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INVESTIGATION OF THE BANDGAP EFFECT IN LOCALLY RESONANT METASTRUCTURES

Daniely Amorim das Neves
Pedro Henrique de Melo Casado Matos
Adriano Todorovic Fabro
Departament of Mechanical Engineering, University of Brasilia, Brasília, DF, 70910900
e-mails d.amorim.neves2@gmail.com, pedrohcasado94@gmail.com and fabro@unb.br

Abstract. *Metamaterials offer excellent flexibility in material design and bring a new perspective on understanding materials, defined as an arrangement of artificial structural elements, designed to achieve advantageous and unusual properties. They stand out in the control of noise and vibration given its feature of vibration attenuation for certain bands of frequency, called bandgap effect. Parameters such as wavenumber, group velocity, modal density and dissipated power characterize its dynamic performance and can be analyzed by methods based on wave propagation. Experimental techniques for modal analysis are well established and have been successfully applied in recent decades, unlike experimental approaches to wave propagation. This work aims to investigate the effect associated with the attenuation of vibrations in metamaterial beam and to apply the correlation method to the experimental data to identify the wavenumber in beam subjected to flexural vibration. The underlying experimental setup consists of measuring frequency response functions by an impact hammer test from a free-free beams at evenly spaced points. Results are compared to an available analytical solution and showed good agreement.*

Keywords: *Metamaterials, Bandgap, Wavenumbers.*

1. INTRODUCTION

Metamaterials are an arrangements of artificial structural elements designed to obtain advantageous and unusual properties not found in conventional materials (Ruepeng Liu et al., 2015). They were introduced initially by photonic crystals in electromagnetism, where it presents properties of directing and/or manipulating electromagnetic waves (Adriano Fabro et al., 2016). In acoustic metamaterials, when the local resonant microstructures vibrate at the same frequency, a phenomenon known as a stop band appears, in which it prohibits the propagation of elastic waves within the bandgap frequency band. Their applications involve technologies such as electromagnetic camouflage, in antennas to increase the range of radars, absorbers of specific frequency bands.

Periodic structures can be designed to display bandgap behavior by varying the material properties, geometric characteristics, boundary conditions and addition of periodically distributed resonators to the main axis (Pai, 2010). Usually wave propagation approaches are being used to describe the dynamic behavior of different structures in terms of dispersion curves, power, wave modes, and transmission and reflection coefficients. However, metamaterials have several applications in passive noise and vibration control through bandgap (Danilo Beli, 2019).

There are many experimental technics for this dynamical analysis succeeded in the literature including Discrete Fourier Transform (DFT) (Arruda, 2010), Inhomogeneous Wave Correlation (IWC) (Niel Ferguson, 2002) (Ichchou et al. 2008) and Prony's series (Gregory McDaniel, 2000), and most concentrated on obtaining the dispersion curve for this analysis. By the dispersion curve, the wavenumber can easily be obtained and through its analysis the characteristics of the propagation wave, energy transfer and vicoelastic properties of the structure. The focus of this paper is the experimental investigation the flexural wavenumbers and wave amplitudes in a beams from response measurements. Applications can be found in literature with sandwich beams (Niel Ferguson et al. 2002), periodically undulated beams (Trainiti et al. 2015), metamaterial beams (Filippo Casadei and Katia Bertoldi, 2014) and locally resonant acoustic metamaterials (Lewinska et al., 2017) using wave propagation for to describe local properties, mostly using analytical and numerical models.

There are many problems in applying experimental methods, mostly related to noise, nonlinearities, equipment calibration, evanescent waves, truncation, frequency and space resolutions causing distortion in acquired signals. In this work, the dynamic response is obtained by transductores equaly spaced along the beam for measuring transversal acceleration while a modal hammer is used to excited it (Michal Kalkowski et al. 2017). The acquired data is treated by classical FRF (*Frequency Response Function*) estimation approaches (H_1 and H_2) and the ordinary coherence function indicating the quality of the estimation. Since the beam has finite length, there will be wave overlap, especially at low frequency and in the contour regions, but at such frequency there will be a dominant response where the wavenumbers will be relatively easy to measure. And especially in metamaterials the periodic locally resonant structures contribute to the rise of unique wave dispersive behavior.

The objective of this work is to investigate experimentally the effect associated to the attenuation of vibrations in metamaterials produced from 3D printing. The attention is focused on the mechanism that prevents harmonic waves from propagating in the metamaterial when the frequency of the waves is close to the local resonance frequency. This work investigates experimentally the characteristics of the waves in two profiles of metamaterial beam, in I and rectangular profile, using the correlation method to estimate wavenumber parameters and are compared to analytical models. The attenuation performance is compared with beams with no resonators attached and the experimental limitations are discussed.

In the next section, the dispersion equation of a beam with resonators is described. In sec. 3, the experimental method of obtaining the wavenumber is described. In sec. 4 the beams used and the experimental methodology are presented. These experimental data are compared to the analytical ones and the results discussed in Sec. 5. And in Sec. 6 a brief conclusion is presented.

2. DISPERSION EQUATION THE BEAM WITH RESONATORS

Considering a continuous system with resonators periodically added in the structure, as shown in Fig. 1, where each unit cell (n) presents interface (n-1), hence, the metastructure has 15 unit cells. It is possible to determine the wavenumber equation by wave propagation analysis (Surgino et al., 2017).

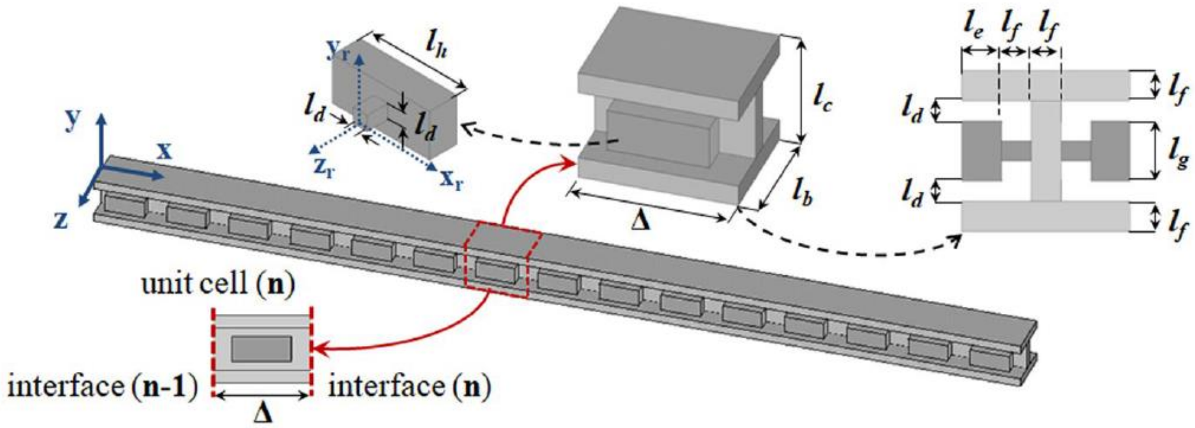


Figure 1. Metamaterial beam and unit cell with the nominal geometric dimensions (Beli, et al., 2019).

The general differential equation governing the vibration *u (x, t)* of the distributed parameter under force *F (x, t)* has the following form,

$$EI\,\frac{\partial^4 u(x,t)}{\partial x^4} + \mu\frac{\partial^2 u(x,t)}{\partial t^2} - \sum_{p=1}^{S} k_p u_p(t)\delta(x - x_p) = F(x,t), \quad (1)$$

where $\mu = \rho A$ *is* density per unit length, *S* the total number of resonators and $u_p\ (t)$ is the displacement of the resonator, $\delta(x - x_p)$ is the Dirac's delta function, representing the point attachment of the *p*-th resonator at x_p position with stiffness k_p. In addition, considering the relative motion of the resonator mass to its point of attachment, the equation of motion for the *p*-th resonator is,

$$m_p\ddot{u}_p(t) + k_p u_p(t) + m_p\ddot{u}(x_p, t) = 0, \quad P = 1, 2, \ldots, S, \quad (2)$$

where m_p is the mass of the *p*-th resonator. Assuming time harmonic motion of the resonators $u_p(t) = U_p e^{i\omega t}$ and the host beam $u(x,t) = U(x)e^{i\omega t}$ then,

$$m_p U_p (i\omega)^2 e^{i\omega t} + k_p U_p e^{i\omega t} + m_p U(x_p)(i\omega)^2 e^{i\omega t} = 0, \quad (3)$$

where $\omega_p^2 = k_p/m_p$ is the resonance frequency of the *p*-th resonator and the amplitude transmissibility of the resonators to the host beam is obtained

$$U_p = \frac{\omega^2}{\omega_p^2 - \omega^2} U(x_p), \quad (4)$$

which can then be replaced back in Eq. (1), yielding,

$$EI\frac{\partial^4 u(x,t)}{\partial x^4} + \mu\frac{\partial^2 u(x,t)}{\partial t^2} - \sum_{p=1}^{S} k_p U_p e^{i\omega t}\delta(x - x_p) = F(x,t). \quad (5)$$

Given that $m_p = \epsilon\mu\Delta l$ where ϵ is the ratio between the total mass of the beam and the resonators and Δl is the spacing between resonators, replacing the relation *Up* from Eq. (4) into Eq. (5) yields

$$EI\frac{\mathrm{d}^4U(x)}{\mathrm{d}x^4} - \omega^2\mu U(x) - \epsilon\mu\omega^2\sum_{p=1}^{s} m_p\frac{\omega_p^2}{\omega_p^2-\omega^2}U(x_p)\delta(x-x_p)\Delta l = F(x,t). \tag{6}$$

Assuming identical resonators $\omega_p = \omega_r$,a sufficiently large number of resonators such that $S \to \infty$, then the summation term turns into a integral in the Riemman sense, which can be straightforwardly evaluated due to the Dirac's delta function. Finally, assuming space-harmonic movement, i.e. $U(x) = Ae^{ikx}$, then

$$EIk^4 - \mu\omega^2 - \epsilon\mu\omega^2\frac{\omega_r^2}{\omega_r^2-\omega^2} = 0. \tag{7}$$

Considering free waves, it is possible to obtain the dispersion relation of a beam with resonators,

$$k = \sqrt[4]{\frac{\omega^2\mu}{EI}\left(1+\epsilon\frac{\omega_r^2}{\omega_r^2-\omega^2}\right)}. \tag{8}$$

3. IDENTIFICATION OF WAVENUMBER BY CORRELATION METHOD

Also called Inhomogeneous Wave Correlation (IWC), this method is most commonly used to estimate wavenumber in one-dimensional and two-dimensional structures (Ichchou et al. 2008). Considering the propagation of harmonic waves $\hat{w}(x)$ in a structure, whose *Fourier Transform* is given by,

$$w(x,\omega) = \int_0^{+\infty}\hat{w}(x)e^{i\omega t}d\omega. \tag{9}$$

The correlation coefficient defined by,

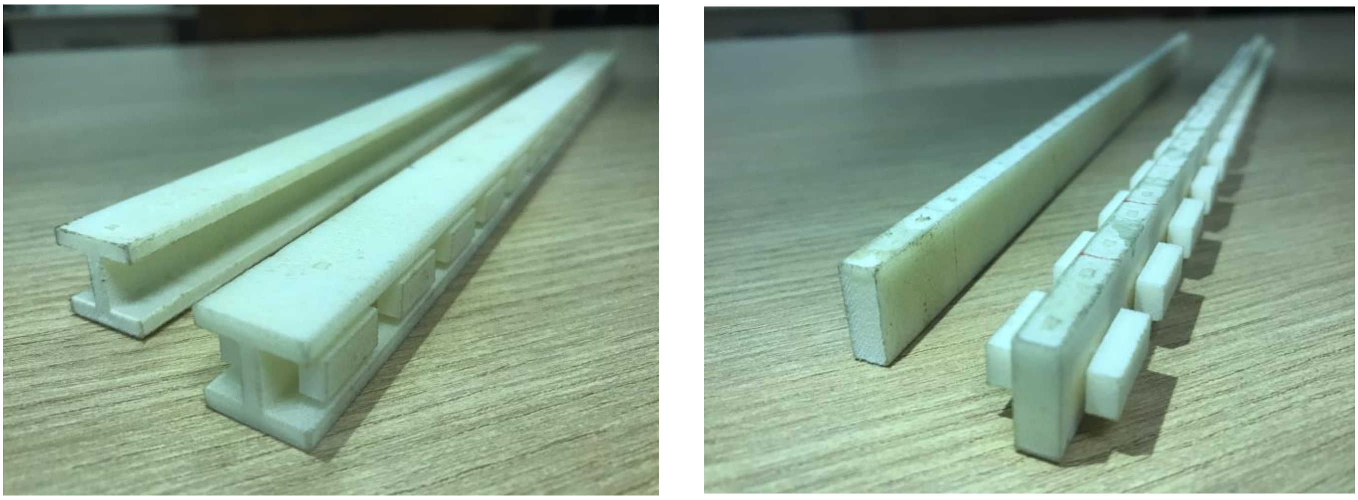
$$\psi(k,\omega) = \frac{L}{N}\sum_{i=1}^{N} w(x_i,\omega)e^{-ikx_i}, \tag{10}$$

reaches the maximum when de displacement field $w(x_i,\omega)$ correlated well with the wave field defined by the wavenumber k, where L is the length of the structure, x_i is i-th measurement point in the structure and N is the number of measured points. A normalizing factor can be added such that the maximum correlation is unitary (Raef Cherif and Noureddine Atallia, 2015).

4. EXPERIMENTAL SETUP

For the experimental analysis of wave propagation in each analysed structure to be able to accurately and reliably represent the systems under investigation, it is of fundamental importance that the required instrumentation and configurations be properly adopted. The experimental technique adopted for the beams consists of an impact test where the structure is manually excited by the short impulsive force employed by a modal hammer strike and the dynamic response measured by an accelerometer.

The polyamide beams are produced from 3D printing methods via Selective Laser Sintering (SLS). Two beam profiles are analyzed one with rectangular profiles and one with profile I, as shown in Fig. 2. For the metastructures, mechanical resonators are fixed periodically on both sides of the structure, totaling 15 units.



(a) (b)

Figure 2. Beams synthesized: (a) I profile and (b) rectangular profiles.

The measured values of dimensions and mass of the beam are then used to estimate the beam's density. Then the Young's modulus was estimated by measuring the natural frequencies of the baseline beam, under the soft foam, i.e. free-free boundary conditions. The properties and geometries of the beams are described in Tab. 1.

Table 1. Geometrical and material properties of the beams.

Properties and Geometries	**Beam (I profile)**	**Beam (rectangular profile)**	**Resonator**
Young´s Modulus [GPa]	1.28	0.84	0,96
Density[kg/m³]	792.02	798.12	1000
Width [mm]	16.05	6	14
Height [mm]	15.94	15.94	4
Length [mm]	330	330	6
Mass [kg]	0.03375	0.02519	-

The experimental technique adopted for the beams consists of an impact test where the structure is manually excited by short-term impulsive force employed by a modal hammer (Model PCB084A17). The dynamic response measured by an accelerometer (Model 352A21) attached to the structure by means of a thin layer of beeswax. A data acquisition board (Model Polytec Vib-E-200) was used for collect accelerometers readings and the signal processing software Vibsoft 5.5. The repeatability of the strike point was guaranteed by means of an apparatus mounted with a universal support to which the hammer was attached, shown in Fig. 3. The free-free boundary condition was applied by placing the beams on a flexible polyurethane foam. The analyzed frequency range was 0 to 5000 Hz for beam of rectangular profiles and 0 to 2100 Hz for beam I profile.

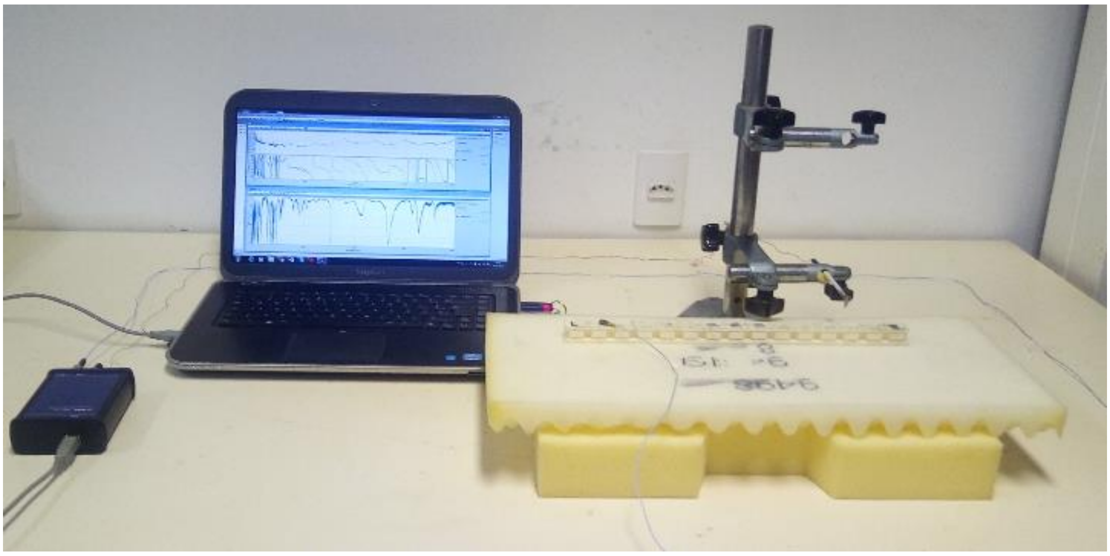


Figure 3. Experimental apparatus for polyamide beams.

For the output signal, an accelerometer was used an exponential window duo to the transient natural of the excitation. For the input signal from the modal hammer, a force window was used eliminating all the force signal before and after the impulse, which are only due to the instrumentation noise. As shown in Fig. 4, the excitation position was 14 mm and the minimum spacing between measurements was 11 mm for the all beams and metamaterial. And that the displacement at the extremities were not analyzed due to the occurrence of evanescent waves; a total of 23 measurement points for all beams and metamaterial was defined.

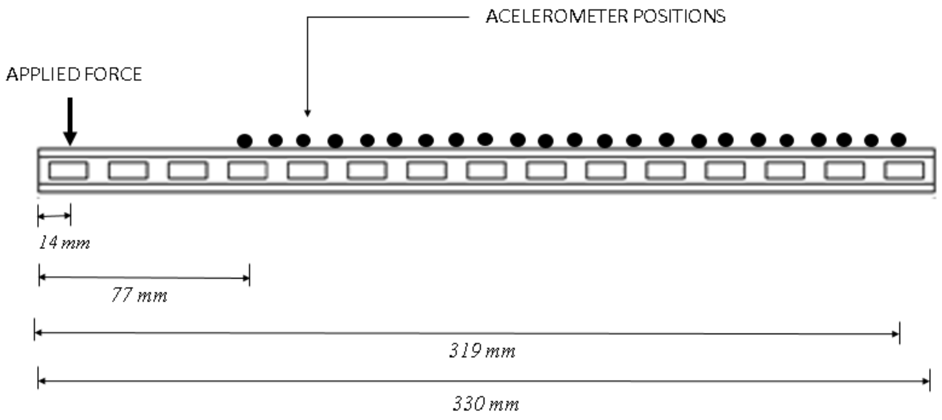


Figure 4. Schematic representation of the measurement points for all beam and metamaterial.

The discretization of a signal in the space influences the maximum measurable wavenumber. Given a spacing Δx between two points of acquisition in the beam so the maximum numberwave is determined by,

$$k_{max} = \frac{\pi}{\Delta x}. \tag{16}$$

Considering that the spacing between the acquisition points taken for all beams was 11 mm, then the maximum measurable wavenumber is 285.60 rad/m. The minimum wavenumber measurable is limited by the presence of evanescent waves near the boundaries, which decay exponentially with position.

5. RESULTS AND DISCUSSION

Fig. 5 shows the amplitudes measured 18 points in the I-beams. It is possible to observe the attenuation of amplitudes in the frequencies between 1500 Hz to 1800 Hz. For the metamaterial with rectangular profile, it is possible to note the attenuation of vibrations between 2200 Hz and 2500 Hz, as shown in Fig. 6.

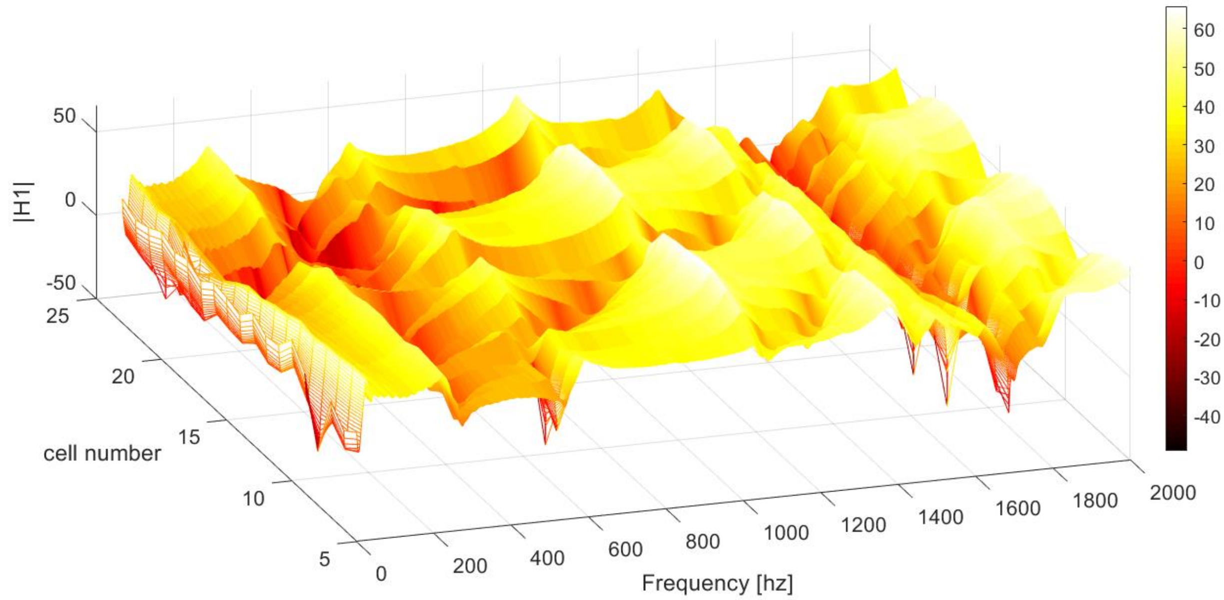


Figura 5. Experimental frequency response amplitudes for metamaterial beam of I profile.

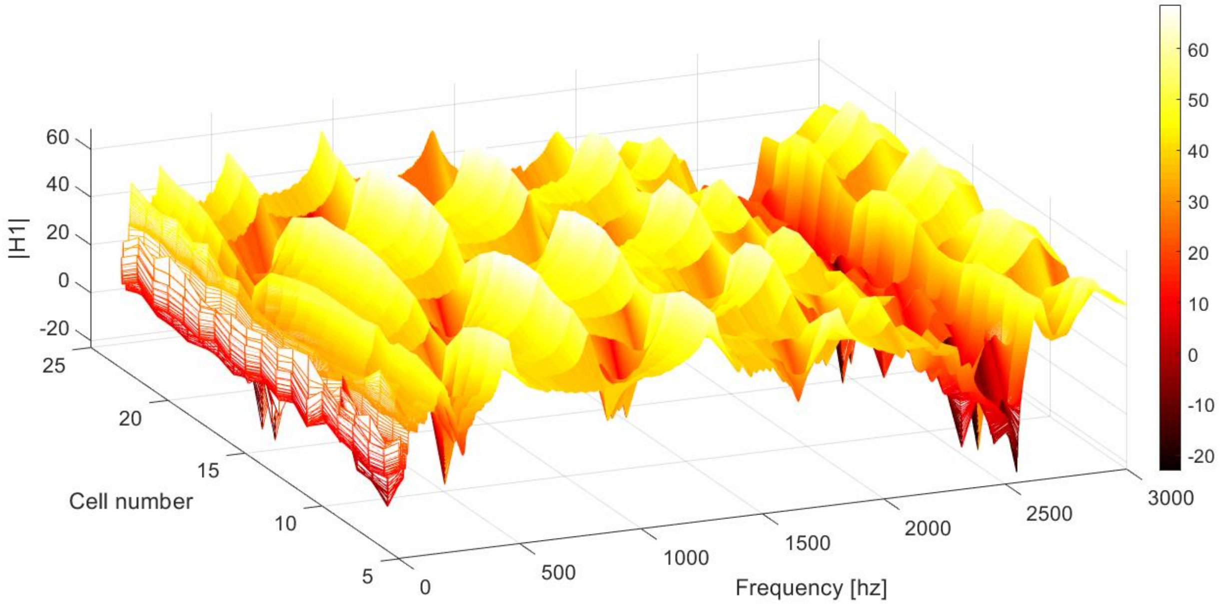
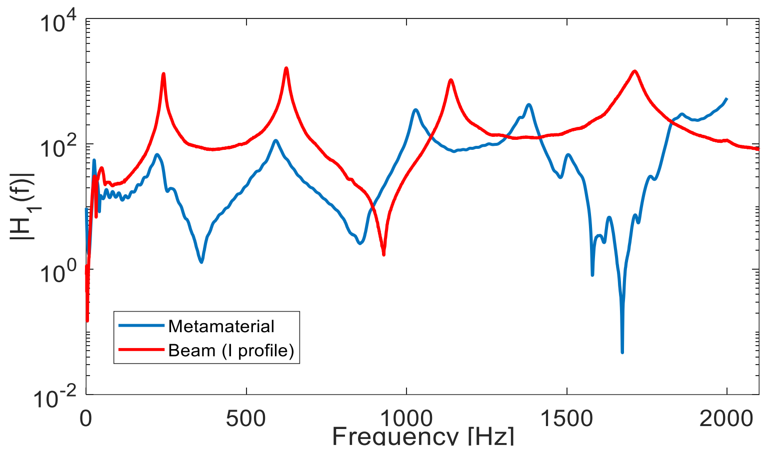
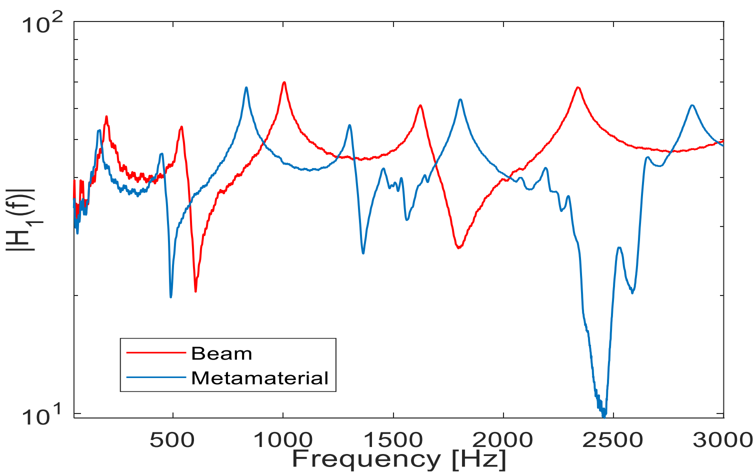


Figura 6. Experimental frequency response amplitudes for metamaterial beam of rectangular profiles.

Comparing the experimental frequency response of the bare I-beam with the I-beam metamaterial, it is possible to observe the maximum attenuation between 1500 Hz to 1800 Hz, as shown in Fig 7 (a). Similarly, for the rectangular profile beams it is possible to observe the resonance peak attenuation between 2200 Hz and 2500 Hz as shown in Fig 7 (b).



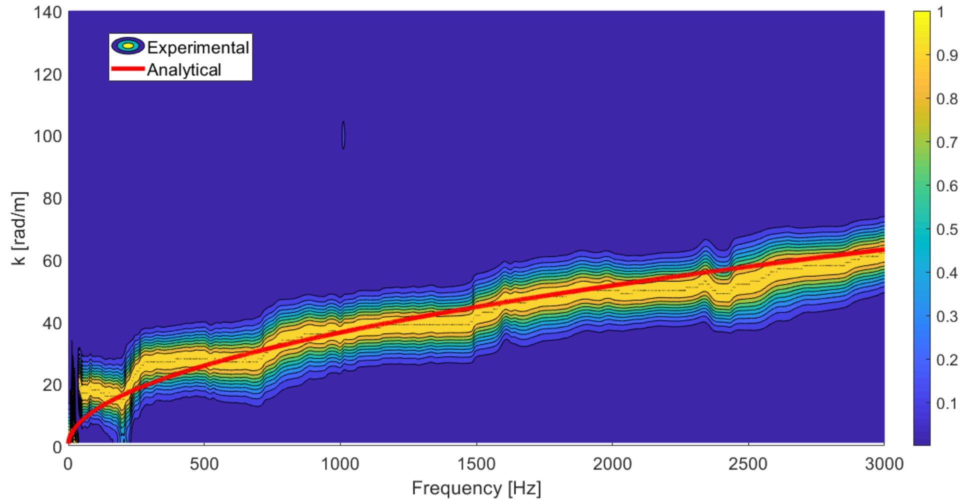
(a)



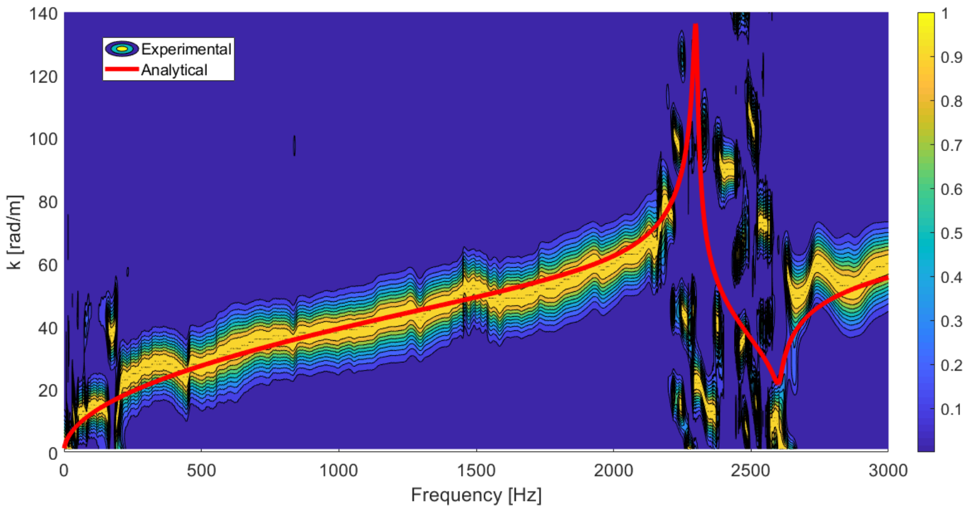
(b)

Figure 1.Comparison between the experimental frequency response of beam: (a) I profile and (b) rectangular profiles.

The wavenumber was estimated by the correlation method using Eq. (10), where $w(x_i, \omega)$ was substituted by the experimental frequency response points measured and maximum wavenumber correlation k_{max} used was 90 rad/m. The maximum wavenumber found was 140 rad/m for the beam with rectangular profile, as shown in Fig. 8. For both beams the experimental results were consistent with the analytical result, being possible to identify the frequency range in which the vibration attenuation occurs between 2200 Hz and 2500 Hz.



(a)



(b)

Figure 2.Wavenumbers estimated by the correlation method: (a) bare beam and (b) metamaterial beam with rectangular cross-section.

The wavenumbers estimated for the I- beam are shown in Fig. 9, the maximum correlation value used was 100 rad / s. It is possible to identify the bandgap frequency between 1500 and 1700 Hz in accordance with the calculated analytical value. However, there is a discontinuity in experimental data between 300 Hz and 500 Hz, possibly due to experimental setup.

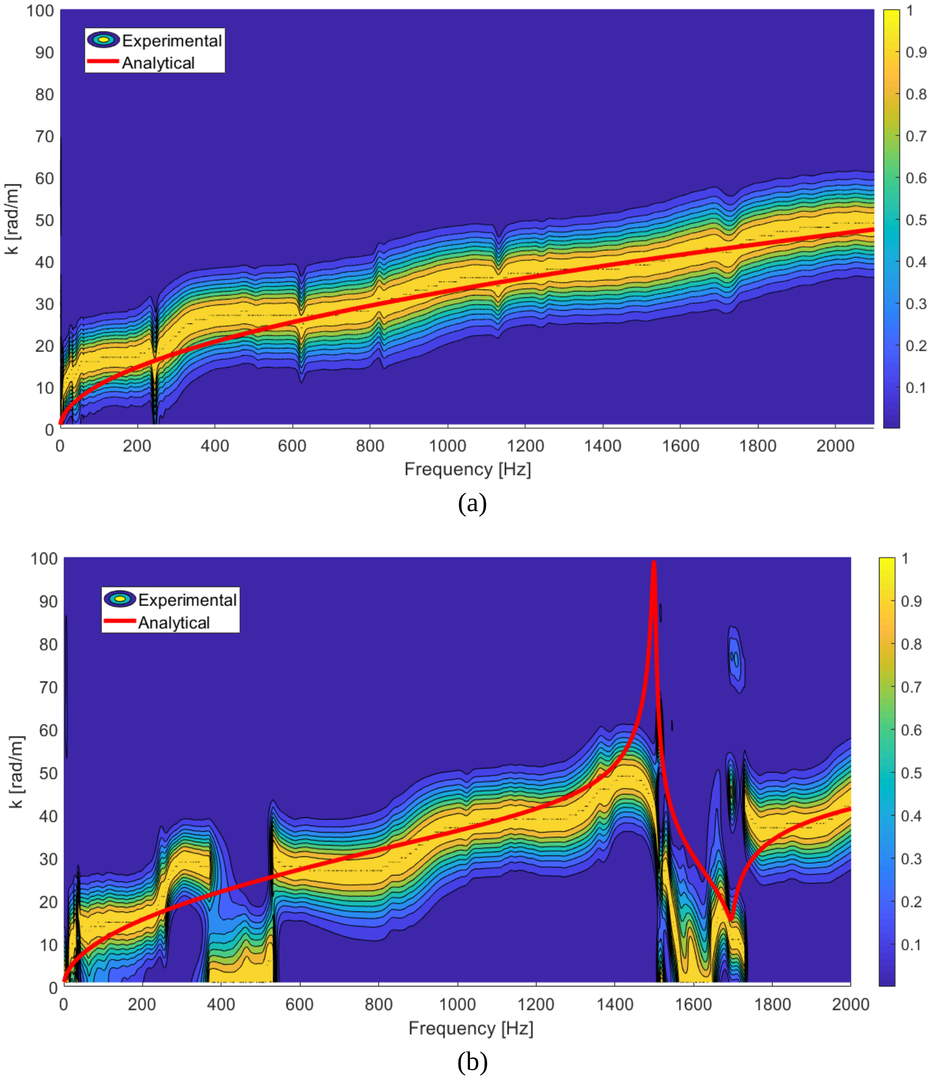


Figure 3. Wavenumbers estimated by the correlation method: (a) bare beam and (b) metamaterial beam with I cross-section.

6. CONCLUDING REMARKS

In this work, the wavenumber of beams undergoing flexural vibration was estimated using a analytical expression and a small number of FRF measurements at evenly spaced locations. A technique was presented called the correlation method, which is used to identify a complete dispersion equation. This method deals with measurement data from an arbitrary mesh of a 1D structure and its feasibility on metamaterial beam was shown. And the results appear to be independent of geometry, boundary conditions and source locations.The experimentally dispersion curves obtained shown a good agreement with the analytical solution and was possible to identify the bandgap effect. Therefore, it was observed that the rectangular profile presented better wavenumber estimates than the I profile, this aspect should be better investigated in future works, since the resonator geometry and attenuation frequency are the same. However, it intends to investigate the data obtained and applying uncertainty analysis methods, such as WKB (after Wentzel, Kramers and Brillouin), to represent the randomness and quantify them in order to investigate the performance of the vibration isolation.

7. ACKNOWLEGEMENTS

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