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INTELLINGENT POSITION CONTROL IN A MULTIVARIABLE SYSTEM WITH TWO DEGREE OF FREEDOM

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Abstract. *In this paper, the control of the position of a multivariate mechatronic system with two degrees of freedom is performed by an intelligent controller, which has its performance compared to a PD controller. The system consists of two servo motors (SVR02) of Quanser interconnected by four rigid bars. The tuning of the Fuzzy controller is done empirically and the tuning of the PD controller is done by approaching the transfer function of the system to the canonical form of a second order linear system. The results show that the Fuzzy controller obtained a lower Integral Absolut Error (IAE) and Integral of the Time weighted Absolute Error (ITAE), whereas the PD controlled obtained a lower Goodhart Index (GI). The Fuzzy controller proved to be more efficient in following the previously established trajectory, it presented a smaller error in transient regime and steady regime, with a less oscillatory control signal.*

Keywords: *Position Control, MIMO System, Fuzzy Controller*

1. INTRODUCTION

Nowadays servomechanisms appear prominently in the world industry, provided by its high precision and the need for a simple control system that have facilitated the massification of this electromechanical mechanism in the market (McComb, 2002). In the midst of this industrial universe, a large number of this servo mechannisms of simple functionality are coupled to perform more complex functions. The combination of these machines provides a system with several degrees of freedom and with multiple control variables. From the servo it is possible to create a closed-loop system, meaning the output of these devices is coupled to a control circuit which controls their response in the form of a precise angular rotation in view of a digital input applied to this device resulting in many advantages, such as in the field of robotics (McComb, 2002).

Among the controllers capable of operating in these systems mentioned above, the Proportional, Integral and Derivative (PID) controller and its derivations remain the most used even with the development of a wide range of control strategies in the last decades (Tang *et al.*, 2001). Its low number of tunable parameters, applicability and robust performance in various scenarious corroborate its great usage. In 1942, Ziegler and Nichols presented works that used empirical methods to aid the tuning of these controllers in systems with single input and single output (SISO) (Ziegler and Nichols, 1942) and later other manual and automatic methods were devoloped for this same tuning (Åström and Hägglund, 2001). Thus, with the advancement of computing, PID controlers are the type of controller most used in industry, since they are basically developed and tuned by using a programable logic controler (PLC), which with its flexibility, low cost, robustness and availability of functional hardware blocks making the PID controller just another computer program in the PLC memory (Simoes and Shaw, 2007).

However, the designer of a controller always has the following disadvantages when devoloping a PID: the imprecise parameter and dimensional values, the lack technical qualification and nonlinearities identified in real systems undermine the performance of the PID controller for each project where it is used (Simoes and Shaw, 2007).

The existence of a distance between man and machine is notorious even nowadays (Kirby *et al.*, 2010) and therefore the divergence between the creative capacity of human beings with the possible solutions presented by computers is still felt because in the real world everything is a matter of point of view (Simoes and Shaw, 2007). This gap exists because human reasoning is uncertain, inaccurate, diffuse and hazy, while machines work precisely and binary. One way to diminish this divergence is by using the concept of Fuzzy logic that is able to capture vague information in general, described in natural language and therefore full of uncertainties as well as in human communication and convert them to a numerical set of easy manipulation (Wagner, 2003) to make computers "reason like humans" (Kirby *et al.*, 2010; Simoes and Shaw, 2007)

The use of Fuzzy logic in solutions to problems of control systems began with the work of Mamdani (Mamdani, 1974; Mamdani and Assilian, 1975). Later Takagi and Sugeno proposed a new simplified method of decision making, also based on Fuzzy logic (Takagi and Sugeno, 1983). These papers ground the current studies of the Fuzzy controllers as solutions to problems of position control (Dursun and Durdu, 2016; Precup and Hellendoorn, 2011). In this context where the Fuzzy logic incorporates the of way of thinking in a control system, a typical Fuzzy controller can be designed to behave according to the deductive reasoning, in which people infer conclusions based on information already known. This way human operators can control industrial processes and control plants with non-linear characteristics and even with little known dynamic behavior, through experence and inference in relation of the process control variables (Simoes and Shaw, 2007).

Scientific contributions have been generated with the application of modern control techniques, as is still the case with Fuzzy. However, because the widely usage of PID controllers, this kind of controller continues to be a reference when the ontroller designer is proposing the use of a more advanced control technique, so if possible, a comparison is made with a PID family controller. This paper presents a proposal of using a Fuzzy controller to meet the performance specifications of the servo type problem in a multivariable plant with two degrees of freeedom that was tuned heuristically. A classic PD controller was also designed manually and implmented so that comparisons can be made with the performance of the Fuzzy controller.

The structure of the paper is arranged as follows: this section presents a brief introduction about servo motors, classical controllers and inteligent controllers. In section two the model of the plant used in this paper will be presented. In section three it is presented the methodology and tuning of the used controllers. In section four the results of each of the controllers are presented and discussed. Finally, section five concludes the article and leave suggestions for possible future work.

2. SYSTEM MODELING

The two degree of freedom (2-DOF) robot is a mechanism with two servo motors (A and B) manufactured by Quanser (SRV02) that are interconnected by four rigid bars (Fig. 1), system multivariable with two inputs and two outputs and two degrees of freedom. The SRV02 consists of a DC motor equipped with sensors (encoders) capable of measuring the angular position of the load gear and form the direct kinematics of the system so it is possible to find the junction point of these bars called the point E or the point of interest.

Figure 1: Physical system.
Available from: (Quanser, 2011)

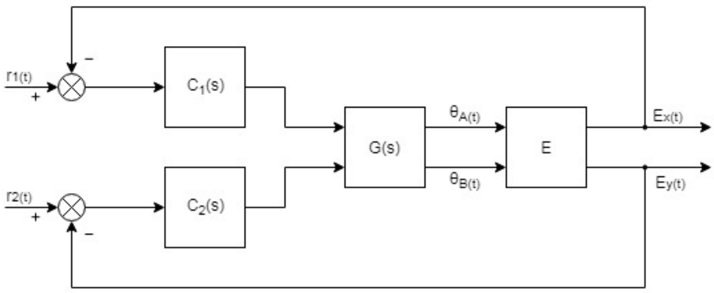

Figure 2: Closed-loop block diagram.
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Figure 2 represents the closed-loop system. Each servo motor is controlled by a controller (C_1 and C_2) in which its input is given by the error generated between the coordinates of the measured position of point E ($E_x(t)$ and $E_y(t)$) and the coordinates of your desired position ($r_1(t)$ and $r_2(t)$). The transfer function of the variation of the angle of rotation of each motor as a function of the tension applied in open mesh is defined by Eq. (1).

$$G'(s) = \frac{k_m}{s(\tau s + 1)}, \tag{1}$$

where $k_m = 1.53\ rad(Vs)^{-1}$ and $\tau = 0.0414\ s$ (Quanser, 2011). The angular position of each of the gears is changed and a new point E is obtained through the direct kinematics each time the controller acts on the plant.

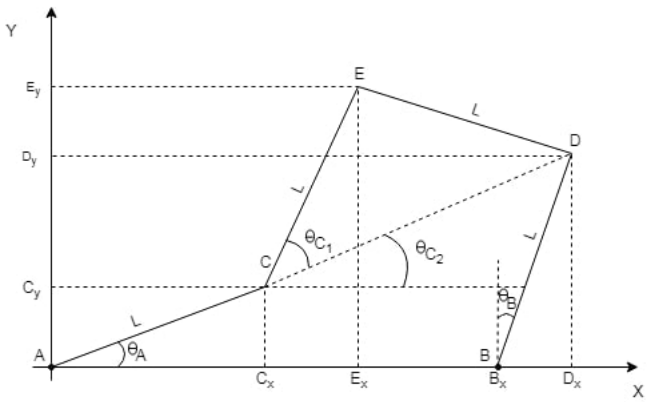

Figure 3: Direct kinematics.
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Figure 3 is a schematic representation of Fig. 1, the angles θ_A and θ_B represent the angular position of each of the gears. The parameter L represents the length of each of the bars and θ_{C_1} and θ_{C_2} are angles that aid in the construction of direct kinematics. It is possible to find that the position of the point E in the Cartesian plane is defined by Eq. (2) and Eq. (3) using the angles of rotation of each servo (θ_A and θ_B) if we consider that all bars have the same length L (12.7 cm) and that the distance between servos are always constant (25.4 cm).

$$E_x = Lcos(\theta_A) + Lcos(\theta_{C_1} + \theta_{C_2}) \tag{2}$$

$$E_y = Lsin(\theta_A) + Lsin(\theta_{C_1} + \theta_{C_2}) \tag{3}$$

Where θ_{C_1} and θ_{C_2} are described by,

$$\theta_{C_1} = cos^{-1}\left(\frac{\sqrt{(Lcos(\theta_B) - Lsin(\theta_A))^2 + (2L - Lcos(\theta_A) - Lsin(\theta_B))^2}}{2L}\right) \tag{4}$$

$$\theta_{C_2} = tg^{-1}\left(\frac{Lcos(\theta_B) - Lsin(\theta_A)}{2L - Lcos(\theta_A) - Lsin(\theta_B)}\right) \tag{5}$$

3. TUNER OF CONTROLLERS

The tuning of a controller is directly connected to the system and the type of problem to which it will be inserted. In this paper, a plant starting from repose must follow a previously stipulated trajectory according to some performance specifications. Initially the controllers C_1 and C_2 will be of the Fuzzy type with the same parameters in each of them and later other results will be obtained for the controllers C_1 and C_2 of the Proportional Derivative (PD) type with the same gains in each of them. The trajectory of the point E has the shape of the letter F when drawn in the Cartesian plane and is constructed of multiple steps stimulated for each servo motor.

The performance specifications help to delineate what criteria the controller must achieve in order for the control to be classified as satisfactory, for example the transient and steady regime characteristics. It is defined that the rise time until the reference will be up to 0.15 seconds with maximum overshoot of 2% and accommodation time of 0.25 seconds for the transient regime. In the permanent regime will be adopted the criterion of the 2% (Campos and Teixeira, 2006) that considers an acceptable response within the region of ± 2% around the reference.

The effectiveness of each of the two types of controllers will be attributed to how close to the ideal trajectory the plant was presented and which of them proved to be less harmful to the physical integrity of the servo motors and the evaluations will be aided by performance indexes. The first index used will be the Integral Absolut Error (IAE), the system performance is evaluated by Eq. (6) and does not have any weighting in the whole error signal (Campos and Teixeira, 2006).

$$IAE = \frac{1}{N}\sum_{k=1}^{N} |e(k)| \tag{6}$$

The second index will be the Integral of the Time weighted Absolute Error (ITAE) evaluated by Eq. (7) an initial error in the response to the step is weighted with small weight and errors that occur later are greatly penalized (Campos and Teixeira, 2006).

$$ITAE = \frac{1}{N}\sum_{k=1}^{N} t|e(k)| \tag{7}$$

And the third index will be the Goodhart Index (GI) evaluated Eq. (8) an importance is given to the control signal and the system error (Goodhart *et al.*, 1994).

$$GI = \alpha_1\epsilon_1 + \alpha_2\epsilon_2 + \alpha_3\epsilon_3, \tag{8}$$

where α_1, α_2 and α_3 are weights assigned to ϵ_1 (medium control effort) Eq. (9), ϵ_2 (variance of the control signal) Eq. (10) and ϵ_3 (mean of the quadratic error) Eq. (11).

$$\epsilon_1 = \frac{1}{N}\sum_{k=1}^{N} |u(k)|, \tag{9}$$

$$\epsilon_2 = \frac{1}{N}\sum_{k=1}^{N} (|u(k)| - \epsilon_1)^2, \tag{10}$$

$$\epsilon_3 = \frac{1}{N}\sum_{k=1}^{N} (r(k) - y(k))^2, \tag{11}$$

where $u(k)$ is the control signal, $r(k)$ is the reference signal and $y(k)$ is the output signal. For this paper the parameters of Eq. (8) are given by Tab. 1.

Table 1: Parameters Goodhart Index.

α_1	α_2	α_3
2.0	0.75	0.25

3.1 FUZZY CONTROLLER

In classical theory, an x element of the universe in speech belongs or does not belong to a set and in the theory of Fuzzy sets this element may belong partially to a set. From a universe of discourse U (numerical range of all possible values that a variable can assume) a Fuzzy set A is defined by a membership function μ that ranges from zero to one within this universe (Zadeh *et al.*, 1965).

$$\mu_A : U \rightarrow [0, 1] \tag{12}$$

In the literature there are several forms of function of pertinence, for example trapezoidal and triangular, its parameters are determined by the experience of the designer in controlling the plant. The functions of pertinence have the objective of matching a value or a linguistic variable in Fuzzy sets. The values of the linguistic variable are names of Fuzzy sets defined in 5 parts (x, $A(x)$, U, G, M), x is the name of the variable, $A(x)$ is the set of terms (the set of language value names of x), U is the universe of discourse, G is the grammar to generate the names and M is a set of semantic rules to associate each x with its meaning (Bauer *et al.*, 1996). Figure 4 illustrates the linguistic variable $error$ with the nebulous terms given by $Negative$, $Zero$, $Positive$.

A Fuzzy controller is composed of a fuzzyfication interface, a database and rules (knowledge base) and a defuzzification interface (Zadeh *et al.*, 1965) Fig. 5. In the literature other variations appear depending on the purpose of the project.

The fuzzification interface is responsible for mapping the actual input values to Fuzzy domain values, the mapped values are transformed into linguistic variables defined by the membership functions. The knowledge base consists of a database and a rule base, containing all the knowledge required to control the system. The data and rules are represented in an easily understood linguistic form from premises of the type: If <condition> Then <conclusion>. The database stores the definitions of the membership functions and the discretization and normalization of the discourse universes and the rule base is formed by implication operators, composing the rules that represent the knowledge represented in the Fuzzy system. The procedure of inference consists of verifying the degree of compatibility between the facts and the clauses in

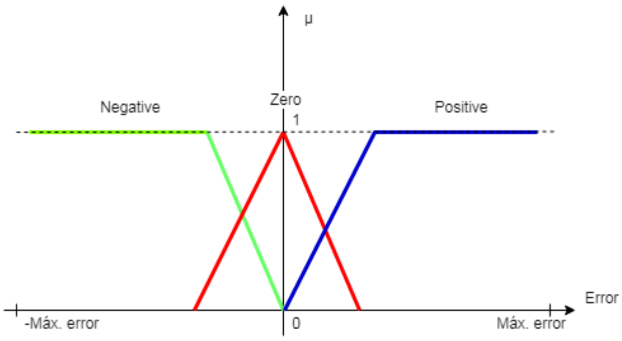

Figure 4: Linguistic terms map variable error.
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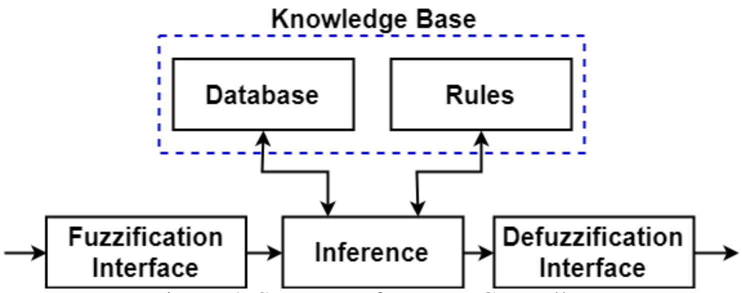

Figure 5: Structure of a Fuzzy Controller.
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the premises of the rules, determine the degree of total compatibility of the premise of each rule, determine the value of the conclusion according to the degree of compatibility of the rule with the data and the action and constant control in the conclusion, aggregate the obtained values as conclusion in the various rules obtaining a global control action (Simoes and Shaw, 2007).

The literature presents several models of Fuzzy systems, classified basically as the classic models, emphasizing the Mamdani model (Mamdani, 1974) and interpolation, especially the Tagaki-Sugeno-Kang model (TSK) (Sugeno and Kang, 1988). Each Fuzzy model will present an approach for the representation of the premises and representation of control actions. In the TSK model all the nebulous terms of the premise must be monotonic functions and the consequent of each rule is represented by a function of the input variables ($f_i(x_1, x_2, ..., x_n)$) that for each of the rules a corresponding weight is obtained (ω_i). In the Fuzzy interpolation systems models there is no defuzzification interface, the output is obtained directly and for the TKS model the output is, commonly, defined by a weighted average of the outputs generated by each of the rules (Driankov *et al.*, 1993),

$$y = \frac{\sum_{i=1}^{k} \omega_i f_i(x_1, x_2, ..., x_n)}{\sum_{i=1}^{k} \omega_i}, \tag{13}$$

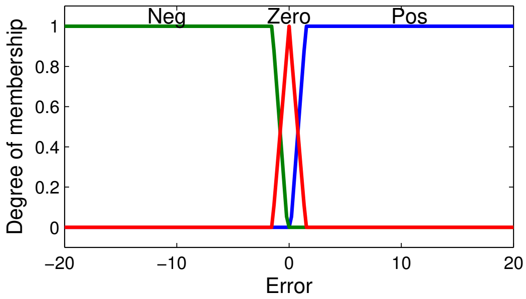
when y is the output of the controller, $x_1, x_2, ..., x_n$ is the inputs, k is the number of rules, ω_i is the result of the inference made for the i-th rule based on the values generated by the Fuzzyfication interface for the inputs $x_1, x_2, ..., x_n$ and $f_i(\cdot)$ is the i-th sugeno output function,

$$f_i(x_1, ..., x_n) = c_{1,i}x_1 + ... + c_{n,i}x_n + c_{n+1,i}, \tag{14}$$

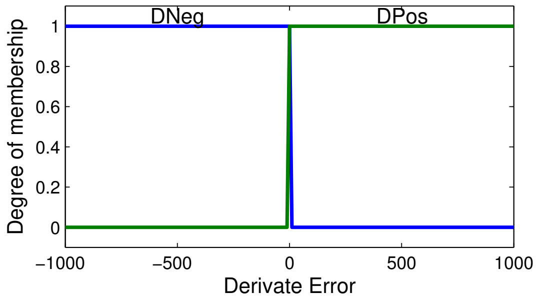
where each c_k is a constant. The parameter values of the membership functions and the values of the sugeno output function were obtained empirically until the performance specifications presented were met. Figure 6a presents the functions of pertinence tuned for the input $Error$ and Fig. 6b presents the functions of pertinence tuned for the input $Derivate\ Error$.

After defining the parameters of the pertinence functions for both inputs ($Error$ and $Derivate\ Error$), the sugeno output functions quantified by Tab. 2.

Finally, after defining the parameters of the membership functions for both system inputs and the parameters of the sugeno output functions, it was possible to construct the Fuzzy Associative Matrix (FAM) as shown in Tab 3.



(a) Error Membership Function.



(b) Derivate Error Membership Function.

Figure 6: Membership Function.
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Table 2: Sugeno output functions.

Output Functions	c_1	c_2	c_3
Small Positive (SP)	8.00	0.25	0
Small Negative (SN)	5.00	0.25	0
Big Positive (BP)	15.00	0.10	0
Big Negative (BN)	9.00	0.10	0

Table 3: Fuzzy Associative Matrix.

Rules	**Derivate Error**	
Error	DNeg	DPos
Neg	BN	BN
Zero	SN	SP
Pos	BP	BP

3.2 PD CONTROLLER

In an alternative parallel Proportional Derivative (PD) controller, the Proportional action equals the product of the error signal by proportional gain K_P and the Derivative action is equivalent to the product of the derivative of the error signal by the inverse of the derivative gain K_D (Campos and Teixeira, 2006) where $K_D = \frac{1}{\tau_D}$.

$$G_c(s) = \frac{U(s)}{E(s)} = K_P + T_D s \tag{15}$$

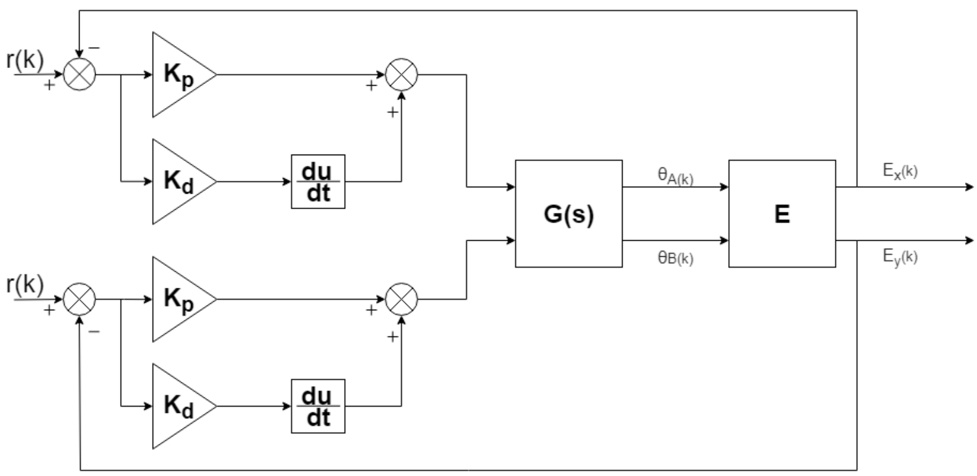

Figure 7: Closed-loop block diagram to PD controller.
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All tests were performed on the MATLAB/Simulink software according to Fig. 7. From Eq. (1), coupling a PD controller Eq. (15), defines that the angular position of the servo in relation to the closed-loop reference is given by,

$$\frac{\theta_l(s)}{\theta_d(s)} = \frac{k_m K_p(1 + \frac{1}{K_p K_d} s)}{\tau s^2 + s(1 + \frac{k_m}{K_d}) + k_m K_p}, \tag{16}$$

where $k_m = 1.53\, rad(Vs)^{-1}$ and $\tau = 0.0414\, s$ (Quanser, 2011). One way of quantifying the gains of each action of this controller is to approximate Eq. (16) to the canonical form of a second-order linear system (Åström and Hägglund, 1995),

$$C(s) = \frac{\omega_n^2}{s^2 + s2\xi\omega_n + \omega_n^2}, \tag{17}$$

where ω_n is the natural frequency of the system and ξ is the damping factor. From the established performance specifications, K_p and K_d values were tuned and in sequence by making real plant tests, a fine-tuning of the parameters to correct nonlinearities and other non-modeled plant parameters. Finally, the values of K_p and K_d that meet the criteria in the real plan are shown in the Tab. 4.

Table 4: Parameters PD controller.

Controller	**Kp**	**Kd**
PD	9.20	0.22

4. RESULTS

After tuned the two controller types, a trajectory was stipulated for the point E in order to analyze the performance of the Fuzzy and PD controllers. Figure 8a and Fig. 8b present the trajectory employed by the point E in the Cartesian plane using the Fuzzy controller and the PD controller respectively. Figure 9a and Fig. 9b present the responses of the servo motor A for each of the controllers, Fig. 10a and Fig. 10b present the responses of the servo motor B. Figure 11a and Fig. 11b present the signals generated by each controller for servo A, Fig. 12a and Fig. 12b present the signals generated by each controller for servo B and Tab. 6 present the performance indexes calculated for the Fuzzy controller and PD controller respectively.

Using the Fuzzy controller for the servo motor A the highest overshoot was 1%, the slowest rise time was 0.05 seconds and the longest accommodation time was 0.17 seconds. Also for the servo motor A, the PD controller had an overshoot of 1%, the slowest rise time was 0.15 seconds and maximum accommodation time was 0.2 seconds. For the servo motor B, using the Fuzzy controller, a maximum overshoot of 2% was recorded, the slowest rise time was 0.1 seconds and the longest accommodation time was 0.25 seconds while the PD controller had a maximum overshoot of 2%, the slowest rise time was 0.15 seconds and maximum accommodation time was 0.5 seconds.

From the results presented, it is possible to observe that only the Fuzzy controller has succeeded in following the trajectory within the previously imposed limits. Although the results presented by the PD controller for the servo motor A are in accordance with the performance specifications, the same does not occur for the servo motor B. The PD controller presented a rise time of up to three times higher when compared to the Fuzzy controller and its greater accommodation time exceeded the imposed limit of 0.25 seconds, not being able to stabilize the plant before the same one changed of reference as can be observed in Fig. 10b. The sum of the errors presented by the Fuzzy controller was 10% lower than that presented by the PD controller for the servo motor A and for the servo motor B as demonstrated in the Tab. 5 and Tab. 6. The best continuous regime performance is also generated by the Fuzzy controller which presented 26% (servo A) and 9% (servo B) of smaller error when compared to the PD controller according to ITAE (Tab. 5 and Tab. 6). However, as can be observed in the GI (Tab. 5 and Tab. 6 and Fig. 11 and Fig. 12 the Fuzzy controller presented a more aggressive behavior than the PD controller, it reached more than the saturation limit voltage of the system, but in a less oscillatory way.

5. CONCLUSION

This paper explored the use of controllers in a multivariate system with two degrees of freedom in order to control the position of the junction of four rigid bars between two servo motors of Quanser. The classical Proportional Derivative controller presented problems to meet all the established performance criteria. The Fuzzy controller was capable of controlling this position in order to contemplate all performance criteria.

The results presented by the Fuzzy controller prove to be better than the PD controller. The intelligent controller presented transient errors and lower steady state when compared to the classical controller and a less oscillatory control signal, on the other hand, presented a higher number of voltage peaks. It is concluded that the proposed Fuzzy controller is shown to be more appropriate to use than the PD controlled for 2-DOF robot system. Future work intends to explore techniques to replace the use of two controllers (C_1 and C_2) by a single controller (multivariable) in which it is able to control the position in a MIMO system with two degrees of freedom.

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7. RESPONSIBILITY NOTICE

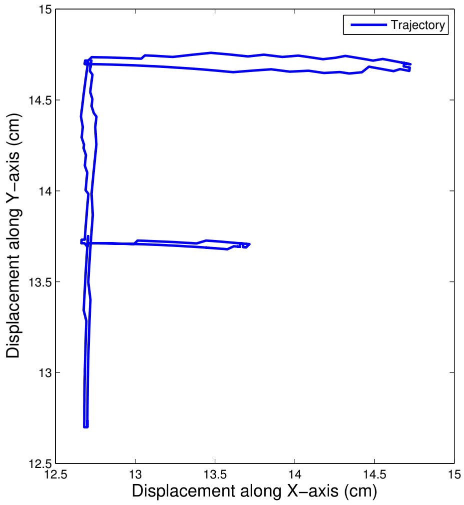
The authors are the only responsible for the printed material included in this paper.

Table 5: Index Fuzzy Controller.

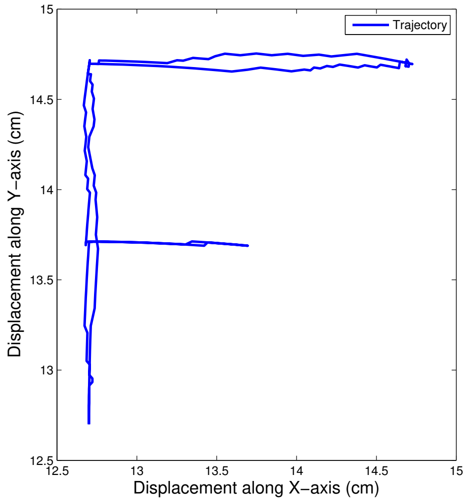
Fuzzy Controller	**Servo A**	**Servo B**
IAE	$2.8908\ 10^{-3}$	$4.1320\ 10^{-3}$
ITAE	0.8283	0.9568
GI	1.2872	0.9184

Table 6: Index PD controller.

PD Controller	Servo A	Servo B
IAE	$3.2301\ 10^{-3}$	$4.6110\ 10^{-3}$
ITAE	1.1128	1.0604
GI	0.7011	0.9428



(a) Point E Trajectory by using Fuzzy Controller.



(b) Point E Trajectory by using PD Controller.

Figure 8: Trajectory of Point E.
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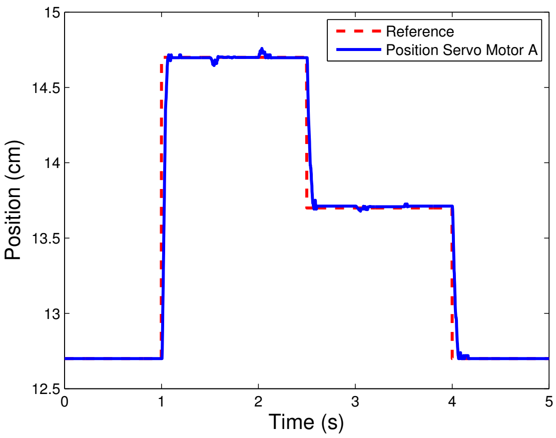
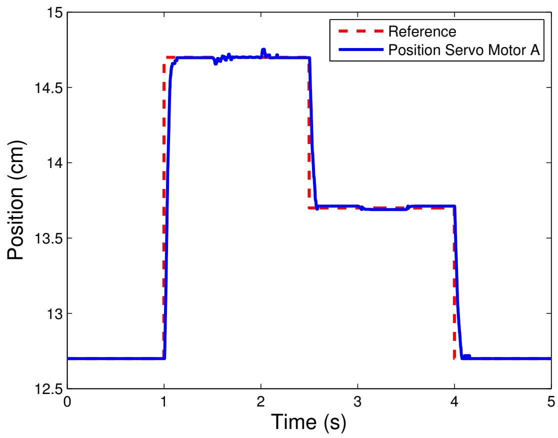
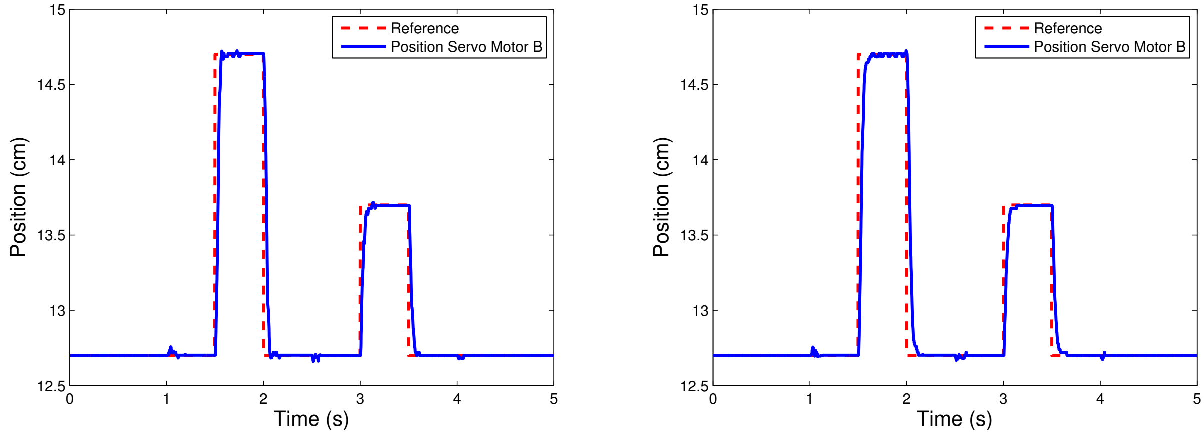


(a) Output Servo Motor A by using Fuzzy Controller.



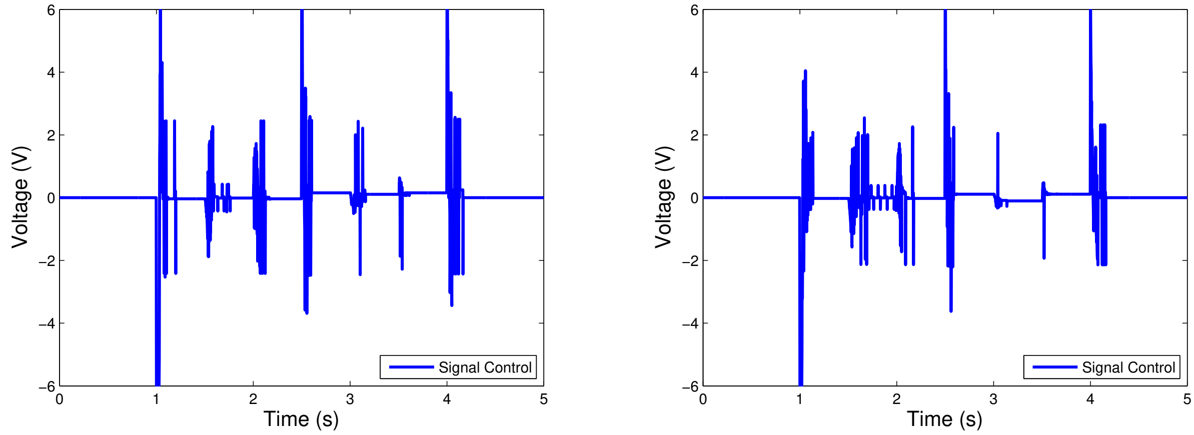
(b) Output Servo Motor A by using PD Controller.

Figure 9: Output Servo Motor A.
Available from: Author



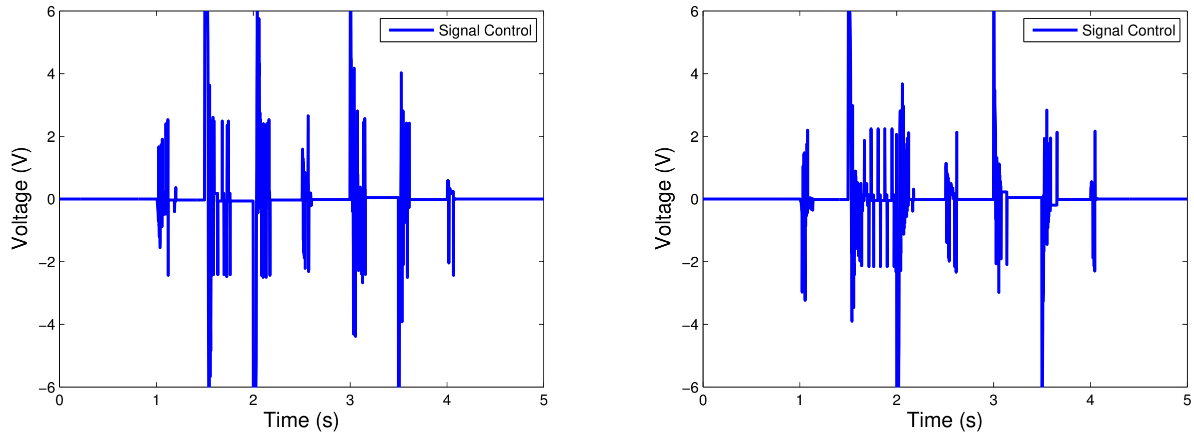
(a) Output Servo Motor B by using Fuzzy Controller. (b) Output Servo Motor B by using PD Controller.

Figure 10: Output Servo Motor B.
Available from: Author



(a) Control Signal Servo Motor A by using Fuzzy Controller. (b) Control Signal Servo Motor A by using PD Controller.

Figure 11: Control Signal Servo Motor A.
Available from: Author



(a) Control Signal Servo Motor B by using Fuzzy Controller. (b) Control Signal Servo Motor B by using PD Controller.

Figure 12: Control Signal Servo Motor B.
Available from: Author