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A STUDY OF THE LOAD SHARING OF KNEE LIGAMENTS

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Abstract. *The research in knee ligaments field has been developing quickly. Although this area has accumulated reasonable medical knowledge; the mechanical description of knee ligaments is still gathering contributions to generate a comprehensive knowledge about this matter. In fact, certain medical conditions could be better explained using mechanical modeling. In this work, a preliminary study is implemented to access the mechanical response of knee ligaments. In this study, four knee ligaments are used: anterior cruciate ligament (ACL), lateral collateral ligament (LCL), medial collateral ligament (MCL) and posterior cruciate ligament (PCL). Both analytical and numerical models were proposed. The analytical model comprehends the geometric, the constitutive and the share of loads of porcine ligaments.*

Keywords: *knee, ligaments, biomechanics*

1. INTRODUCTION

The study of knee ligaments can be done through the utilization of different approaches. Whereas some authors are concentrated in the description of the ligaments behavior, others are more focused in the modeling the knee as a whole, with a mechanical description of knee ligaments, that includes geometric parameters as: angles, forces, displacements and strains.

It can be mentioned some authors, that are concentrated in the description of the ligaments behavior, as Ristaniemi *et al*. (2018) which studied the characterization and comparison of material properties between the knee ligaments and patellar tendon. Dumbbell-shaped tensile test samples were obtained from bovine knee ligaments (ACL, LCL, MCL, PCL) and patellar tendon (PT) and they were subjected to tensile testing. Sinusoidal loading tests were also performed. The results indicate that LCL was more viscous than other ligaments at low-frequency loads. MCL was the stiffest and toughest, and its modulus increased most steeply at the toe-region, possibly a result of a greater amount of collagen. This study improves the knowledge about elastic, viscoelastic and failure properties of the knee ligaments and PT. Woo *et al*. (2006), made a good review paper providing an overview of the up-to-date of biological and biomechanical knowledge on normal knee ligaments, as well as ligament healing and its reconstruction after injury. Aalbersberg *et al.* (2005) made an interesting research investigating the effect of muscle activation on tendon orientation in vivo, with the utilization of magnetic resonance imaging (MRI) images of the knee, that were made during relaxation and isometric knee extensions and flexions. The orientation angles of six tendons, in sagittal and frontal plane were calculated and the angles were tabulated.

Authors, that models the knee as a whole, as Ragajopal *et al*. (2016) create an open-source software, using OpenSim, presenting a three-dimensional musculoskeletal model with high-fidelity representations of the lower limb musculature of healthy young individuals thought to simulations of gait. Its results provide an important source of muscle and ligaments behaviour during gait.

Zheng *et al*. (1998) proposed an analytical model of the knee joint to estimate the forces at the knee during exercise. Muscle forces were estimated based upon electromyographic activities during exercise and during maximum voluntary isometric contraction (MVIC). That proposed model provided a good estimation of knee forces during exercises, preventing significant overestimates of tibiofemoral compressive forces and cruciate ligament tensions. Shelburne and Pandy (1997) proposed analytical models, where a quadriceps force was applied to extend an intact cadaveric knee quasi-statically, for a given angle of knee flexion, which defined a system of five equations with five unknowns. The unknowns were the anterior-posterior tibia1 translation, the proximal-distal tibia1 distraction, the patellar-ligament force, the tibiofemoral contact force, and the quadriceps force, needed to equilibrate the lower leg. The solution of this system of equations used, iteratively, a nonlinear programming algorithm.

Numerical models of knee ligaments have been proposed to estimate the acting forces, as in Galbusera *et al*. (2014), which did a literature review about Finite Element approaches in this area. Peña *et al*. (2006), presents both analytical and finite element models in a very detailed way, with complete results using principal stresses developed in the knee ligaments. Experimental approaches are also have been utilized, as in Slane *et al*. (2017), which uses an ultrasound-based method to characterize non-uniform strains in large energy-storing tendons.

The current work present simple models, for both analytical and numerical approaches. Also, an exploratory experimental approach has began, using a strain gage, to measure a porcine PCL strains during tensile loading.

2. ANALYTICAL MODEL

The proposed analytic model uses mechanics of solids to estimate the load share between four porcine ligaments, namely: ACL, LCL, MCL, PCL. The model considers a 2D representation of the knee of porcine ligaments, in the coronal plane. It is supposed that the ligaments are in a parallel arrangement, being submitted to the same displacement and sharing forces in function of its own stiffnesses. They are renamed as follows: ACL = 1, LCL = 2, MCL = 3 and PCL = 4. Figure 1 shows the geometric representation of the ligaments of a porcine knee.

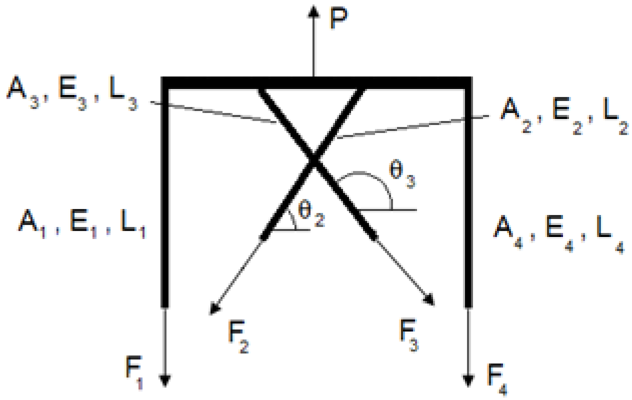


Figure 1. Simplified analytical model.

The model uses (1) as vertical equilibrium condition and (2) as compatibility condition.

$$F_1 + F_2\sin(\theta_2) + F_3\sin(\theta_3) + F_4 = P \tag{1}$$

$$\delta_1 = \delta_2 = \delta_3 = \delta_4 \tag{2}$$

Note that $\delta_i = \frac{F_i L_i}{A_i E_i} = \frac{F_i}{K_i}$ where $K_i = \frac{A_i E_i}{L_i}$ for i = 1, 2, 3 or 4. So, (2) can be re-written as:

$$\frac{F_1}{K_1} = \frac{F_2}{K_2} = \frac{F_3}{K_3} = \frac{F_4}{K_4} \tag{3}$$

Where, F_i are forces, P is the resultant of forces F_i, θ_2 and θ_3 are angles, δ_i are displacements, A_i are transversal areas, E_i are elasticity modulus, L_i are ligament lengths, K_i are stiffnesses of the ligaments. For the ACL and PCL, respectively:

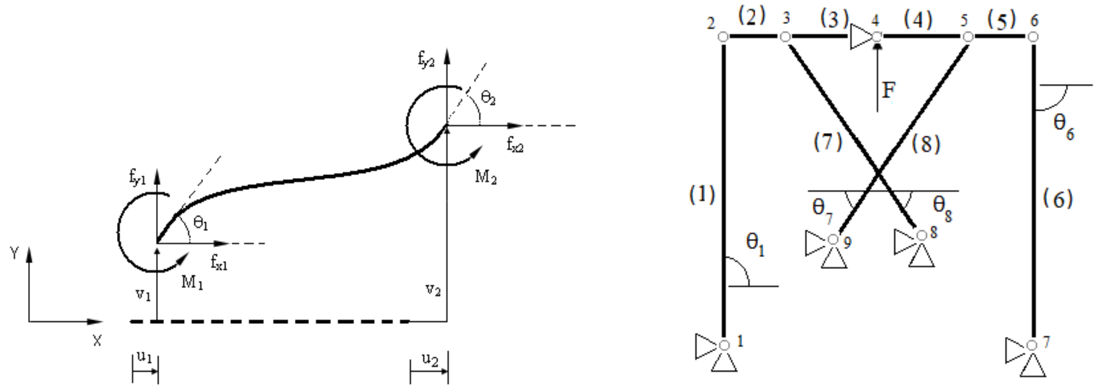
$$F_1 = \frac{P}{1+\frac{K_2}{K_1}\sin(\theta_2)+\frac{K_3}{K_1}\sin(\theta_3)+\frac{K_4}{K_1}} \qquad F_4 = \frac{P}{\frac{K_1}{K_4}+\frac{K_2}{K_4}\sin(\theta_2)+\frac{K_3}{K_4}\sin(\theta_3)+1} \tag{4}$$

For the LCL and MCL, respectively:

$$F_2 = \frac{P}{\frac{K_1}{K_2}+\sin(\theta_2)+\frac{K_3}{K_2}\sin(\theta_3)+\frac{K_4}{K_2}} \qquad F_3 = \frac{P}{\frac{K_1}{K_3}+\frac{K_2}{K_3}\sin(\theta_2)+\sin(\theta_3)+\frac{K_4}{K_3}} \tag{5}$$

3. NUMERICAL MODEL

The numerical model use plane frame elements, one per one ligament ((1), (6), (7) and (8)) and four elements to represent the bone ((2), (3), (4) and (5)), were the ligaments are attached, with a total of eight elements and nine nodes. Figure 2 show the element utilized, as well the numerical model representation of the four ligaments.



(a) (b)

Figure 2. (a) Plane frame element with 6 degrees of freedom e (b) finite element representation of the four ligaments model.

The plane frame element has two nodes (*i* and *j*), with three degrees of freedom per node (longitudinal and transversal displacements, and rotation). Nodes 1, 7, 8 and 9 were restrained in global *x* and *y* directions, but not in its rotations. Also, the global *x* direction of node 4 is restrained, but not in global *y* direction (because the force *F* acts in this direction) or rotation. Equation (6) show the transformation from local to global element stiffnesses.

$$K_{g\_e} = T_e^T K_e T_e \qquad \text{where e = from 1 to 8} \qquad (6)$$

Where subscript *e* represents the element number. The matrices of the local stiffness element K_e and the local to global transformation T_e are available in Appendix. As the numerical model has a small number of elements, it was possible to use a mathematical software, as Mathcad, to implement the numerical model. Equation (7) show the obtainment of the global displacement vector U_g through the multiplication of the inverse of the stiffness assembled matrix K_g (this matrix assembles, properly, all K_{g_e}) and the global vector force F_g, both shown at Appendix.

$$U_g = K_g^{-1} F_g \qquad (7)$$

Equation (8) is used to transform the element global displacement vectors U_k (picked, properly, from the global displacement vector U_g) to element local displacement vector $u(k)$:

$$u(k) = T_k U_k \qquad \text{where k = 1, 6, 7 or 8} \qquad (8)$$

Where *k* represents the ligament element number. Note that the T_k matrix is the same of T_e matrix, only the indices are different (*e* = from 1 to 8 encompasses all elements and *k* = 1, 6, 7 or 8 refers only to the ligament elements). Equation (9) relates the difference of local displacement nodes of an element and the correspondent element local force (aligned with the element length - local *x* axis):

$$F_k = \frac{A_k E_k}{L_k}\left(u2(k)_m - u1(k)_m\right) \qquad \text{where k = 1, 6, 7 or 8 \quad and \quad m = from 0 to 5} \qquad (9)$$

Where $u1$(k) and $u2$(k) are, respectively, the first and the second node of local displacement of each ligament element *k*, in *x* axis local direction (longitudinal). The subscript *m* of eq. (9), inform the position of the displacement value in the local displacement vector *u*(k). For the node 1, the position 0, 1 and 2 are filled, respectively, with longitudinal, transversal and angular local displacements; for the node 2, the position 3, 4 and 5 are filled, respectively, with longitudinal, transversal and angular local displacements. Finally, the ligament forces of the model shown in Fig.2.b can be accessed through the following final calculations:

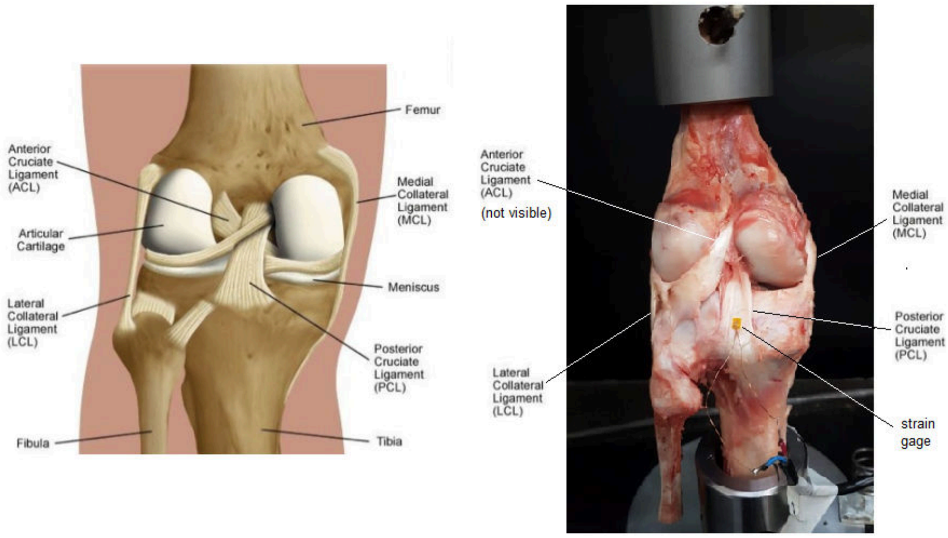
$$F_1 = \frac{A_1 E_1}{L_1}\left(u2(1)_3 - 0\right) \qquad F_6 = \frac{A_6 E_6}{L_6}\left(0 - u1_0(6)\right) \qquad F_7 = \frac{A_7 E_7}{L_7}\left(0 - u1_0(7)\right) \qquad F_8 = \frac{A_8 E_8}{L_8}\left(0 - u1_0(8)\right) \qquad (10)$$

In the next item it is described the preliminary experimental approach.

4. EXPLORATORY EXPERIMENTAL APPROACH

A porcine knee was used, since it is quite similar to the human knee, to implement the preliminary experimental study of knee ligaments. A refrigerated porcine knee was prepared to be coupled, through the utilization of transversal pins, to a 10 kN INSTRON material testing machine, existing in LADES laboratory in CEFET/RJ. This material testing machine was responsible for applying increasing axial loads, with 3 mm/min rate, up to ligament failure. The PCL was instrumented, with a strain gage EXCEL model PA-06-060BA120-L, to measure the longitudinal strains developed during the test. The data acquisition system consisted of a SPIDER A/D converter, with quarter Wheatstone bridge signal conditioning, with gain of 300. A three wires option was used to connect the strain gage with A/D system. A 2.5 V of voltage excitation was also selected.

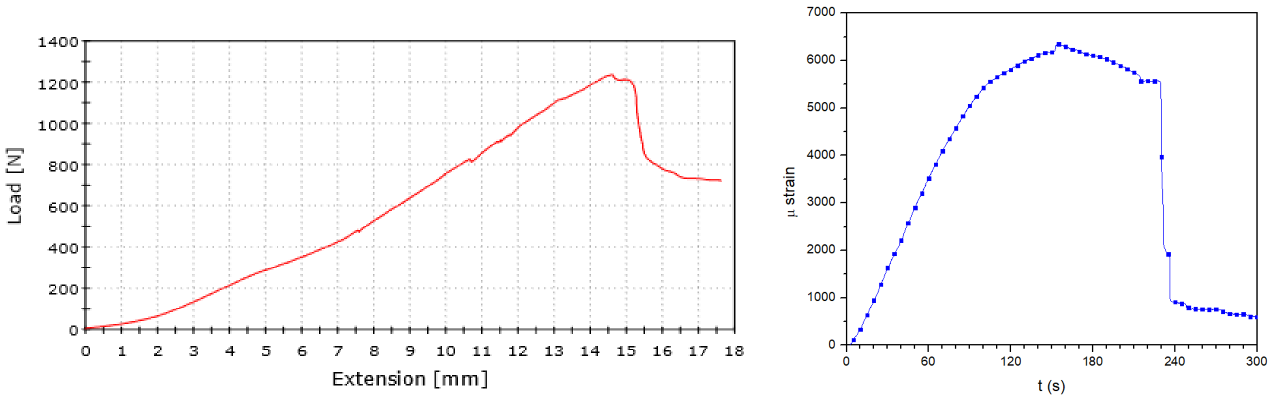
Figure 3.a shows a graphical representation of a human knee and Fig.3.b shows a photo of a porcine knee positioned at the material testing machine, with a strain gage attached, with cyanoacrylate adhesive, to porcine's PCL. Note the striking resemblance between knees.



(a) (b)

Figure 3. Left knee, view from behind: (a) graphical representation of a human knee, available from: Stanford Healthcare (2019) and (b) Porcine's knee with strain gage installed on PCL (photo).

The load versus displacement results of the porcine knee are shown in Fig. 4.a and the strain results obtained from strain gage glued to PCL surface are shown in Fig. 2.b.



(a) (b)

Figure 4. Experimental results: (a) load versus extension graphic of porcine knee, (b) strain gage results of porcine PCL.

Note that the load versus extension graphic, of Fig. 4.a shows a relatively linear response of the porcine knee until the failure around 1200 N. Also, note that after the failure the load was apparently redistributed by other structures, remaining some residual strength left. A relatively large displacement was imposed before failure was even noted. Figure 4.b shows the response only of PCL, which have also a relatively linear response up to 5000 με.

5. CONCLUSIONS

An analytical and a numerical model of the ACL, LCL, MCL and PCL of a porcine knee were presented to estimate the load share between ligaments in function of its stiffnesses. Relevant ligament mechanical variables as lengths, transversal areas, elastic material properties and angles were used. For both models, it was considered that the ligament material behaves as linear elastic, nevertheless the inclusion of viscoelastic behavior for the ligaments is underway. Also, a 3D development of this 2D model is being implemented.

The preliminary experimental study was successful executed with a refrigerated porcine knee positioned in a material testing machine, with a strain gage attached, with a regular cyanoacrylate adhesive, to PCL surface, and obtaining the experimental strain results. With the research progress, it is planned to attach one strain gage on each ligament (ACL, LCL, MCL and PCL) of refrigerated porcinis knee, to experimentally describe the load share between them, as well as the load redistribution after the sequential failure of ligaments.

6. ACKNOWLEDGEMENTS

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8. RESPONSIBILITY NOTICE

The authors are the only responsible for the printed material included in this paper.

9. APPENDIX

The element stiffness matrix K_e and the transformation matrix T_e:

$$Ke := \begin{pmatrix} \frac{Ae\cdot Ee}{Le} & 0 & 0 & \frac{Ae\cdot Ee}{Le} & 0 & 0 \\ 0 & \frac{12\cdot Ee\cdot Ie}{Le^3} & \frac{6\cdot Ee\cdot Ie}{Le^2} & 0 & \frac{12\cdot Ee\cdot Ie}{Le^3} & \frac{6\cdot Ee\cdot Ie}{Le^2} \\ 0 & \frac{6\cdot Ee\cdot Ie}{Le^2} & \frac{4\cdot Ee\cdot Ie}{Le} & 0 & \frac{6\cdot Ee\cdot Ie}{Le^2} & \frac{2\cdot Ee\cdot Ie}{Le} \\ \frac{Ae\cdot Ee}{Le} & 0 & 0 & \frac{Ae\cdot Ee}{Le} & 0 & 0 \\ 0 & \frac{12\cdot Ee\cdot Ie}{Le^3} & \frac{6\cdot Ee\cdot Ie}{Le^2} & 0 & \frac{12\cdot Ee\cdot Ie}{Le^3} & \frac{6\cdot Ee\cdot Ie}{Le^2} \\ 0 & \frac{6\cdot Ee\cdot Ie}{Le^2} & \frac{2\cdot Ee\cdot Ie}{Le} & 0 & \frac{6\cdot Ee\cdot Ie}{Le^2} & \frac{4\cdot Ee\cdot Ie}{Le} \end{pmatrix} \qquad Te := \begin{pmatrix} \cos(\theta e) & \sin(\theta e) & 0 & 0 & 0 & 0 \\ -\sin(\theta e) & \cos(\theta e) & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \cos(\theta e) & \sin(\theta e) & 0 \\ 0 & 0 & 0 & -\sin(\theta e) & \cos(\theta e) & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

The global stiffness assembled matrix K_g and the global vector force F_g:

$$Kg = \begin{pmatrix}
K1_{3,3}+K2_{0,0} & K1_{3,4}+K2_{0,1} & K1_{3,5}+K2_{0,2} & K2_{0,3} & K2_{0,4} & K2_{0,5} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
K1_{4,3}+K2_{1,0} & K1_{4,4}+K2_{1,1} & K1_{4,5}+K2_{1,2} & K2_{1,3} & K2_{1,4} & K2_{1,5} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
K1_{5,3}+K2_{2,0} & K1_{5,4}+K2_{2,1} & K1_{5,5}+K2_{2,2} & K2_{2,3} & K2_{2,4} & K2_{2,5} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
K2_{3,0} & K2_{3,1} & K2_{3,2} & K2_{3,3}+K3_{0,0}+K7_{0,0} & K2_{3,4}+K3_{0,1}+K7_{0,1} & K2_{3,5}+K3_{0,2}+K7_{0,2} & K3_{0,4} & K3_{0,5} & 0 & 0 & 0 & 0 & 0 & 0 \\
K2_{4,0} & K2_{4,1} & K2_{4,2} & K2_{4,3}+K3_{1,0}+K7_{1,0} & K2_{4,4}+K3_{1,1}+K7_{1,1} & K2_{4,5}+K3_{1,2}+K7_{1,2} & K3_{1,4} & K3_{1,5} & 0 & 0 & 0 & 0 & 0 & 0 \\
K2_{5,0} & K2_{5,1} & K2_{5,2} & K2_{5,3}+K3_{2,0}+K7_{2,0} & K2_{5,4}+K3_{2,1}+K7_{2,1} & K2_{5,5}+K3_{2,2}+K7_{2,2} & K3_{2,4} & K3_{2,5} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & K3_{4,0} & K3_{4,1} & K3_{4,2} & K3_{4,4}+K4_{1,1} & K3_{4,5}+K4_{1,2} & K4_{1,3} & K4_{1,4} & K4_{1,5} & 0 & 0 & 0 \\
0 & 0 & 0 & K3_{5,0} & K3_{5,1} & K3_{5,2} & K3_{5,4}+K4_{2,1} & K3_{5,5}+K4_{2,2} & K4_{2,3} & K4_{2,4} & K4_{2,5} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & K4_{3,1} & K4_{3,2} & K4_{3,3}+K5_{0,0}+K8_{0,0} & K4_{3,4}+K5_{0,1}+K8_{0,1} & K4_{3,5}+K5_{0,2}+K8_{0,2} & K5_{0,3} & K5_{0,4} & K5_{0,5} \\
0 & 0 & 0 & 0 & 0 & 0 & K4_{4,1} & K4_{4,2} & K4_{4,3}+K5_{1,0}+K8_{1,0} & K4_{4,4}+K5_{1,1}+K8_{1,1} & K4_{4,5}+K5_{1,2}+K8_{1,2} & K5_{1,3} & K5_{1,4} & K5_{1,5} \\
0 & 0 & 0 & 0 & 0 & 0 & K4_{5,1} & K4_{5,2} & K4_{5,3}+K5_{2,0}+K8_{2,0} & K4_{5,4}+K5_{2,1}+K8_{2,1} & K4_{5,5}+K5_{2,2}+K8_{2,2} & K5_{2,3} & K5_{2,4} & K5_{2,5} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & K5_{3,0} & K5_{3,1} & K5_{3,2} & K5_{3,3}+K6_{0,0} & K5_{3,4}+K6_{0,1} & K5_{3,5}+K6_{0,2} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & K5_{4,0} & K5_{4,1} & K5_{4,2} & K5_{4,3}+K6_{1,0} & K5_{4,4}+K6_{1,1} & K5_{4,5}+K6_{1,2} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & K5_{5,0} & K5_{5,1} & K5_{5,2} & K5_{5,3}+K6_{2,0} & K5_{5,4}+K6_{2,1} & K5_{5,5}+K6_{2,2}
\end{pmatrix}$$

$$Fg := \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ F \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

The global displacement vector U_k:

$$U_1 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ Ug_0 \\ Ug_1 \\ Ug_2 \end{pmatrix} \quad U_6 = \begin{pmatrix} Ug_{11} \\ Ug_{12} \\ Ug_{13} \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad U_7 = \begin{pmatrix} Ug_3 \\ Ug_4 \\ Ug_5 \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad U_8 = \begin{pmatrix} Ug_8 \\ Ug_9 \\ Ug_{10} \\ 0 \\ 0 \\ 0 \end{pmatrix}$$