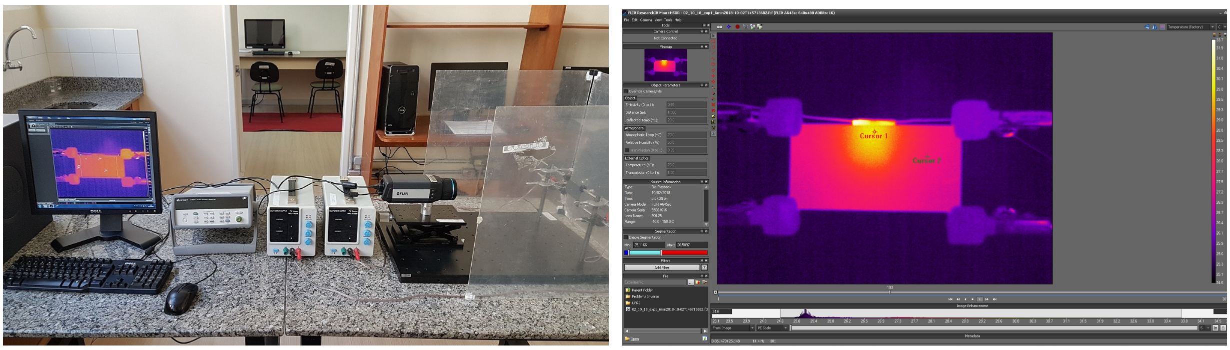


Figure 2. Infrared camera and aluminium plate position setup.

In the experimental set up, shown in Fig. 3(a), a FLIR A645SC camera, with 640×480 pixels, was used along with the Thermacam FLIR ResearchIR software package to obtain and analyze the thermographic images of the plate. This camera is capable of measurements under $20\mu m$. The software calibration approximates variables such as heat losses between the measurement surface and the camera, ambient temperature, among others.

After two minutes of recording the temperature measurements of the plate surface, which is close to the ambient temperature, the power source is turned on in order to create a heat flux. For this purpose three parameters, i. e. electric current, voltage and electrical resistance, are set as $0.2A$, $9V$ and 39.9Ω, respectively.



(a) (b)

Figure 3. Experimental setup.

It is possible to make an analysis of the consistence of the measurements acquired by the camera and processed by the software, as shown in Fig. 3(b). Points were chosen in the center of the plate, in the center of the resistance and over the borders of the plate, in order to make such preliminary analysis. Then the software convert the raw data into a set of matrices, each one representing a frame in the video, to be used as the input for further analyses in MATLAB.

2.3 Inverse Problem

The formulation of the inverse problem is made taking into account the experimental data, being provided by the camera a total of $N_x \times N_y \times N_t = D$ measurements, corresponding to $N_x \times N_y$ pixels and N_t time instants. The experimental data vector can be expressed as

$$\mathbf{Y}^T = (Y_1, Y_2, \ldots, Y_D) \quad (4)$$

where Y_i, $i = 1, 2, \ldots, D$, corresponds to the measurement at each mesh position (pixel) at a given moment in time.

A mesh with $IP_x \times IP_y$ nodes and IP_t steps in time, where $IP_x < N_x$, $IP_y < N_y$ and $IP_t < N_t$, is used to represent the heat flux $q(x, y, t)$, leading to a total of $DP = IP_x \times IP_y \times IP_t$ parameters to be estimated. Notice that $DP < D$. In principle, the experimental and numerical meshes doesn't have to have the same amount of nodes, and so, a interpolation takes place in order to compare the experimental data with the estimated values. The vector of estimates for the sought parameters can be described as

$$\mathbf{P}^T = (q_1, q_2, \ldots, q_{DP}) \tag{5}$$

A Bayesian approach, specifically the Markov Chain Monte Carlo method (MCMC), is used to obtain the solution of the inverse problem. This choice is made to the extent that it makes it possible to incorporate prior information in the calculation of the estimates by means of the Bayes' theorem, as will be shown next. This characteristic is fundamental for the regularization of large inverse problems, in which many parameters are estimated. The solution for the sought inverse problem consists in determining the posterior probability density, expressed by

$$\pi_{post}(\mathbf{P}) = \pi(\mathbf{P}|\mathbf{Y}) = \frac{\pi_{prior}(\mathbf{P})\pi(\mathbf{Y}|\mathbf{P})}{\pi(\mathbf{Y})} \tag{6}$$

where $\pi_{post}(\mathbf{P})$ represents the posterior probability distribution of the parameters $\mathbf{P}$, $\pi_{prior}(\mathbf{P})$ the probability distribution of prior data, $\pi(\mathbf{Y})$ the marginal distribution of probability of the experimental data, which plays the role of a normalization constant, and $\pi(\mathbf{Y}|\mathbf{P})$, the likelihood function, expressed analytically in the form

$$\pi(\mathbf{Y}|\mathbf{P}) = (2\pi)^{-D/2}\mathbf{W}^{-1/2} \times \exp\left[-\frac{1}{2}[\mathbf{Y} - T(\mathbf{P})]^t\mathbf{W}^{-1}[\mathbf{Y} - T(\mathbf{P})]\right] \tag{7}$$

where $\mathbf{W}$ is the covariance matrix of the experimental errors, and $T(\mathbf{P})$ is the vector containing the solution of the direct problem given the values of $\mathbf{P}$, in the same positions and instants in time in which the experimental measurements are obtained.

The implementation of the MCMC is performed by means of the Metropolis-Hastings algorithm, described in detail by Kaipio and Somersalo (2008), to sample the candidates for the posterior distribution. The implementation starts with the selection of an auxiliary probability density of motion $p(\mathbf{P}^*,\ \mathbf{P}(t-1))$, which is used to generate a candidate $\mathbf{P}^*$ for the new state of the chain, given the previous state $\mathbf{P}(t-1)$, thus creating the sequence $\mathbf{P}^1, \mathbf{P}^2, \ldots, \mathbf{P}_n$, which is expected to converge to the sought solution.

Regularization is an important step in solving inverse problems, especially when there are many parameters to be estimated. In order to model the prior information, Markov Random Fields (MRF) approaches are employed, specifically the Total Variation (TV) model in space and time, described as (Kaipio and Somersalo, 2008)

$$\pi(\mathbf{P}) \propto \exp\left[-\frac{\gamma}{2}TV(\mathbf{P})\right] \tag{8}$$

being γ the regularization parameter and, for the present case,

$$\begin{aligned} TV(\mathbf{P}) \;=\; & \sum_{i=2}^{IP_x-1}\sum_{j=2}^{IP_y-1}\sum_{k=2}^{IP_t-1} \Delta x\left[\,\left|q_{i,j}^k - q_{i+1,j}^k\right| + \left|q_{i,j}^k - q_{i-1,j}^k\right|\,\right] + \\ & + \;\Delta y\left[\,\left|q_{i,j}^k - q_{i,j+1}^k\right| + \left|q_{i,j}^k - q_{i,j-1}^k\right|\,\right] + \Delta t\left[\,\left|q_{i,j}^k - q_{i,j}^{k+1}\right| + \left|q_{i,j}^k - q_{i,j}^{k-1}\right|\,\right] \end{aligned} \tag{9}$$

where Δx, Δy and Δt represent the discretization in space and time of the unknown heat flux we want to estimate.

3. RESULTS AND DISCUSSION

It is considered the case of a rectangular plate, with dimensions $0.08m \times 0.04m \times 0.002m$. The simulation was seen for total time of 300 seconds for the heat transfer experiment. The mesh for the flux estimation has $20 \times 20 \times 50$ nodes, which leads to a large amount of 20000 parameters to be estimated, i. e. a hard task. The experimental data produced a Gaussian distribution with mean $26.0270\,^oC$, taken as T_∞, and standard deviation $\sigma = 0.0639\,^oC$. The convective heat transfer coefficient chosen was $h = 15.8\,W/mK$, choice based in a previous analysis, as described in Neves (2019). 2×10^6 states were calculated in the Markov chain, with the first 900.000 being discarded as the burn-in phase. The acceptance factor was about 25%. The TV prior regularization parameters are all set as $\gamma_{tvx} = \gamma_{tvy} = \gamma_{tvt} = 0,01$.

For the analysis below, it was considered a formulation for the expected heat flux experimentally applied over the plate surface, as follows:

$$q_{exp} = \frac{Ri^2}{2q_{lx}q_{ly}} \tag{10}$$

being q_{lx} and q_{ly} the dimensions of the resistance used in order to create the heat flux.

In Figs. 4(a)-(c) it is possible to see the superimposed profiles of cuts of the expected and estimated heat flux (with no prior (NP) and prior (P) use for the heat transfer experiment), respectively, in the directions x, y and over time. It is possible to notice that the regularized thermal flux approximates the expected thermal flux profiles in all cuts. Figures 5(a)-(c) show the relative error calculated between solutions of the inverse problem solved with the use of prior information, and with no use of prior information, and the expected flux, once again, respectively, in the directions x, y and over time. In all cases, it is possible to notice a decrease in the error when the prior information is used.

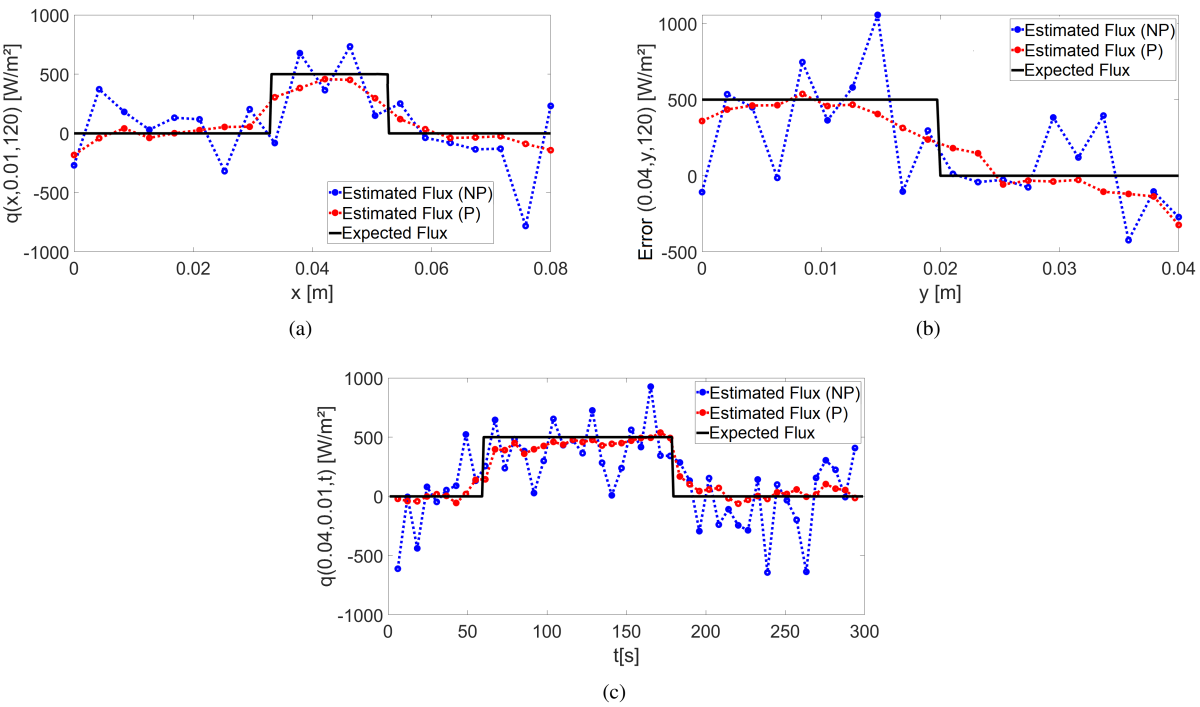


Figure 4. Comparison between estimated and expected flux in directions x, with $y = 0.01\,m$ and $t = 150\,s$ (a), y, with $x = 0.04\,m$ and $t = 150\,s$ (b) and time, with $x = 0.04\,m$ and $y = 0.01\,m$ (c).

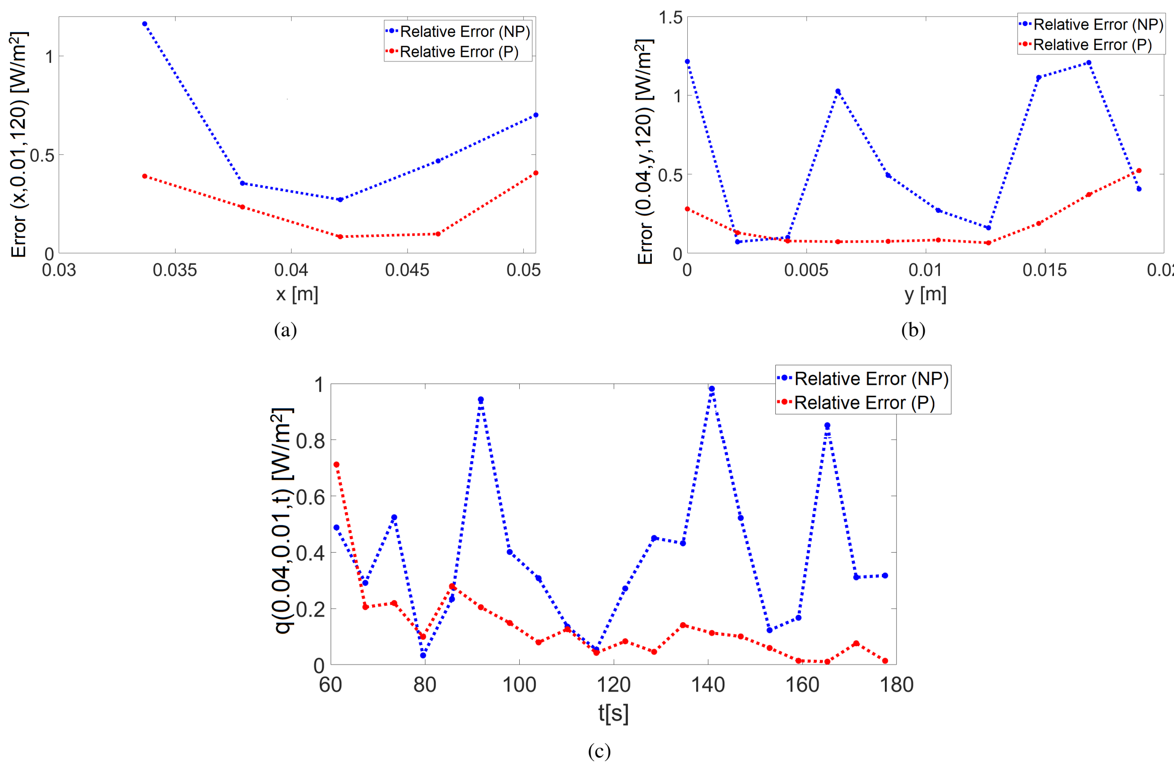
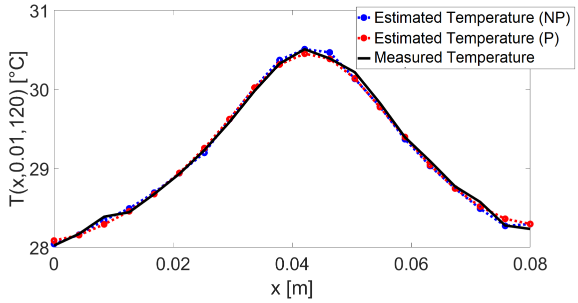
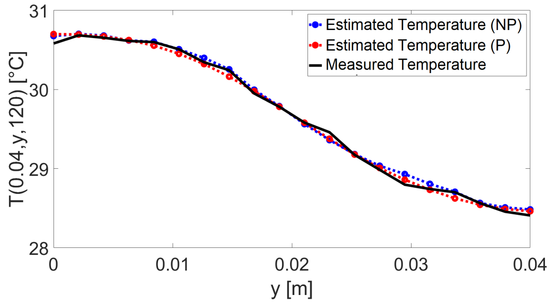


Figure 5. Relative error calculated in directions x, with $y = 0.01\,m$ and $t = 150\,s$ (a), y, with $x = 0.04\,m$ and $t = 150\,s$ (b) and time, with $x = 0.04\,m$ and $y = 0.01\,m$ (c).

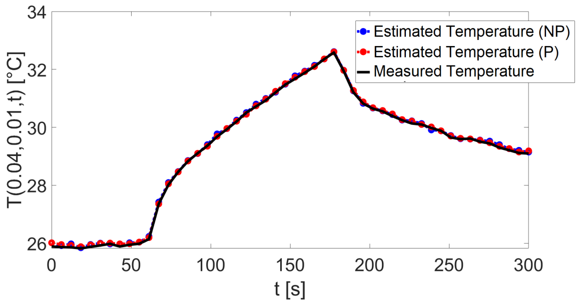
A comparison between the results found with use and no use of prior information shows good match witch the experimental data, as one can see in Fig. 6 along the x direction (Fig. 6(a)), the y direction (Fig. 6(b)) and over time (Fig. 6(c)).



(a)



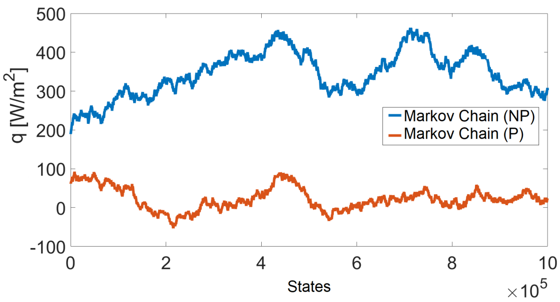
(b)



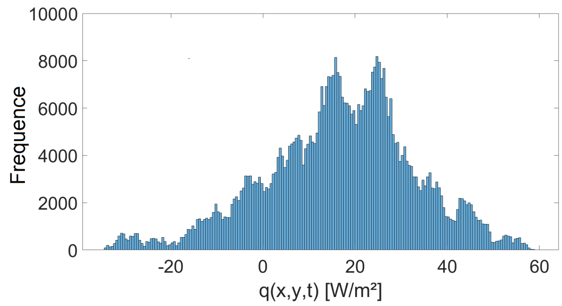
(c)

Figure 6. Comparison between estimated and measured temperature in directions x, with $y = 0.01\,m$ and $t = 150\,s$ (a), y, with $x = 0.04\,m$ and $t = 150\,s$ (b) and time, with $x = 0.04\,m$ and $y = 0.01\,m$ (c).

The convergence of the Markov chain at a point where $q = 0\,W/m^2$ is shown in Fig. 7(a) below. The Markov chain of the inverse problem solved with no prior information shows no convergence to the value expected. On the other hand, the Markov chain of the inverse problem solver with prior information shows a convergence that seems to tends to a value close to the expected one. This perception is strengthened by the analysis of the histogram shown in Fig. 7(b), that demonstrates values of the Markov chain appearing with higher frequencies close to $q = 0\,W/m^2$.



(a)



(b)

Figure 7. Markov chain evolution to a point in mesh where $q = 0\,W/m^2$ (a), histogram for the same point (b).

The expression for the calculated RMS errors are:

$$RMS_T = \sqrt{\frac{1}{N_x \times N_y \times N_t} \sum_{i=1}^{N_x} \sum_{j=1}^{N_y} \sum_{k=1}^{N_t} [T(x_i, y_j, t_k) - T_{ij}^k]^2} \tag{11}$$

$$RMS_q = \sqrt{\frac{1}{N_x \times N_y \times N_t} \sum_{i=1}^{N_x} \sum_{j=1}^{N_y} \sum_{k=1}^{N_t} [q(x_i, y_j, t_k) - q_{ij}^k]^2} \tag{12}$$

Observing the calculated errors, shown in Tab. 2 for each case, it is clear how the modeling of priors in the form of total variation fulfills the role of regularizing the problem in a satisfactory way, producing an 56.24% decrease in calculated errors.

Table 2. Calculated error for values of estimated fluxes and calculated temperatures.

Case	RMS_q $[W/m^2]$	RMS_T $[^oC]$
No Prior	9.8078	0.0022
Prior	4.2915	0.0021

4. CONCLUSIONS

In this study, an effective approach was presented to solve the problem of reconstruction of a heat flux with two-dimensional spatial and temporal variation, using, for its formulation and purpose, real experimental data measured by a infrared thermography camera. The studied regularization technique, Total Variation, showed good results, producing a 56.24% decrease in calculated errors if compared with the solution computed with the no regularization technique.

5. ACKNOWLEDGMENTS

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