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HORIZONTAL AXIS WIND TURBINE MODELLING USING MODAL APROACH FOR LOW FREQUENCY VIBRATION BEHAVIOR ANALYSIS

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Abstract
The most effective strategy for reducing production cost of wind power is the wind turbine upscale. Such unavoidable trend leads to serious vibrations problems which need to be studied to ensure safe operations under these conditions and to provide higher power generation efficiency at lower costs. This paper aims to model and simulate the low frequency vibrations behavior of a 5-MW horizontal wind turbine using the modal approach method. The technical specifications of an offshore 5-MW baseline wind turbine were provided by NREL internal report. The structural model was tested and validated by comparing modal characteristics from the authors simulation with the values given by this report. Furthermore, the animation of the wind turbine modal forms has shown good agreement with the modal forms presented by the literature.

Keywords: *Wind Turbine; Low Frequency Vibratory Behavior; Modal Approach.*

1. INTRODUCTION

The most effective strategy for reducing wind power production costs is the wind turbine upscale. From the 1980s the wind industry has been through a period of intense and fast transitions related to this upscale process, recently, wind turbines greater than 10MW such as 12MW HALIADE-X (wind turbine produced by GE Wind Power, which is 260 meters high and has 220 meters rotor diameter) have been safely tested and even larger wind turbines are expected to arrive in the market soon.

However, aeroelasticity studies performed by the authors Thompsen et *al.* (2000), Chaviaropoulo (2001), Riziotis et *al.* (2004) and Hansen (2003, 2004, 2006, 2007), agreeingly point towards an imminent problem: vibration in wind turbines can be strongly amplified through upscaling. To address this concern, this paper aims to model and simulate the lower frequency vibrations behavior of a horizontal wind turbine using the modal approach method.

2. MODAL APPROACH

The modal approach method is based on the hypothesis that the deflected shape of the structure $v(x)$ can be expressed as the sum of a specific series of shape functions $\psi_n(x)$ which are compatible with the geometric support conditions and maintain the continuity of the internal displacement of the structure. Therefore, these functions become the displacement coordinates of the structure. For any shape function adopted, the deflected shape of the structure depends also of the amplitude term Z_n (generalized coordinate) (Clough and Penzien, 1975).

The deflected shape of the structure is:

$$v(x) = \sum_n Z_n \psi_n(x) \tag{1}$$

In general, a better accuracy of deflected shape of the structure can be achieved using the modal approach idealization than lumped mass approach because using the first method, at least theoretically, the structure can be idealized with infinite DOFs (degrees of freedom), what means that if there are *n* shapes functions available, than the structural system can be idealized with *n* DOFs as well. Although, must be pointed that the modal approach requires greater computational effort than the lumped mass approach (Clough and Penzien 1975).

The mathematical development that supports this theory can be found in Clough and Penzien (1975). Using modal approach, the generalized properties of the structure can be approximated by:

$$\begin{cases} m^* = \int_0^L \mu(x)\psi(x)^2 dx + \sum m_i {\psi_i}^2 + \sum I_{0i} {\psi'_i}^2 \\ c^* = \int_0^L c(x)\psi(x)^2 dx + \sum c_i {\psi_i}^2 \\ k^* = \int_0^L EI(x)\big(\psi(x)\big)^2 dx \int_0^L EI(x)\big(\psi''(x)\big)^2 dx + \sum k_i {\psi_i}^2 \\ k_g^* = F_{ext} \int_0^L \big(\psi'(x)\big)^2 dx \\ p_{eff}^*(t) = \int_0^L \dot{p}(x,t)\psi(x)\ \ dx + \sum p_i {\psi_i}^2 \end{cases} \tag{2}$$

being:

- m^*: generalized mass
- c^*: generalized damping
- k^*: generalized flexural stiffness
- k_g^*: generalized geometrical stiffness
- p_{eff}^*: generalized effective load
- EI: flexural stiffness
- μ: mass per unit of length
- F_{ext}: external force

Addressing to wind turbine model in this work: the blades' 1st bending vibration modes in flapwise and edgewise directions of wind turbine at standstill has been used as shape functions related to the blade's bending movements and the tower shape functions has been approximated using cubic Hermite's functions.

3. STRUCTURAL MODEL

The wind turbine is idealized by 13 DOF model, in which the blades' conicity are neglected. All 13 DOF are distributed in four coordinate systems (two of them showed in Fig. 1): one global coordinate system X_1, X_2, X_3 and three local coordinate system, x_1, x_2 , x_3 (each of them is fixed in each blade). Moreover, x_1 and x_2 are located at the plane of the rotor and X_2, X_3 are fixed the mid plane of the tower. The position of the local coordinate system attached to *b-th* blade is specified by the azimuthal angle $\Psi_b(t)$,which is considered positive when rotating clockwise seen from an upwind position (Zhang, 2015).

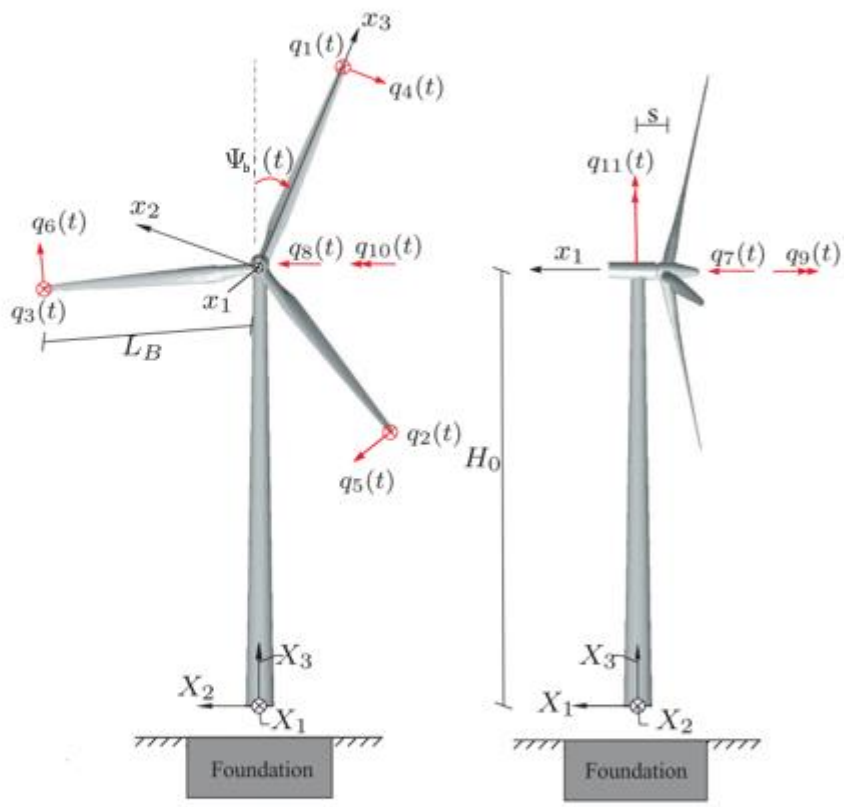

Figure 1 13-DOF Aeroelastic wind turbine model. Available from: Zhang, 2005. (Adapted)

The flapwise and edgewise displacements of the *b-th* blade are represented by the degrees of $q_b(t)$ and $q_{b+3}(t)$, $b = 1,2,3$, described in the local coordinate systems. The next five degrees of freedom, $q_7(t)$ to $q_{11}(t)$, described in the global coordinate system, refer to the tower movements, being $q_7(t)$ and $q_8(t)$ the tower displacements at hub height, and $q_9(t)$, $q_{10}(t)$, $q_{11}(t)$ the angles about X_1, X_2 e X_3 axes, respectively. The tower height measured from the ground to the nacelle is denoted by H_0 and the horizontal distance from the center of the tower top to the origin of the moving coordinate systems is given by s (Zhang, 2015).

The flexible powertrain model is represented by the degrees of freedom $q_{12}(t)$ e $q_{13}(t)$, in which $q_{12}(t)$ indicates the angular displacement of the hub, $q_{13}(t)$ the angular displacement of the generator. N is the gear ratio (Zhang, 2015).

Then, the azimuthal angle $\Psi_b(t)$ of blade *b-th* is given by:

$$\Psi_b(t) = \Omega t + \frac{2\pi}{3}(b-1) + q_{12}(t),\ b = 1,2,3 \tag{3}$$

3.1 Blade Analysis

Figure 2 shows the velocities field at a cross section of a blade. Let $\bar{u}_{1,b}(x_3,t)$ and $\bar{u}_{2,b}(x_3,t)$ be the components of the elastic deformation seen by the observer fixed at moving coordinate system, using the modal approach method, one can model the blade displacement as described in Eq. (4). Where $q_b(t)$ and $q_{b+3}(t)$ are the tip displacements of each blade related to the flapwise and edgewise direction respectively; and Φ_f and Φ_e are the undamped fundamental eigenmodes of the blade, also in flapwise and edgewise direction. The mode shapes are normalized to one at the tip, i.e. $\Phi_f(L) = \Phi_e(L) = 1$.

$$\begin{cases} \bar{u}_{1,b}(x_3,t) \simeq \Phi_f(x_3)q_b(t), \\ \bar{u}_{2,b}(x_3,t) \simeq -\Phi_e(x_3)q_{b+3}(t), \end{cases} b = 1,2,3 \tag{4}$$

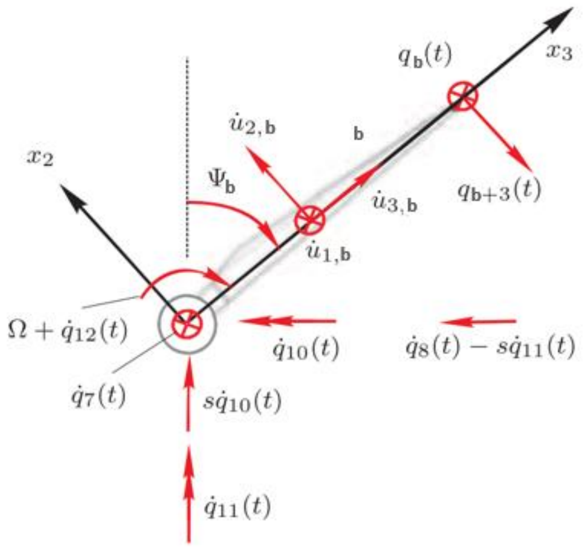

Figure 2 Velocities at a cross-section of one blade. Available from: Zhang, 2015. (Adapted)

The velocity vector of a cross-section blade, in the moving coordinate system, is the result of the sum of all velocity contributions acting in the cross-section:

$$\begin{cases} \dot{u}_{1,b}(x_3,t) = \Phi_f(x_3)\dot{q}_b(t) + \dot{q}_7(t) + x_3\cos\Psi_b\,\dot{q}_{10}(t) + x_3\sin\Psi_b\,\dot{q}_{11}(t) \\ \dot{u}_{2,b}(x_3,t) = -\Phi_e(x_3)\dot{q}_{b+3}(t) + \sin\Psi_b\, s\dot{q}_{10}(t) + \cos\Psi_b\left(\dot{q}_8(t) - s\dot{q}_{11}(t)\right) - x_3\left(\dot{q}_{12}(t) + \Omega\right) \\ \dot{u}_{3,b}(x_3,t) = -\Omega\Phi_e(x_3)\dot{q}_{b+3}(t) + \cos\Psi_b\, s\dot{q}_{10}(t) - \sin\Psi_b\left(\dot{q}_8(t) - s\dot{q}_{11}(t)\right) \end{cases} \tag{5}$$

3.2 Tower Analysis

The tower shape functions were approximated using cubic Hermite interpolation functions:

$$\begin{cases} N_1(X_3) = 3\left(\frac{X_3}{H_0}\right)^2 - 2\left(\frac{X_3}{H_0}\right)^3 \\ N_2(X_3) = H_0\left(\frac{X_3}{H_0}\right)^3 - H_0\left(\frac{X_3}{H_0}\right)^2 \end{cases} \tag{6}$$

in which, H_0 is the hub height.
The displacement component of the tower in the global coordinate system, shown in Fig 3, is given as:

$$\begin{cases} u_{X_1}(X_3,t) = N_1(X_3)q_7(t) - N_2(X_3)q_{10}(t) \\ u_{X_2}(X_3,t) = N_1(X_3)q_8(t) + N_2(X_3)q_9(t) \end{cases} \quad (7)$$

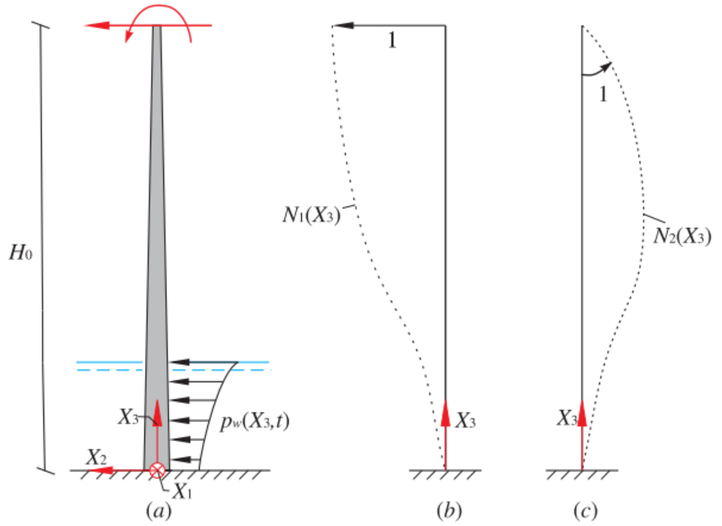

Figure 3 Modeling of the tower vibrations. (a) Two DOFs model for the fore-aft tower vibration or lateral tower vibration. (b) Hermite function $N_1(X_3)$. (c) Hermite function $N_2(X_3)$

3.3 Structural Stiffness and Damping Matrices

The torsion's DOFs has been modeled using lumped mass approach and the bending's DOFs (1 to 10) using modal approach. The structural stiffness and damping parameters related to the DOFs idealized using modal approach is result of the direct application of Eq. (2).

The structural stiffness matrix $\boldsymbol{K_s}$, which has a local stiffness matrix component $\boldsymbol{K_{s,ll}}$ and a global stiffness matrix component $\boldsymbol{K_{s,gg}}$, is:

$$\boldsymbol{K_s} = \begin{bmatrix} \boldsymbol{K_{s,ll}} & 0 \\ 0 & \boldsymbol{K_{s,gg}} \end{bmatrix} \quad (8)$$

The $\boldsymbol{K_{s,ll}}$ is given as:

$$\boldsymbol{K_{s,ll}} = \begin{bmatrix} \begin{bmatrix} k_1 & 0 & 0 \\ 0 & k_1 & 0 \\ 0 & 0 & k_1 \end{bmatrix} & [Z]_3 \\ [Z]_3 & \begin{bmatrix} k_2 & 0 & 0 \\ 0 & k_2 & 0 \\ 0 & 0 & k_2 \end{bmatrix} \end{bmatrix} \quad (9)$$

where $[Z]_3$ is 3x3 zeros matrix and k_1, $k_2$are the blade stiffness in flapwise and edgewise directions (related to q_b and q_{b+3}, respectively). These stiffnesses have a structural component and a centrifugal component, where the second one results of centrifugal force action in the blades.
These stiffnesses is given by:

$$\begin{cases} k_1 = \int_0^{L_B} \left(EI_1(x_3) \left(\frac{d^2\Phi_f(x_3)}{dx_3{}^2} \right)^2 + F(x_3) \left(\frac{d\Phi_f(x_3)}{dx_3} \right)^2 \right) dx_3 \\ k_2 = \int_0^{L_B} \left(EI_2(x_3) \left(\frac{d^2\Phi_e(x_3)}{dx_3{}^2} \right)^2 + F(x_3) \left(\frac{d\Phi_e(x_3)}{dx_3} \right)^2 \right) dx_3 \end{cases} \quad (10)$$

in which, EI_1 and EI_2 are the blade flexural stiffness in flapwise and edgewise directions respectively, and $F(x_3) = \Omega^2 \int_{x_3}^{L_B} \mu(x_3) x_3 dx_3$ is the centrifugal axial force per unit of length along the blade.

The matrix $\boldsymbol{K_{s,gg}}$ is given as:

$$\boldsymbol{K_{s,gg}} = \begin{bmatrix} \begin{bmatrix} k_{7,7} & 0 & 0 & k_{7,10} \\ 0 & k_{8,8} & k_{8,9} & 0 \\ 0 & k_{9,8} & k_{9,9} & 0 \\ k_{10,7} & 0 & 0 & k_{10,10} \end{bmatrix} & [Z]_{4x3} \\ [Z]_{3x4} & \begin{bmatrix} k_{11,11} & 0 & 0 \\ 0 & k_0 & \frac{-1}{N} k_0 \\ 0 & \frac{-1}{N} k_0 & \frac{-1}{N^2} k_0 \end{bmatrix} \end{bmatrix} \tag{11}$$

where $[Z]_{4x3}$ is 4x3 zeros matrix.

The stiffnesses parameters ($k_{7,7}$, $k_{8,8}$, $k_{9,9}$, $k_{10,10}, k_{7,10}$, $k_{8,9}$, $k_{9,8}$ and $k_{10,7}$) related to $q_7(t)$, $q_8(t)$, $q_9(t)$ and $q_{10}(t$ (t) result of the direct application of Eq. (2) and Eq.(7) and are defined as follows:

$$\begin{cases} k_{7,7} = k_{8,8} = \int_0^{H_0} EI_0(X_3) \left(\frac{\partial N_1}{\partial X_3}\right)^2 dX_3 \\ k_{7,10} = k_{10,7} = k_{8,9} = k_{9,8} = \int_0^{H_0} EI_0(X_3) \left(\frac{\partial N_1}{\partial X_3}\right)\left(\frac{\partial N_2}{\partial X_3}\right) dX_3 \\ k_{10,10} = k_{9,9} = \int_0^{H_0} EI_0(X_3) \left(\frac{\partial N_2}{\partial X_3}\right)^2 dX_3 \end{cases} \tag{12}$$

in which, EI_0 is the tower flexural stiffness.

The degrees of freedom $q_{11}(t)$, $q_{12}(t)$ and $q_{13}(t)$ has been modelled using lumped mass approach. $k_{11,11}$ is the torsional stiffness coefficient related to $q_{11}(t)$ and k_0 is the equivalent torsional stiffness of the drivetrain, given as:

$$\frac{1}{k_0} = \frac{1}{k_r} + \frac{1}{N^2 k_g} \Rightarrow k_0 = \frac{N^2 k_r k_g}{k_r + N^2 k_g} \tag{13}$$

in which, k_r and k_g are the rotor and generator stiffnesses respectively.

The structural damping matrix $\boldsymbol{C_s}$ also have components described in the local coordinate system ($\boldsymbol{C_{s,ll}}$) and in the global coordinate system ($\boldsymbol{C_{s,gg}}$) and is defined as:

$$\boldsymbol{C_s} = \begin{bmatrix} \boldsymbol{C_{s,ll}} & 0 \\ 0 & \boldsymbol{C_{s,gg}} \end{bmatrix} \tag{14}$$

in which, $\boldsymbol{C_{s,ll}}$ is given as:

$$\boldsymbol{C_{s,ll}} = \begin{bmatrix} \begin{bmatrix} c_1 & 0 & 0 \\ 0 & c_1 & 0 \\ 0 & 0 & c_1 \end{bmatrix} & [Z]_3 \\ [Z]_3 & \begin{bmatrix} c_2 & 0 & 0 \\ 0 & c_2 & 0 \\ 0 & 0 & c_2 \end{bmatrix} \end{bmatrix} \tag{15}$$

and $\boldsymbol{C_{s,gg}}$ is:

$$\boldsymbol{C_{s,gg}} = \begin{bmatrix} \begin{bmatrix} c_{7,7} & 0 & 0 & c_{7,10} & 0 \\ 0 & c_{8,8} & c_{8,9} & 0 & 0 \\ 0 & c_{9,8} & c_{9,9} & 0 & 0 \\ c_{7,10} & 0 & 0 & c_{10,10} & 0 \\ 0 & 0 & 0 & 0 & c_{11,11} \end{bmatrix} & [Z]_{5x2} \\ [Z]_{2x5} & [Z]_2 \end{bmatrix} \tag{16}$$

The damping factors, ζ, of all DOFs of are available in the technical report published by de Jonkman et al. (2009), therefore the damping parameter can be calculated by $c = 2\zeta\sqrt{mk}$.

3.4 Equation of Motion

The kinetic energy of the wind turbine system (including blades, tower and powertrain) is given as:

$$T(\boldsymbol{q}(t), \dot{\boldsymbol{q}}(t)) = \frac{1}{2}\sum_{j=1}^{3}\int_0^{L_B}\mu_3(x_3)\left(\dot{u}_{1,b}^2(x_3,t) + \dot{u}_{2,b}^2(x_3,t) + \dot{u}_{3,b}^2(x_3,t)\right)dx_3 + \int_0^{H_0}\mu_0(X_3)\left(\dot{u}_{1,b}^2(x_3,t) + \dot{u}_{2,b}^2(x_3,t)\right)dX_3 + \frac{1}{2}M_0(\dot{q}_7^2(t) + \dot{q}_8^2(t)) + \frac{1}{2}J_g\left(N\Omega + \dot{q}_{13}(t)\right)^2 \quad (17)$$

in which, $\mu_3(x_3)$ and $\mu_0(X_3)$ are the mass per unit length of the blade and tower respectively and M_0 is the nacelle mass.

Analyzing Eq. (17), the first term refers to the kinetic energy stored in the rotor, the second and third terms represents the kinetic energy stored in the tower and the nacelle related to the globally defined degrees of freedom; the last one is the kinetic energy related to the generator.

The total potential energy (strain energy) of the wind turbine is given by:

$$U(\boldsymbol{q}(t)) = \frac{1}{2}\boldsymbol{q}^T(t)\boldsymbol{K_s}\boldsymbol{q}(t) \quad (18)$$

in which, $\boldsymbol{K_s}$ is defined by Eq. (8).

Using Lagrangian Mechanics the motion equation is obtained:

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\boldsymbol{q}}}\right) - \frac{\partial T}{\partial \boldsymbol{q}} + \frac{\partial U}{\partial \boldsymbol{q}} = \boldsymbol{f}_a(\dot{\boldsymbol{q}}(t), \beta(t), t) - \boldsymbol{C}_s\dot{\boldsymbol{q}}(t) \quad (19)$$

in which, $\boldsymbol{f}_a(\dot{\boldsymbol{q}}(t), \beta(t), t)$ is the aerodynamic load, which cares no further discussion once this load is not necessary to perform the analysis addressed to this paper.

Developing Eq. (19), the equation of motion is given as:

$$\boldsymbol{M}(t)\ddot{\boldsymbol{q}}(t) + (\boldsymbol{C_s} + \boldsymbol{G}(t))\dot{\boldsymbol{q}}(t) + \boldsymbol{K}(t)\boldsymbol{q}(t) = \boldsymbol{f}_a(\dot{\boldsymbol{q}}(t), \beta(t), t) \quad (20)$$

in which, $\boldsymbol{M}(t)$ is the mass matrix that is symmetric and positive definite, $\boldsymbol{C}(t)$ is the damping matrix, $\boldsymbol{G}(t)$ is the gyroscopic matrix and $\boldsymbol{K}(t)$ is the stiffness matrix.

Equation (20) represents a time variant system; in which the time dependence results of the blade rotation given its angular displacement represented in Eq. (3). Therefore, one can notice that if $q_{12}(t)$ is ignored in Eq. (3) the system matrices become periodic with a constant period $T = 2\pi/\Omega$.

Analyzes based in time independent systems assumptions, such as solve the eigenvalue problem, can't be directly perform once the system is periodic time dependent. To address this problem, the Multiblade Coordinate Transformation (Bir, 2010) has been used. This transformation is presented in the next section.

4. MULTIBLADE COORDINATE TRANSFORMATION

The Multiblade Coordinate Transformation (MBC) or Coleman Transformation is a special case of Lyapunov- Floquet Transformation for an isotropic rotor. MBC is a linear transformation which relates the rotating DOFs to new degrees of freedom fixed in the inertial frame i.e. converts the rotor's rotating coordinates to non-rotate ones. Therefore, enabling eigenvalue analysis (SKJOLDAN.; HANSEN, 2009).

The azimuth equation – Eq. (3) – can be generalized as

$$\Psi_b(t) = \Psi + \frac{2\pi}{N_b}(j-1) \quad (21)$$

The equations are developed for a 3-blade-rotor because it is more pertinent for the present case (BIR, 2008,2010):

$$\begin{cases} q_0 = \frac{1}{3}\sum_{b=1}^{3} q_b \\ q_c = q_{1c} = \frac{2}{3}\sum_{b=1}^{3} q_b \cos\Psi_b \\ q_s = q_{1s} = \frac{2}{3}\sum_{b=1}^{3} q_b \sin\Psi_b \end{cases} \quad (22)$$

where, q_{1c} is the cosine-cyclic mode and q_{1s} is the sine-cyclic mode; these two modes together with the q_0 mode lead to coupling of the rotor with the rest of the wind turbine. These new degrees of freedom are expressed in nonrotating frame and are called “nonrotating degrees of freedom”. They can also be referred as “rotor coordinates” because they express the cumulative behavior of all rotor blades (and not individual blades) in the fixed frame.

The inverse transformation, yielding the blade coordinate given the rotor coordinates, is presented in Eq. (23).

$$q_b = q_0 + q_c \cos \Psi_b + q_s \sin \Psi_b \,, b = 1,2,3. \tag{23}$$

5. RESULTS

To obtain reliable information in scientific studies, the use of realistic physical inputs to the model is required. The report *"Definition of a 5-MW Reference Wind Turbine for Offshore System Development"*, by Jonkman et *al.*, 2009, describes the technical specifications of a Wind Turbine called *"NREL offshore 5-MW Baseline Wind Turbine"*. Such data are representative of a typical multimegawatt turbine and was, therefore, adopted as inputs for the simulations in the present work.

5.1 Blade Input Modes Shapes

The blade modes shapes were obtained by the NREL (National Renewable Energy Laboratory) open-source software BMODES (BIR, 2007) which is a finite-element code that provides dynamically coupled modes for the wind turbine rotor, Fig. 4.

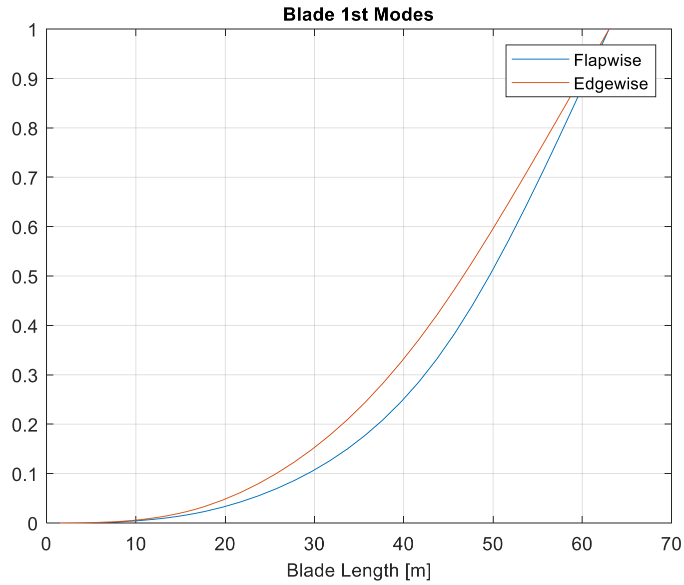

Figure 4 Blade’s 1st Modes

5.2 Model Validation

In order to validate the 13 DOF structural model, the lower natural frequencies obtained by the eigenvalue analysis from the 13DOF model were compared to the values given by the simulations performed in FAST and ADAMS presented at *"Definition of a 5-MW Reference Wind Turbine for Offshore System Development"* report, Tab.1.

Table 1 Natural Frequencies at Standstill

Modes	MODAL APPROACH (Hz)	FAST (Hz)	ADAMS (Hz)
1º Tower Fore-and-Aft	0.3341	0.3240	0.3195
1º Tower Side-to-Side	0.3390	0.3120	0.3164
1º Flapwise Asymmetric A	0.6673	0.6664	0.6296
1º Flapwise Symmetric	0.6690	0.6675	0.6686
1º Flapwise Asymmetric B	0.6876	0.6993	0.6993
1º Edgewise Asymmetric A	1.0718	1.0793	1.0740
1º Edgewise Asymmetric B	1.0805	1.0898	1.0877

Although the complete wind turbine system was idealized with only 13 degrees of freedom including a very simple powertrain model, the eigenfrequencies provided by the modal approach model were very close to the ones provided by more detailed models.

Figures 5 and 6 show the Coleman Diagram and Damping Ratio. The Coleman Diagram has a good agreement which the one published by Hansen (2007).

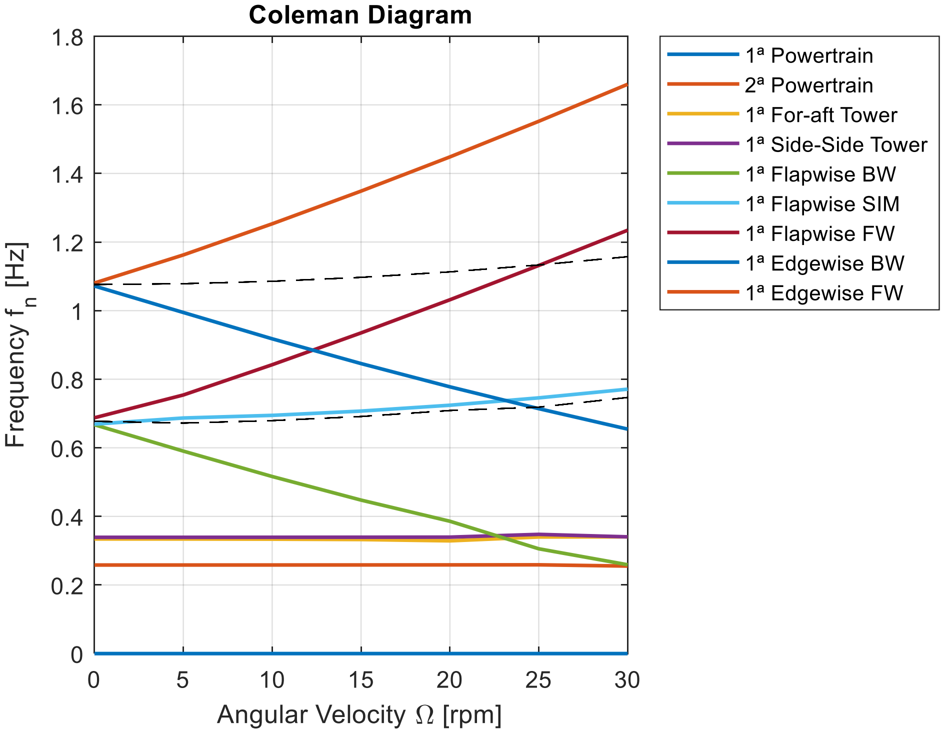

Figure 5 Coleman Diagram

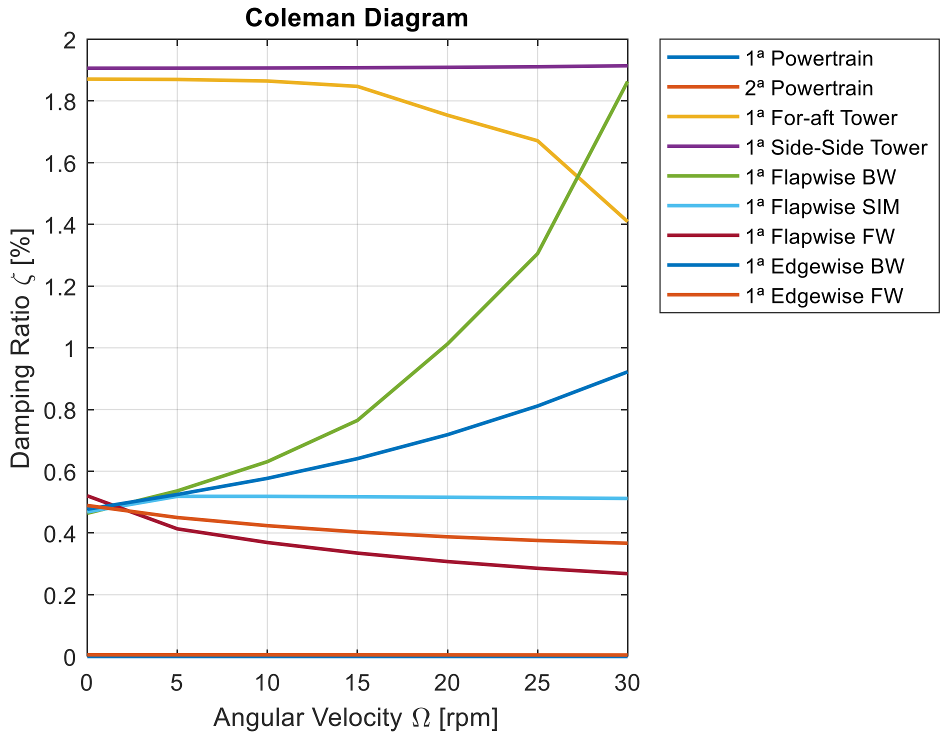

Figure 6 Damping Ratio

Furthermore, the modal forms at standstill also resulted in a good agreement with the modal forms presented by Ramamurthy in 2011, Fig. 7 and 8.

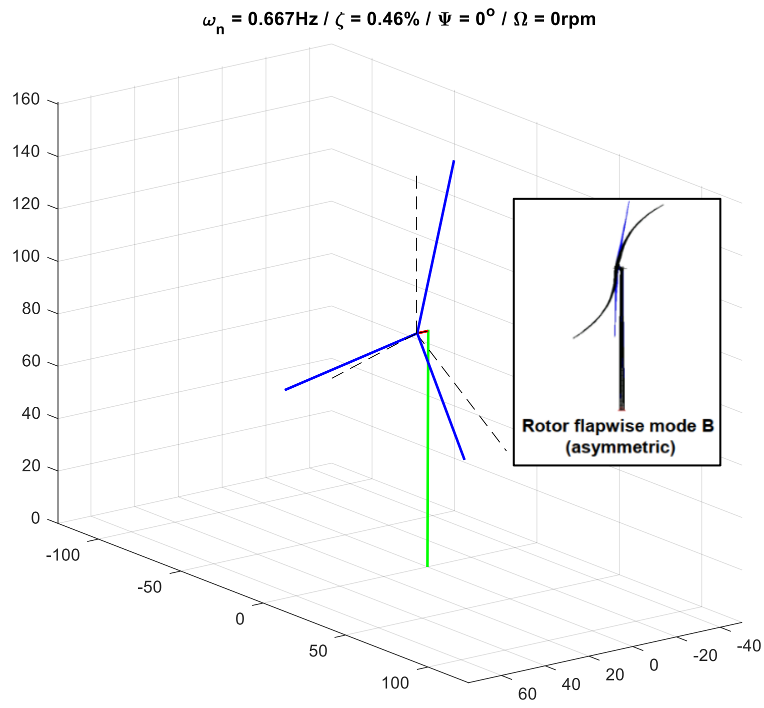

Figure 7 Mode Animation – 1st Flapwise Asymmetric B

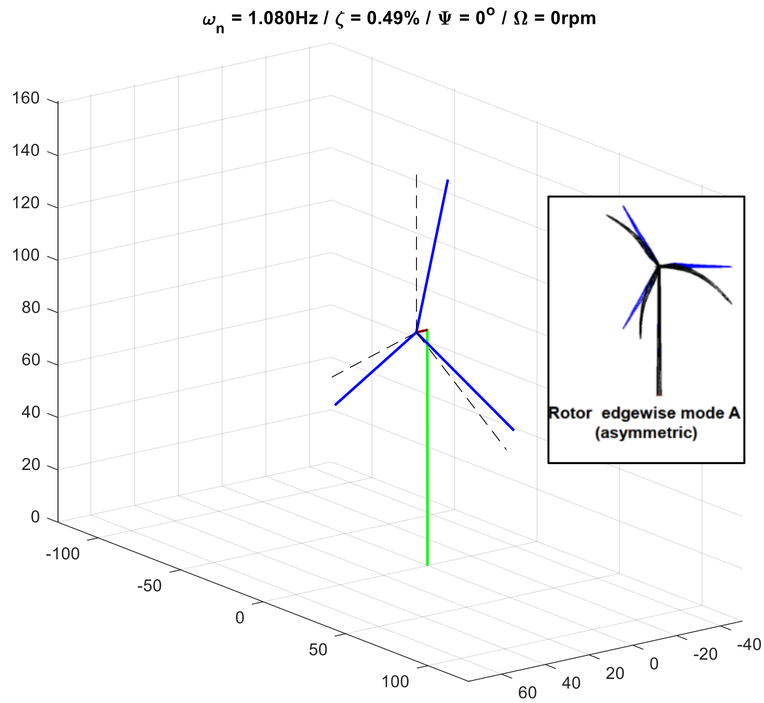

Figure 8 Mode Animation – 1st Edgewise Asymmetric A

6. CONCLUSIONS

This paper aimed to model, simulate and validate a 13DOF wind turbine model, which was idealized using modal approach method focusing in low frequencies. The wind turbine model equations (i.e. the coupled tower-nacelle-rotor equations) were derived using mixed degrees of freedom (some of which were described in the rotating frame and others in the nonrotating frame) resulting in a periodic time dependent system. Therefore, the Multiblade Coordinate Transformation has been used to solve the eigenvalue problem, obtaining the Coleman Diagram.

Although the model has only 13 DOF including a very simple powertrain model, the natural frequencies obtained show a very good agreement when compared to more complex models such the ones done in FAST and ADAMS. Moreover, the modal forms also presented good agreement with the literature, which indicates that the modal approach method is can be used to model wind turbines resulting in relatively non-complex models and with very good results.

7. ACKNOWLEDGEMENTS

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