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SLIDING MODES CONTROL OF AN ELECTROHYDRAULIC ACTUATOR USING COMPENSATION BY GAUSSIAN PROCESS

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Abstract. *The development of precise control systems for electrohydraulic actuators (EHA) depends on adequate compensation of the unknown dynamic effects. In this work, a Sliding Modes controller is combined with a Gaussian Process compensator to provide adequate trajectory tracking. Gaussian process is a well-known machine learning strategy that can be used in the recognition of functions. The closed-loop convergence properties of the system are analyzed by the Lyapunov Stability Theory. Numerical results confirm a strong improvement in the performance of the controller when the proposed compensator is inserted.*

Keywords: *control of mechanical and robotic systems, electrohydraulic actuators, sliding modes control, gaussian process regression, machine learning.*

1. INTRODUCTION

Electrohydraulic actuators (EHA) have been used in many applications because they can not only apply high forces and torques, but also have a high power-to-weight ratio (Sun and Chiu, 1999; Liu and Alleyne, 1999; Yim *et al.*, 2014). These devices can either be incorporated into automotive systems (Balau *et al.*, 2011) or applied to exoesquelestos (Yang *et al.*, 2018) or aerospace (Altare and Vacca, 2015).

Due to the EHA mentioned advantages, in applications that require high precision operation it is necessary to develop a controller capable of providing a good performance. However, the EHA has a high degree of non-linearity due to the fluid compressibility effects, valve actuation, friction, saturation, dead zone, among others (Yim *et al.*, 2014,?). In relation to the controllers already implemented for electrohydraulic systems the literature presents several examples with control techniques and compensators to deal with the nonlinearities present in the actuators. (Bonchis *et al.*, 2002) experimentally compared several control techniques, including Proportional-Derivative (PD) and predictive control. (Bessa *et al.*, 2010) used Fuzzy Logic as an adaptive strategy within a Sliding Modes Controller (SMC) to estimate the uncertainties due to the effect of an unknown dead zone on the control effort. (Skarpetis and Koumboulis, 2013) applied a robust strategy to a Proportional-Integral-Derivative (PID) controller. (Liem *et al.*, 2016) also used PID as an ally controller this time with Neural Networks in a *feedforward* context to improve controller performance.

In this work, a new type of compensator is presented inside a robust controller based on the *Gaussian Process* (GP), that comes within the functions regression problem context, in the case of unknown system dynamics and the perturbations.

The GP has been applied in the most diverse situations. For example, within machine learning, (Doerr *et al.*, 2017) has used GP to learn the system dynamics by applying it to the robustness improvement problem of a *learning-by-reinforcement* based model. (Xiloyannis *et al.*, 2017) has also used GP to replace the relationship between muscle activity and a hand intended movement by an approach that takes into account previous movements and instantaneous hand muscular action to predict future movements. Already GPR has been applied to model the rigid body dynamics using as input data the Euler angles (Lang and Hirche, 2017). (Doerr *et al.*, 2018) shows how GPR can be adopted as a non-parametric model to estimate plant uncertainties. Finally, GPR with Robust Sliding Control law has already been used for the design of a diesel engine controller (Aran and Unel, 2018). However, unlike the methodology of this last work, here the training of the GP will be done *online*.

The robust SMC is proposed in this paper. The stability analysis of this controller will be done by the Lyapunov Theory. Numerical simulations with the implementation of this SMC controller with GPR compensation will be made to a model of an electrohydraulic actuator in order to verify the proposed controller performance.

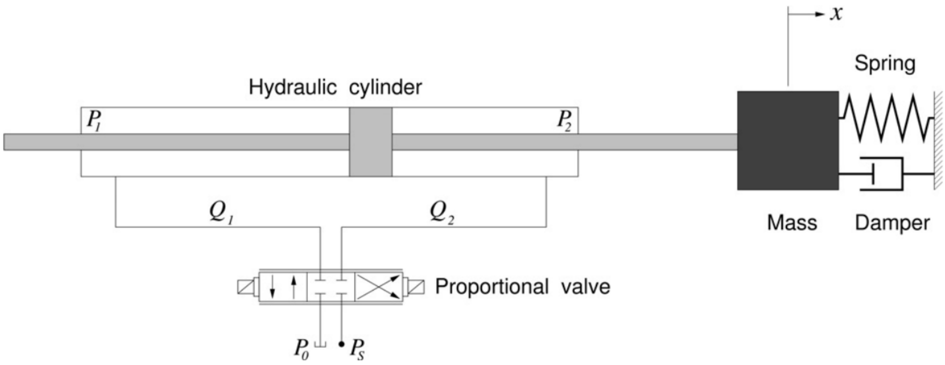


Figure 1: Schematic diagram of the EHA system.

2. SYSTEM MODEL

The system considered consists in a hydraulic cylinder, a four-way proportional valve and a mass-spring-damper system to represent the submitted variable load. The methodology used to determine the system dynamics expression and all the considerations involved is presented by (Bessa *et al.*, 2010). Therefore the third order differential equation that represents the EHA dynamics is:

$$\dddot{x} = -\boldsymbol{a}^\top \boldsymbol{x} + b(\boldsymbol{x}, u)[u - d(u)] \tag{1}$$

where $\boldsymbol{x} = [x, \dot{x}, \ddot{x}]^\top$ is the states vector whose coefficients $\boldsymbol{a} = [a_0, a_1, a_2]^\top$ are given by:

$$a_0 = \frac{4\beta_e C_{tp} K}{V_t M} \tag{2a}$$

$$a_1 = \frac{K}{M} + \frac{4\beta_e A_p^2}{V_t M} + \frac{4\beta_e C_{tp} B}{V_t M} \tag{2b}$$

$$a_2 = \frac{B}{M} + \frac{4\beta_e C_{tp}}{V_t} \tag{2c}$$

and,

$$b(\boldsymbol{x}, u) = \frac{4\beta_e A_p}{V_t M} C_d \omega k_v(u) \times \sqrt{\frac{1}{\rho}\left[P_s - \frac{\operatorname{sgn}(u)(M\ddot{x} + B\dot{x} + Kx)}{A_p}\right]} \tag{3}$$

3. GAUSSIAN PROCESS

A GP can be defined as a probability distribution over the function space h since a finite set of input and output, that is, $(s, h(s))$, have a joint Gaussian distribution Williams and Rasmussen (2006). Because it is a probability distribution, a GP is specified through its mean mu and covariance functions k *a priori*:

$$h(s) \quad \sim \quad \mathcal{GP}(\mu(s), k(s, s')) \tag{4}$$

where, the mean and variance are defined as,

$$\mu(s) = \mathbb{E}[h(s)] \tag{5a}$$

$$\begin{aligned} k(s, s') &= \mathbb{V}(s, s') \\ &= \mathbb{E}[(h(s) - \mu(s))(h(s') - \mu(s'))] \end{aligned} \tag{5b}$$

GPR is to establish a non-parametric model to estimate functions as long as they can be described through a GP. For this, consider that the function h to be estimated by GPR can be represented in the following form:

$$\bar{h}(s) = h(s) + \varepsilon, \qquad \varepsilon \sim \mathcal{N}(0, \sigma_\epsilon^2) \tag{6}$$

whose noise ε follows a normal distribution with null mean and variance σ_ϵ^2.

The process of learning by GPR consists of estimating the value of the function $h^* = h(s^*)$ for arbitrary entries s^* based on the training set. Lima *et al.* (2018) and Doerr *et al.* (2018) suggest that both the mean and the a *posteriori* variance are calculated based on the following equations:

$$\mathbb{E}(h^* \mid \mathcal{D}_N) = \mu(s^*) + k(s^*, s)(\boldsymbol{K}_N + \sigma_\epsilon^2 \boldsymbol{I})^{-1}\tilde{h}(s) \tag{7a}$$

$$\mathbb{V}(h^* \mid \mathcal{D}_N) = k(s^*, s^*) - k(s^*, s)(\boldsymbol{K}_N + \sigma_\epsilon^2 \boldsymbol{I})^{-1}k(s, s^*) \tag{7b}$$

where $\boldsymbol{K}_N$ is defined by

$$\boldsymbol{K}_N = \begin{bmatrix} k(s_1, s'_1) & \dots & k(s_1, s'_N) \\ \vdots & \ddots & \vdots \\ k(s_N, s'_1) & \dots & k(s_N, s'_N) \end{bmatrix} \tag{8}$$

and $\tilde{h}(s) = [\bar{h}(s_1) - \mu(s_1), \bar{h}(s_2) - \mu(s_2), \dots, \bar{h}(s_N) - \mu(s_N)]^\top$. $\bar{h}(s_{i-1})$ is the output related to the input s_{i-1}, which belongs to the training set and is used to calculate the *posteriori* mean at the i instant.

4. CONTROL

Since the EHA model is given by a 3rd order system, then consider that this model can be generically described as follows:

$$\dddot{x}(t) = f(\boldsymbol{x}, t) + b(\boldsymbol{x}, t)u(t) + \xi \tag{9}$$

where $\dddot{x}$ represents a n-order derivative in time of the state variable x and u the input variable, the functions $f, b : \mathbb{R}^3 \to \mathbb{R}$ are non-linear functions and ξ is a term that groups the occasional disturbances that may affect the system including the dead zone effect.

By the SMC control method, a *sliding* surface is able to map $s : \mathbb{R}^3 \to \mathbb{R}$,

$$s(\boldsymbol{x}, t) = \ddot{e} + 2\lambda\dot{e} + \lambda^2 e \tag{10}$$

where λ is a strictly positive constant and $e = x - x_d$ is the tracking error.

Considering that the f and b functions can not be precisely determined, that is to say, $f = \hat{f} + \Delta f$ and $b = \hat{b} + \Delta b$, where $\hat{f}$ and $\hat{b}$ are f and b estimations, Δf and Δb are the each function estimate errors, the system shown by the Eq. (9) may be reformulated as follows:

$$\dddot{x} = \hat{f} + \hat{b}u + d \tag{11}$$

where d is a function that groups the system disturbances and the functions f e b associated uncertainties, $d = \Delta f + \Delta bu + \xi$. In this paper this is called term only disturbance.

Consideration 1. The disturbance d is an unknown function, but limited $|d| \leq \delta$.

The system control law may then be written as follows, using the saturation function to avoid the *chattering* effect on the system:

$$u = \hat{b}^{-1}\left(-\hat{f} - \hat{d} + \dddot{x}_d - 2\lambda\ddot{e} - \lambda^2\dot{e} - \kappa \text{sat}\left(s/\phi\right)\right) \tag{12}$$

where $\hat{d}$ is the disturbance estimate by the GPR and κ is a term that ensures that the system reaches the sliding surface.

Observation 1. Although Consideration 1 does not formally consider that the GP, because it is a probability distribution, has, in theory, infinite extent, in real cases unlimited perturbations usually does not occur. Because of this, it is possible to establish an interval where the controller robustness is guaranteed.

Consideration 2. The disturbance limit δ corresponds to a 95% confidence interval obtained by GP, that is $\delta = |\hat{d}| + 2\sigma_d$, onde $\sigma_d^2 = \mathbb{V}(h^* \mid \mathcal{D}_N)$.

By Lyapunov stability theory it is possible to demonstrate the system convergence properties in closed loop. Therefore, the controller gain $\kappa \geq \eta + |\hat{d}| + \delta$, it is possible to ensure, by applying Lyapunov stability theory

$$\dot{V}(t) \leq -\eta|s_\phi| \tag{13}$$

that implies $V(t) \leq V(0)$ and s_ϕ is limited. Thus, once in the boundary layer $s \leq \phi$. So it is possible to demonstrate that the tracking error converges to a closed region deefined by $\Phi = \left\{\boldsymbol{e} \in \mathbb{R}^3 \mid |e^{(i)}| \leq (i+1)!\lambda^{i-2}\phi,\ i = 0,\ 1,\ 2\right\}$.

Finally, the algorithm 1 summarizes the controller proposed in this work.

Algoritmo 1: SMC controller with GPR compensation.

Set control parameters
Specify *a priori* GP (mean μ, covariance k)
Set initial states: $\boldsymbol{x}_0$
Initialize dataset: $\mathcal{D}_0$
while $t \leq t_{sim}$ **do**
    Calculate desired trajectory: $\boldsymbol{x}_{\mathrm{d}}$
    Calculate tracking error: $\boldsymbol{e}$
    $s \leftarrow \ddot{e} + 2\lambda\dot{e} + \lambda^2 e$
    $\hat{d} \leftarrow \mathbb{E}[h(s^*)|\mathcal{D}_{i-1}]$
    $\sigma_d^2 \leftarrow \mathbb{V}[h(s^*)|\mathcal{D}_{i-1}]$
    $\delta \leftarrow |\hat{d}| + 2\sigma_d$
    $\kappa \leftarrow \eta + |\hat{d}| + \delta$
    $u \leftarrow \hat{b}^{-1}\big(-\hat{f} - \hat{d} + \dddot{x}_d - 2\lambda\ddot{e} - \lambda^2\dot{e} - \kappa\mathrm{sat}\,(s/\phi)\big)$
    Apply u in the system
    Update states: $\boldsymbol{x}$
    Update *posterior* GP: $\mathbb{E}[h(s^*)|\mathcal{D}_i],\ \mathbb{V}[h(s^*)|\mathcal{D}_i]$
end

5. NUMERICAL RESULTS

The programming language used was C++ with 100 Hz and 500 Hz sampling rate for the controller and the simulator respectively. The 4th-order Runge-Kutta was implemented to solve numerically the system model. The EHA system parameters used were $P_s = 7$ MPa, $\rho = 850$ kg/m^3, $C_d = 0.6$, $\omega = 2.5 \cdot 10^{-2}$ m, $A_p = 1.1 \cdot 10^{-3}$ m^2, $C_{tp} = 2 \cdot 10^{-12}$ m^3/(s.Pa), $\beta_e = 700$ MPa, $V_t = 4.4 \cdot 10^{-4}$ m^3, $M_t = 50$ kg, $B = 100$ Ns/m, $K = 75$ N/m, $k_l = 1.8 \cdot 10^{-6}$ m/V, $k_r = 2.2 \cdot 10^{-6}$ m/V, $\delta_l = -1.1$ V e $\delta_r = 0.9$ V.

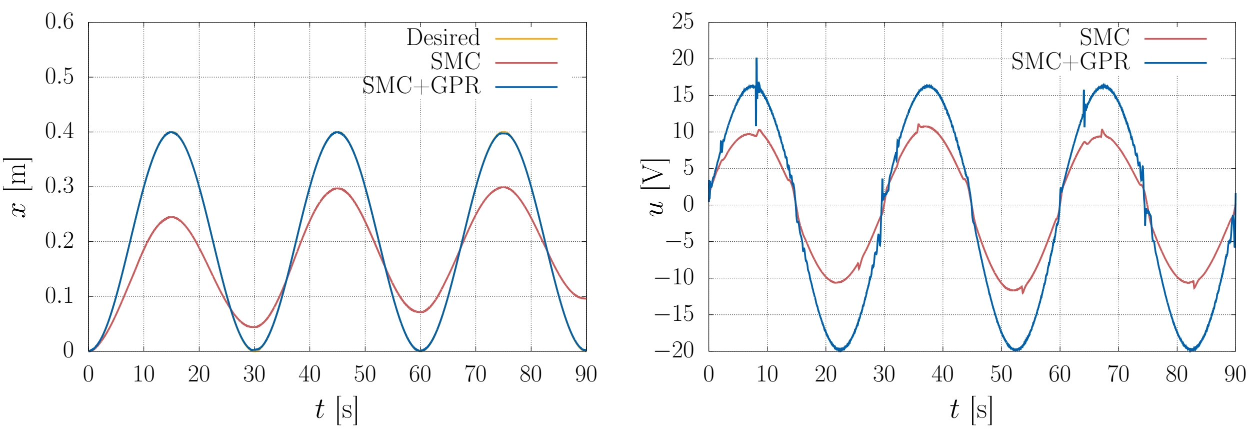
The controller parameters were $\lambda = 25$, $\phi = 0.8$ e $\eta = 0.5$. The b term estimate was $\hat{b} = 1.0$ and also was considered that there was no previous knowledge about f term, that means $\hat{f} = 0$.

The distance to the sliding surface was used as GPR input s and the a priori mean was null ($\mu(s) = 0$). As done by (Lima *et al.*, 2018), the Radial Basis Function kernel was used as covariance function defined bellow:

$$k(s, s') = \sigma_f^2 \exp\left(-\frac{\| s - s' \|^2}{2\gamma^2}\right) \tag{14}$$

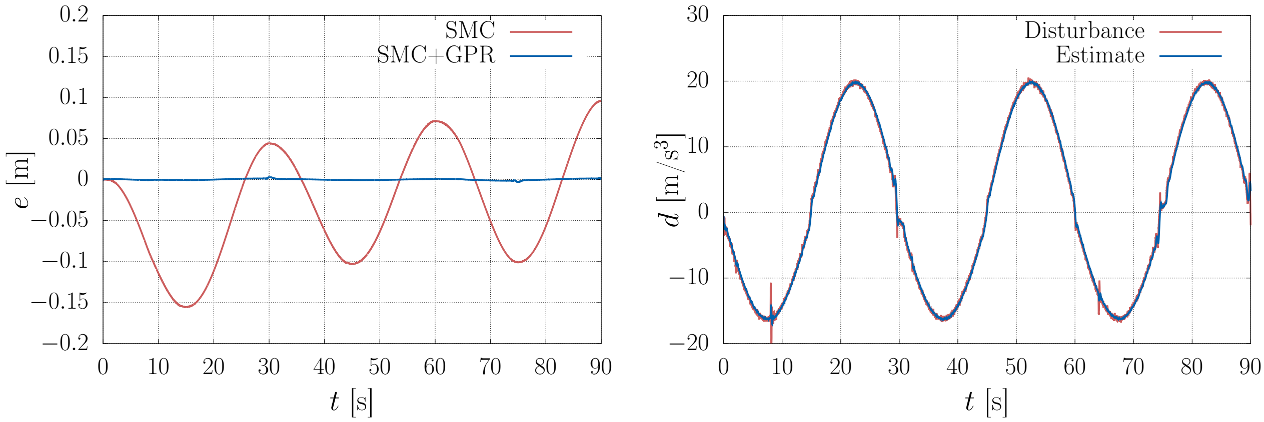
where σ_f^2 e γ are hyper-parameters. The considered parameters were $\sigma_\epsilon = 0.6$, $\sigma_f = 3.0$ e $\gamma = 50$. The trajectory to be tracked by the actuator was $x_d = 0.2[1 - \cos(\pi t/15)]$ for $t_{\text{sim}} = 90$ s.

It was considered that only the position of the actuator could be measured, so the other states were determined by means of a (Levant, 2003) second-order Sliding Modes differentiator.



(a) Trajectory tracking (b) Control effort.

Figure 2: Trajectory tracking with the SMC and SMC+GPR approaches.



(a) Tracking error (b) Disturbance and GPR.

Figure 3: SMC and SMC+GPR approaches tracking errors and disturbance.

As can be seen in Fig. 2a and Fig 3a, the proposed control (SMC + GPR) for the electrohydraulic actuator was able to improve trajectory tracking with small tracking error compared to the conventional controller). For this, the inclusion of the GP compensator increased the control effort, Fig. 2b, with an excellent performance in estimating the disturbance, Fig. 3b.

6. CONCLUSIONS

In this paper, a sliding mode controller is combined with a Gaussian Process regression function to perform trajectory tracking of an electrohydraulic actuator. GPR was used as a non-parametric model to compensate for the system unmodulated dynamics. The Lyapunov Stability Theorem was used to demonstrate the convergence properties of the closed-loop system. Through numerical simulations, a significant improvement in controller performance compared to the conventional approach (SMC) has been demonstrated.

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