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# MATHEMATICAL MODELING OF SMOKE FILLING IN DIFFERENT FIRE SCENARIOS

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**Abstract.** *In a fire situation, two main aspects are determining causes of human death: the high temperature and the intoxication by smoke. In present work, it is analyzed the transient evolution of smoke layer thickness in different fire situations. It was performed comparisons between different experimental fire data such as: a large hall with only one opening of 0.2 m obtained by Li et al. (1999); a tunnel with two distinct Heat Release Rates (HRR) performed by Gao et al. (2016); a hall through the experiment performed by Lai et al. (2013) that have a vertical opening (door) of 2 m × 0.8 m, and a laboratory through the experiment performed by Yi et al. (2005). In all cases the results are obtained using the fundamental solution of the smoke equation and its analytical solution proposed by Novozhilov (2012), both applied to the same fire scenario. The results of the fundamental smoke equation presented good agreement with the experimental data for most of the fire scenarios tested.*

**Keywords:** fire, smoke filling, compartment, smoke equation, analytic solution.

## 1. INTRODUCTION

During a fire in a compartment, a separation is formed between two distinct gas layers, a rich in oxygen (cold, fresh air, positioned lower) and a rich in toxic gases (hot, combustion products, positioned upper).

The developing smoke layer is normally treated like a distinct control volume, having uniform properties of modeling. It is supposed that the control volumes of higher and bottom layer are separated by a distinct thermal discontinuity. The higher layer descends due the fresh air drag from bottom layer and the leak paths. Once the smoke layer descends until the fire source elevation, this becomes immersed at smoke layer and the fresh air additional drag from bottom layer stop. After this point, the smoke layer continues descending due to the gas expansion and the fire intensity can decrease in this point due to the oxygen exhaustion at higher layer.

Numerical simulations play an important role to investigate propagation of smoke in large buildings, existing two kinds of simulation methods, the zone model and the field method.

When the fire is at the ground of a compartment, a hot smoke plume rises to the ceiling through drag force. When the smoke plume reaches the ceiling, the plume moment have an effect of jet, scattering the smoke for all ceiling. Once the smoke is covering all ceiling, the smoke layer thickness increase until that all compartment is full of smoke or the smoke layer becomes steady (or even decays) as a result of some exhaustion device. Due to that, the smoke exhaustion system design is important against fire hazards in buildings of any size.

Smoke in compartments was studied by Mower (1999), Huo et al. (2005), Qin et al. (2009), Bennardo and Inzaghi (2010), Novozhilov (2012), Zhang et al. (2012) and Zhou (2017).

The present work give emphasis to the performance of a smoke filling equation solution applied in compartment fires, analyzing the mathematical methodologies to the determination of smoke filling time in an environment in fire situation, where it is used as base the equation develop by Karlsson and Quintiere (1999) and Novozhilov (2012). The objective of this work is to verify the accuracy of the mathematical solutions (fundamental and analytical) when applied to different fire scenarios.

## 2. SMOKE FILLING EQUATION

The smoke filling equation can be written as (Karlsson and Quintiere, 2000):

$$\frac{dy}{d\tau} + \dot{Q}^* + \gamma \cdot y^{5/3} = 0, \quad y(0) = 1 \quad (1)$$

with the non-dimensional variables and parameters:

$$\dot{Q}^* = \frac{\dot{Q}}{\rho_{\infty} c_{\infty} T_{\infty} \sqrt{g} H^{5/2}}, \quad y = \frac{z}{H}, \quad \tau = \sqrt{\frac{g}{H}} \frac{H^2}{S} t \quad (2)$$

where  $\dot{Q}$  is the heat release rate;  $z$  is the smoke layer position (interface above ground);  $t$  is the time;  $\rho_{\infty}, c_{\infty}, T_{\infty}$  are the environment air properties (density, specific heat and temperature) and  $g$  is the gravity acceleration.

We get like result of the Eq. (1):

$$y(\tau) = \tau + \int_0^y \frac{5\rho_{\infty} c_{\infty} T_{\infty} \sqrt{g} H^{5/2}}{\left(\frac{\dot{Q}}{\rho_{\infty} c_{\infty} T_{\infty} \sqrt{g} H^{5/2}}\right)^{1/3} - a^{5/3} \rho_{\infty} c_{\infty} T_{\infty} \sqrt{g} H^{5/2} + 5\dot{Q}} da \quad (3)$$

$$- \int_0^1 \frac{5\rho_{\infty} c_{\infty} T_{\infty} \sqrt{g} H^{5/2}}{\left(\frac{\dot{Q}}{\rho_{\infty} c_{\infty} T_{\infty} \sqrt{g} H^{5/2}}\right)^{1/3} - a^{5/3} \rho_{\infty} c_{\infty} T_{\infty} \sqrt{g} H^{5/2} + 5\dot{Q}} da = 0$$

## 2.1 Analytical solution of the smoke filling equation (Novozhilov, 2012)

Novozhilov (2012) demonstrated that the smoke filling equation admits an analytic solution to the particular case of constant heat release rate. In the cited work, it is considered a compartment with height  $H$  and floor area  $S$ .

Novozhilov (2012) approximated the parameter  $\gamma$  of Eq. (1) as  $\gamma = \frac{1}{5} (\dot{Q}^*)^{1/3}$ . With this approximation, the problem given in Eq. (1) with the parameters given in Eq. (2) becomes,

$$\tau = \frac{1}{\dot{Q}^*} \int_y^1 \frac{du}{\left[1 + (\gamma/\dot{Q}^*) u^{5/3}\right]}. \quad (4)$$

Making the variable separation, we get:

$$\tau = 3(5)^{3/5} (\dot{Q}^*)^{-3/5} \int_{\beta y^{1/3}}^{\beta} \frac{s^2}{(1+s^5)} ds \quad (5)$$

The result of this integral is finding in (Gradshteyn and Ryzhik, 2007). In this manner, after some trigonometric and algebraic manipulations, it is obtained the following analytical solution for Eq. (1):

$$\tau = 3(5)^{-2/5} (\dot{Q}^*)^{-3/5} + \left\{ \ln \left[ \frac{(\dot{Q}^*)^{-2/15} + a}{(\dot{Q}^*)^{-2/15} y^{1/3} + a} \right] + \sum_{i=1}^2 b_i^0 \left[ \arctg \left[ (\dot{Q}^*)^{-2/15} \frac{(1-y^{1/3})}{\left[ b_i^1 ((\dot{Q}^*)^{-2/15} + b_i^2) + b_i^3 (\dot{Q}^*)^{-2/15} ((\dot{Q}^*)^{-2/15} + b_i^4) y^{1/3} \right]} \right] \right] \right\} \quad (6)$$

$$+ H \left[ - \left[ (1 + a^{-2} b_i^4 (\dot{Q}^*)^{-2/15}) + a^{-2} (\dot{Q}^*)^{-2/15} (b_i^4 + (\dot{Q}^*)^{-2/15}) y^{1/3} \right] \right]$$

$$+ \sum_{i=1}^2 c_i^0 \ln \left[ \frac{((\dot{Q}^*)^{-2/15} + c_i^1)^2 + c_i^2}{((\dot{Q}^*)^{-2/15} y^{1/3} + c_i^3)^2 + c_i^4} \right]$$

### 3. RESULTS AND DISCUSSIONS

#### 3.1 Large hall fire

The first compartment considered to the evaluation of the mathematical models has external dimensions 27.6 m × 18.1 m with a height of 30.6m, and internal dimension of 22.4 m × 11.9 m and a height of 27 m, being here denominated as a large hall. This large hall fire scenario is built to study the smoke movement. The compartment has only one little vertical square opening of 0.2 m left open to the fresh air. One oil burner is placed in the floor center.

Figure 1 represents the geometry of this scenario. More details of this fire scenario are available in Li et al. (1999).

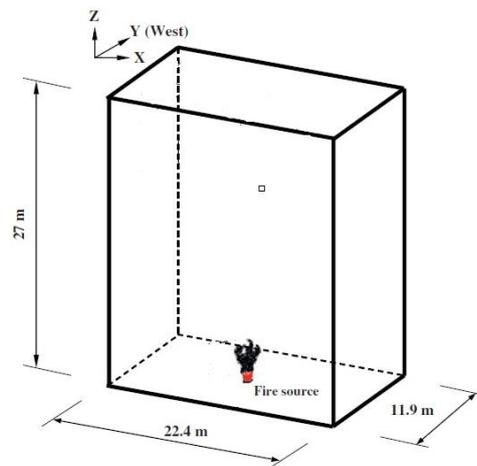


Figure 1. Geometry of problem (Li et al., 1999).

In Figure 2 it is presented the comparison results obtained for the compartment cited above, where the experimental data reported by Li et al. (1999) are compared with the results obtained through the fundamental solution of the smoke equation (Karlsson and Quintiere, 2000) and its analytical solution (Novozhilov, 2012). It is observed that the experimental data and the mathematical results are close to each other when the fundamental solution of the smoke equation is considered, but the results obtained with the analytical solution are not compatible with the experiments, indicating that this analytical solution cannot be used for this kind of compartment (a large hall) or that the fire conditions are not the same as that employed to develop the analytical solution.

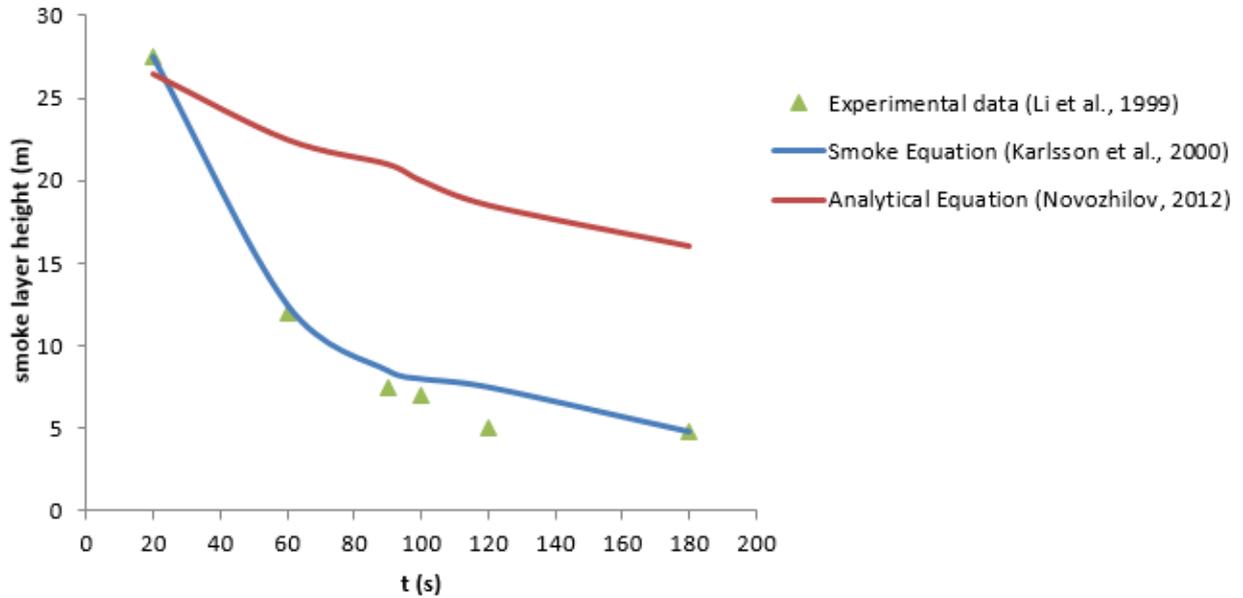


Figure 2. Descent of smoke with high heat release rate of 4 MW.

### 3.2 Tunnel fire

Figure 3 shows the 1/6th scale physical tunnel model of Gao et al. (2016). The tunnel is 6 m long, 2 m wide and 0.9 m high. Its aspect ratio was determined based on a survey of 17 urban road tunnels in Beijing, Nanjing and Shenzhen in China. To estimate the effect of natural ventilation on smoke layer interface height, a vertical shaft, with cross section of 30 cm  $\times$  30 cm, 0.8 m in height, was set 4.2 m from the left opening. The side near the aisle was made up of 6 mm thick fire-resistant glass to observe the experimental phenomena, the top, bottom and the other side of tunnel were 8 mm thick fireproofing board. The two tunnel openings in the experiments are open to the environment and the air supply velocity at the openings is only caused by the fire source and the smoke exhausting shaft (for the cases with natural ventilation). The ambient temperature ranged between 15–17°C in the experiments. More details about this fire scenario are available in Gao et al. (2016).

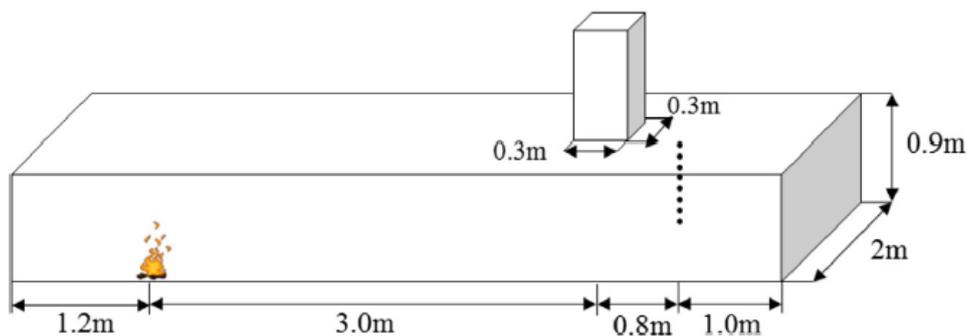


Figure 3. Geometry of problem (Gao et al., 2016).

In this tunnel were performed two experiments, due to the fact that Gao et al. (2016) performed the experiment with distinct HRRs. The results shown in Fig. 4 are for HRR = 82 kW and those in Fig. 5 are valid for HRR = 127 kW. As in the previous scenario, Figures 4 and 5 presents a comparison of experimental data (Gao et al., 2016) and the results obtained through of the fundamental solution of the smoke equation and its analytical solution. Note that in these cases the experimental data and the analytic solution are agrees reasonably well, being this solution indicated to this kind of compartment (with elongated horizontal dimension, i.e., a tunnel), while the results obtained with the fundamental equation was not compatible with the experiment.

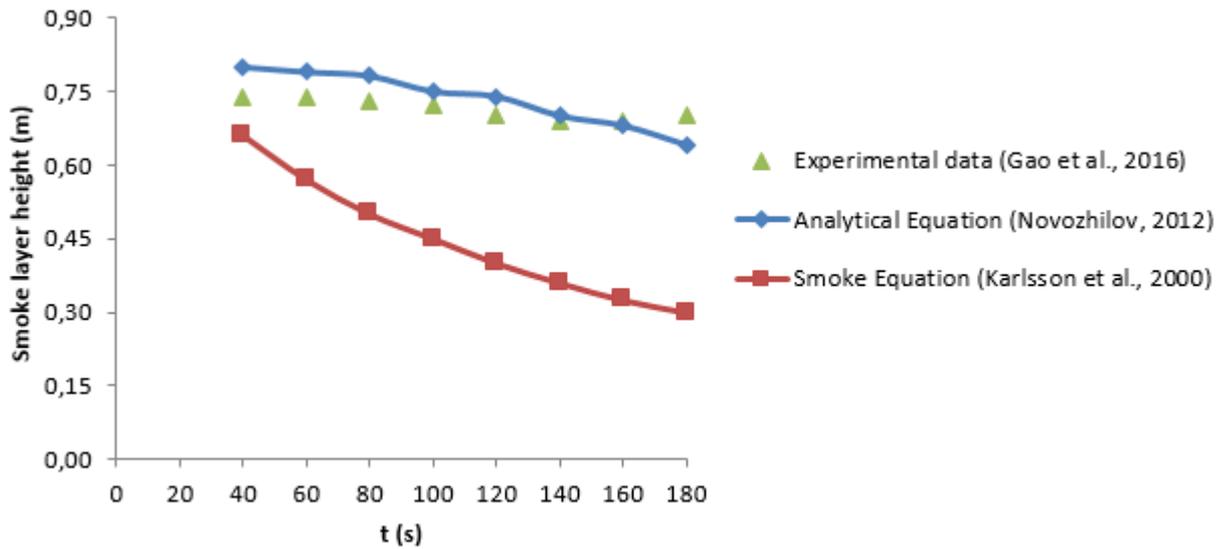


Figure 4. Descent of smoke with high heat release rate of 82 kW.

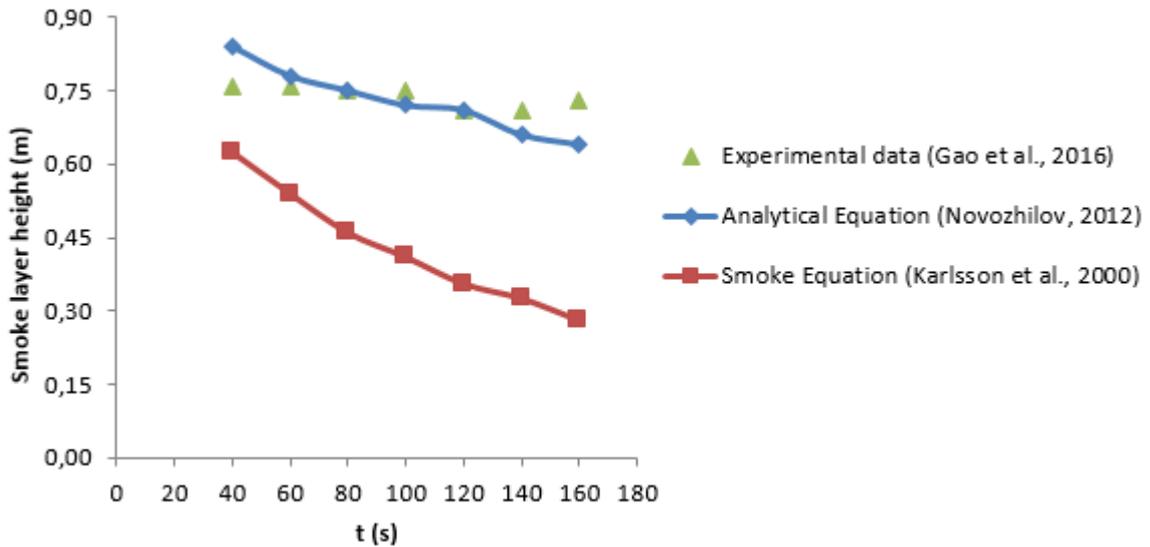


Figure 5. Descent of smoke with high heat release rate of 127 kW.

### 3.3 Room fire

The third compartment employed to the evaluation of the mathematical models have an external dimensions 5.4 m  $\times$  8 m with a height of 3 m as seen in the Room 1 in Figure 6. This hall fire scenario is built to study the smoke movement by Lai et al. (2016). The compartment have only one little vertical opening of 2 m  $\times$  0.8 m left open to the fresh air. One oil burner is placed on the floor center. Figure 6 represents the geometry of the proposed problem. More details of this fire scenario are available in Lai et al. (2016).

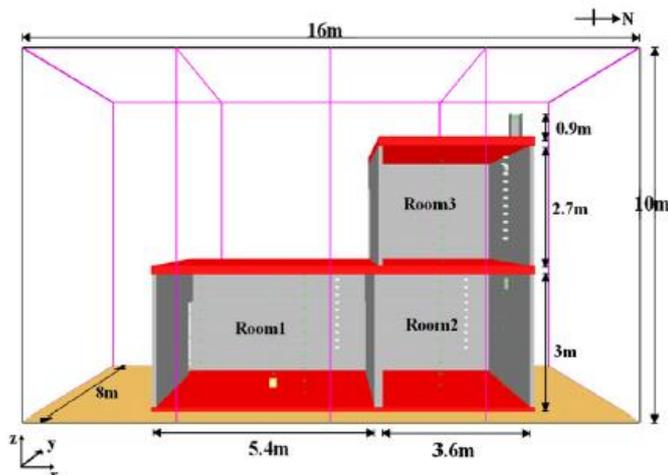


Figure 6. Geometry of problem (Lai et al., 2013).

An analysis in the smoke layer thickness was performed in Room 1, like as previously described. Figure 7 presents a comparison between the experimental data of Lai et al. (2013) and the mathematical solutions. It was obtained a better agreement when comparing the experiment to the fundamental solution of the smoke equation than to its analytic solution. In this fire scenario, the present work considered only Room 1 in the analysis, since that is the room where the oil burner was located. Despite the fundamental solution presented a better agreement with the experiment, note that until 100 s the analytic solution showed a good result when compared with the experiment, but after this time interval the experimental slope of the smoke thickness was not captured by the analytical solution, while the fundamental solution was capable to correctly model such behavior.

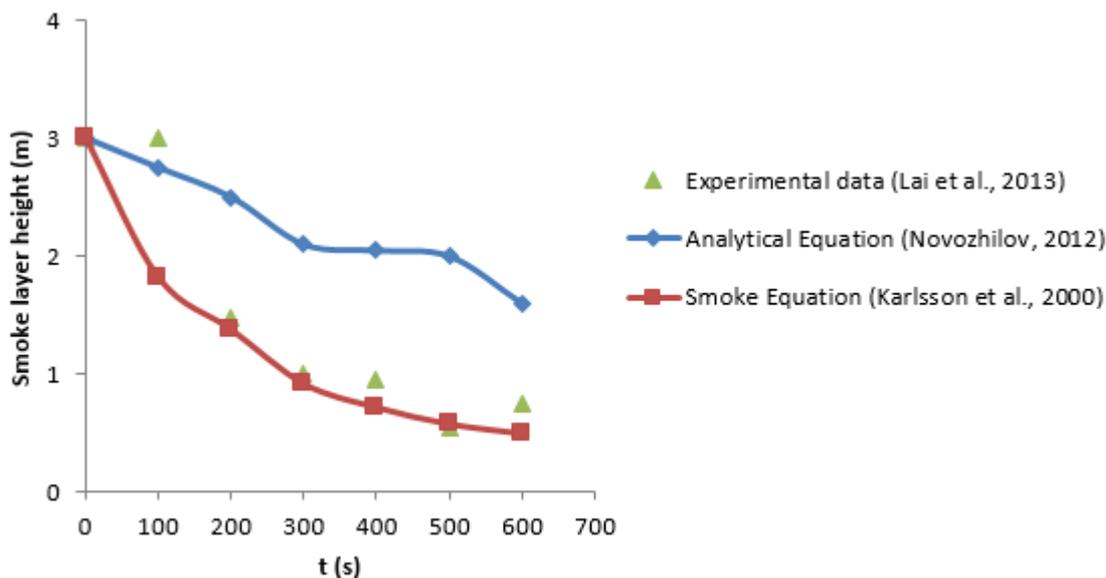


Figure 7. Descent of smoke with high heat release rate of 50 kW.

### 3.4 Laboratory fire

Yi et al. (2005) performed experiments in a laboratory located in China, at The Hong Kong Polytechnic University (PolyU) and at University of Science and Technology of China (USTC). The laboratory atrium have  $22.4 \text{ m} \times 12 \text{ m}$  and 57 m of height. In the compartment, has an opening (window) of  $1.4 \text{ m} \times 1.1 \text{ m}$ , centered in one of the walls of 22.4 m length, with sill of 10.5 m. The environment temperature was  $10^\circ\text{C}$  and the fire duration time was 560 s. Figure 8 represents the geometry of the proposed problem. More details of this fire scenario are available in Yi et al. (2005).

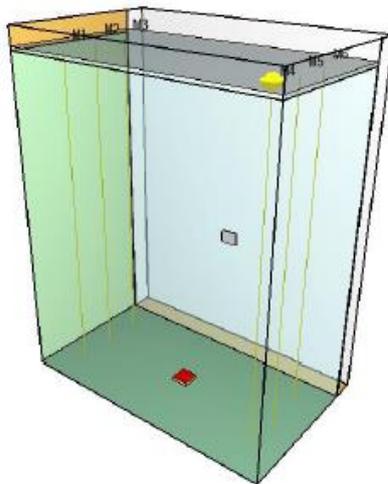


Figure 8. Geometry of problem (Yi et al., 2005).

The experimental data obtained by Yi et al. (2005) for the smoke layer thickness evolution is compared in Figure 9 with the fundamental solution of the smoke equation and its analytical solution. It is observed that the geometry of this fire scenario and that presented in Section 3.1 (Li et al., 1999) are very similar, changing some physical properties due to the oil burner and the HRR. In both fire scenarios, the fundamental solution of the smoke equation agreed well with the experiments, while the analytical solution did not.

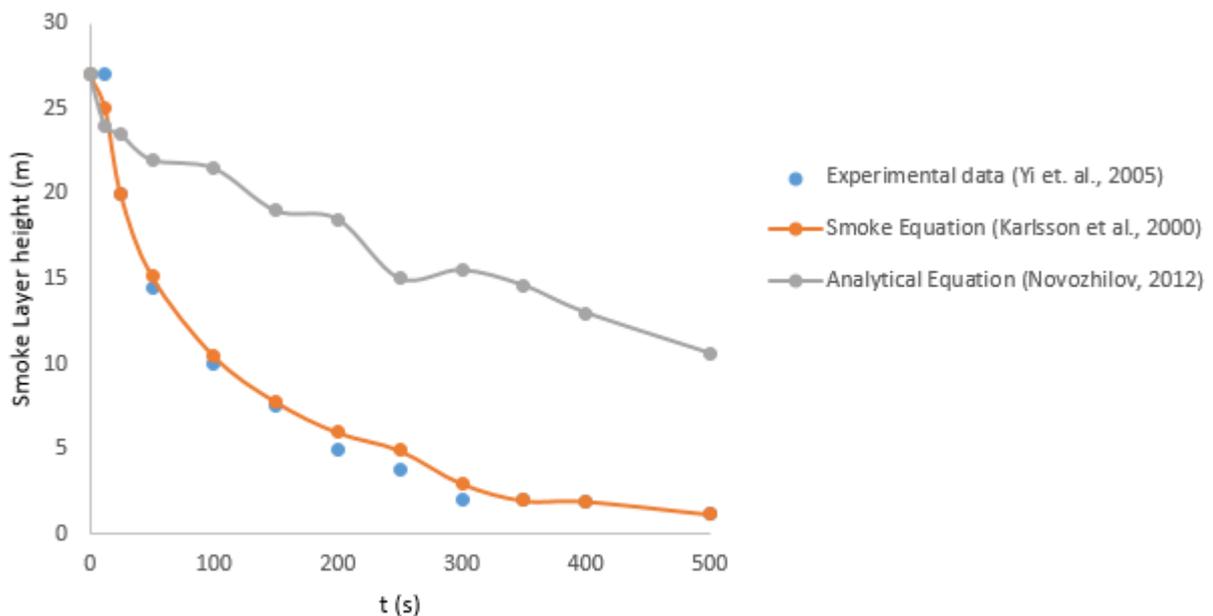


Figure 9. Descent of smoke with high heat release rate of 3 MW.

#### 4. CONCLUSIONS

In this work was performed a study about smoke layer height in different fire scenarios. Through this analysis it was made comparisons of the experimental data to the fundamental solution of the smoke equation (Karlsson and Quintiere, 2000) and to the analytical solution (Novozhilov, 2012). The first case analysed was the experiment performed by Li et al. (1999), where we obtained a good agreement when comparing the experimental data to the fundamental solution of the smoke equation. The second case analysed was the experiment performed by Gao et al. (2016), where we obtained a compatible result when it was made a comparison with the analytical solution. The third case analysed was the experiment performed by Lai et al. (2013), where it was obtained a compatible result when it was made the comparison with the fundamental solution of the smoke equation. Finally, the fourth case analysed was the

experiment performed by Yi et al. (2005), where it was obtained a compatible result when it was made a comparison with the fundamental solution of the smoke equation.

In general, the results of this study can be summarized as follows: (i) concerning the fire scenario geometry, the fundamental solution of the smoke equation presented good agreement with experiments when the fire scenario was compatible with the two zone model, in which the aspect ratio of the compartments are close to a cube, while in the tunnel fire the compartment is elongated, and in that scenario the fundamental solution worked badly (on the other side, the analytical solution agreed well with tunnel experiments, but this outcome must be better studied in order to make it possible to generalize such applicability of the analytical solution) and (ii) concerning the HRR range, the fundamental solution of the smoke equation agreed well with the fire scenarios with the highest and the lowest HRRs, while it was not good for the intermediate HRR (the contrary is valid for the analytical solution).

The continuity of this research is focused on evaluating the fundamental solution of the smoke equation, its analytical solution, and some additional smoke filling models, for a range of other fire scenarios (other tunnels, other room, other atriums, etc.), in order to make it possible to understand in which fire scenarios each smoke filling model is applicable and, if not, develop new models for some specific fire scenarios.

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