25th ABCM International Congress of Mechanical Engineering
October 20-25, 2019, Uberlândia, MG, Brazil

COB-2019-0294
PARAMETRIC OPTIMIZATION OF A LOAD CELL WITH STRAIN/STRESS AND NATURAL FREQUENCY CONSTRAINTS USING A METAHEURISTIC ALGORITHM

Ígor Simon[1]
Herbert M. Gomes[2]
Federal University of Rio Grande do Sul, Graduate Program in Mechanical Engineering, Av. Sarmento Liete, 425, room202, 2°. floor, 90050-170, Porto Alegre, RS, Brazil.
e-mail: isimon.igor@gmail.com[1], herbert@mecanica.ufrgs.br[2]

Abstract. *Load cells are devices capable of converting mechanical loads into proportional electric signal, capable of being read and interpreted by electronic equipment such as Wheatstone bridge and amplifier. Aiming at improving the sensitivity of the load cell to a given applied force, to ensure integrity to a given loading cycle, a parametric optimization of the load cell shape is performed by modifying specific dimensions and proportions. The Ansys software is used as the FEM (Finite Element Method) analysis tool for both static loading and modal analysis, and a PSO (Particle Swarm Optimization) metaheuristic algorithm is implemented in Python language, to perform the optimization. The multiobjective function includes mass minimization and maximization of load cell sensitivity and first natural frequency. Stress/strain iso-surfaces are presented of the optimized model and compared with the initial design using nominal parameter values. The obtained result has 54% of reduction in the objective function value, with gains in the mass and first natural frequency, despite the reduction of the sensitivity.*

Keywords: *load cell, optimization, metaheuristic, finite elements method*

1. INTRODUCTION

A load cell is a device that converts an external load into deformation of a metallic structure, and afterwards to resistance unbalance of strain gages using a Wheatstone bridge and finally measurable variation of electric voltage. By electrical voltage measurement, it is possible to recovery the deformation of the load cell and thus the applied load. In the elastic range of materials, the relation between deformation and the external load can be known *a priori* based on material elastic properties and geometry of the load cell. The load cell design is especially important in order to get the better sensitivity for the load range of interest. The geometry should be correctly sized to increase sensitivity keeping a safety factor for the measured loads.

This paper describes the parametric sensitivity optimization of a bending load cell by a Particle Swarm Optimization algorithm. The objective function comprises to minimize mass, maximize the sensitivity for load-voltage conversion, and the first natural frequency. As constraints, it should be ensured a stress safety factor for the load cell as a whole and a safety factor for strain at the fixing points of the strain gages.

2. BIBLIOGRAPHICAL REVIW

Load cells have been used for a long time for monitoring and acquisition of forces, since then, many studies have been made on these components aiming their improvement and exploring new applications. Vijay and Gore (2012) present a form optimization study in order to minimize the mass of an S-type load cell using the Ansys finite element software. The results of the optimization are checked by photo-elasticity analysis and show values very close to those obtained with Finite Elements. In their work, a maximum limit elongation of 1000 µm/m is applied on the deformation observed at the bonding sites of the strain gages and holes. Special care is taken in modeling the holes for the cable connections. In the end, the authors report a 24.95% improvement in load cell sensitivity with a reduction of 23.77% of the final volume. In their work, Soni and Priyadarshi (2019) analyzed the design of scales for multiple force components from the design of a load cell of the bending beam type. For an initial design, a coupling between the components of forces intended to be measured was observed, but this was reduced by using the design of a cell with the tapered central part, eliminating cross-sensitivities in the other directions. All analysis were performed with Ansys software. The optimization was performed automatically with the Design Xplorer software, version 13.0 by Ansys. Varne *et al.* (2006) used the response surface method to feasibly optimize a pancake-type load cell. Multiple objective functions such as sensitivity and mass of the load cell were used. In this case, Hyperworks software was used to perform finite element analysis. The response surfaces were trained for various load cell configurations such as web width, web thickness and load cell height so that, later in optimization, it can be used as an objective function directly without having to perform numerical analysis delayed by finite elements. In the end, they reach a result that represents a compromise between increasing sensitivity and decreasing load cell volume. It is concluded that the height of the cell is the most important variable to be considered in the optimization.

3. THEORETICAL BASIS

3.1 Strain Gage

Electrical resistance extensometers are resistive elements, which vary their resistance linearly (to some extent) with the deformation they are subjected to, since they are fixed to the elements that undergo mechanical deformations. Physically, the strain gages are sensors that contain a resistor composed of a very thin layer of conductive material. The variations of structure dimensions are transmitted mechanically to the strain gage, which transforms these variations into equivalent variations of their electrical resistance, and are thus defined as transducers. They are used to measure implicitly or explicitly several physical quantities such as pressure, tension or compression, moments and stress/strains. Thus, for the selection one should take care on some points such as: material of the metallic grid, material of the surface being tested, adhesive material, just to name a few. As for its shape, it has different functions and characteristics such as sensitivity factor, grid length, grid orientation and resistivity, among others.

3.2 Wheatstone Bridge

According Beckwith *et al.* (2011), the use of the Wheatstone bridge is meant to increase the accuracy of measurements. The circuit is ideally designed for the evaluation of unknown electrical resistances but it is widely used in instrumentation for the evaluation of mechanical deformations. Fig. 1 shows a typical Wheatstone Bridge, with four resistors.

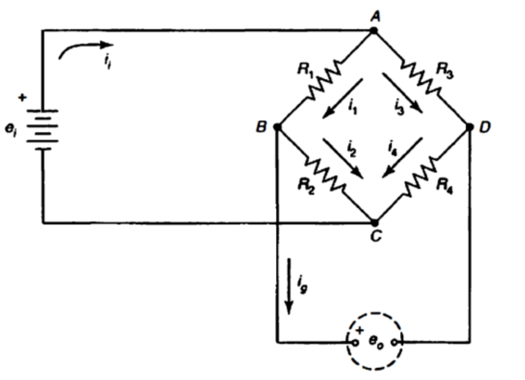

Figure 1. Wheatstone Bridge. Source: Beckwith *et al.* (2011).

The resistors of the system are the strain gages. By the deformation of the load cell, resulted from compression or tension loads, it changes their internal resistance. The generic equation for the Wheatstone Bridge is given by Beckwith *et al.* (2011):

$$\Delta E = \left(\frac{R_1}{R_1+R_2} - \frac{R_4}{R_4+R_3}\right) \cdot V, \tag{1}$$

where ΔE is the measured voltage variation, R_1, R_2, R_3 e R_4 are the four resistances of the strain gages and V is the source voltage. For small variations in each of the resistors, the corresponding deformations can be evaluated by:

$$\varepsilon_i = \frac{\Delta R_i/R_i}{k}, \tag{2}$$

where ε_i is the strain of the i strain gage, ΔR_i is the resistance variation of the i-*th* strain gage, R_i is the initial resistance of the i strain gage and k is the sensitivity of the material (usually close to 2.0) the strain gage is made.

Assuming small individual resistance variations in the generic equation of the Wheatstone Bridge, one arrives at the following equation in terms of the measured deformations:

$$\Delta E = \frac{k}{4} \cdot (\varepsilon_1 - \varepsilon_2 - \varepsilon_3 + \varepsilon_4) \cdot V \ , \tag{3}$$

where the overall Sensitivity $\Delta E/V$ can be evaluated as proportional to:

$$\text{Sensitivity} = \varepsilon_1 - \varepsilon_2 - \varepsilon_3 + \varepsilon_4 \tag{4}$$

3.3 Optimization Method

A general optimization problem can be stated as (Bhatti, 2000):

$$\begin{aligned} &minimize\ f(\boldsymbol{x}) \\ s.t. &\begin{cases} g_i(\boldsymbol{x}) \leq 0 \\ h_j(\boldsymbol{x}) = 0 \\ x_{Lower_k} \leq x_k \leq x_{Upper_k} \end{cases} \end{aligned} \ , \tag{5}$$

where $f(\boldsymbol{x})$ is the objective function, $\boldsymbol{x}$ is the optimizations variables $(x_1, x_2, x_3, \ldots, x_k)^T$, $g_i(\boldsymbol{x})$ and $h_i(\boldsymbol{x})$ are respectively the inequality and equality constraints, x_{Lower_k} and x_{Upper_k} are the bounds to the optimization variables.

In case of geometric modifications with stress/strain and frequency constraints, it can be considered as a nonlinear constrained problem. To solve it, the use of a penalty method is usual, since the objective function and constraints are not explicitly stated. It consists in transform a constrained problem into an unconstrained one, by penalizing the objective function when some of the constraints do not comply (Bhatti, 2000). So, a penalized objective function $pf(\boldsymbol{x})$ is defined as:

$$pf(\boldsymbol{x}) = f(\boldsymbol{x}) + P\left[\sum_{i=1}^{m}\left(max\left(0, g_i(\boldsymbol{x})\right)\right)^2 + \sum_{j=1}^{n}\left(h_j(\boldsymbol{x})\right)^2\right] \tag{6}$$

A penalized part is created adding to the original one $f(\boldsymbol{x})$ a penalization factor P multiplied by some measure of degree of constraint violation, resulting in an unconstrained optimization problem. Thus, this unconstrained $pf(\boldsymbol{x})$ is intended to be solved by the Particle Swarm Optimization (PSO) algorithm.

The Particle Swarm Optimization (PSO) is a metaheuristic method originally developed by Kennedy and Eberhart (1995) and widely used to optimize nonlinear functions. This method is based on initial values for position and velocity for a set of particles that represent potential solutions of the problem. These positions and velocities are updated along iterations by the value of the functions evaluated in the past iterations as indicated by Eq. (7) and Eq. (8).

$$v_{ij}(t+1) = w(t) \cdot v_{ij}(t) + c_1 \cdot r_1 \cdot \left[pbest_{ij} - x_{ij}(t)\right] + c_2 \cdot r_2 \cdot \left[gbest_i - x_{ij}(t)\right] \ , \tag{7}$$
$$x_{ij}(t+1) = x_{ij}(t+1) + v_{ij}(t+1) \tag{8}$$

where $x_{ij}(t)$ and $v_{ij}(t)$ are the position and the velocity, respectively, of the variable i of the particle j on the iteration t. w is the inertia parameter, r_1 and r_2 are two random values between 0 and 1, c_1 and c_2 are the cognitive and the social parameter, respectively. $pbest_{ij}$ is the value of the design variable i and particle j of the best iteration and $gbest_i$ is the value of the design variable i of the best particle so far from all iterations. The evaluation occurs asynchronously as describe by Esposito and Gomes (2013), which means the update of $pbest_{ij}$ and $gbest_j$ is readily available to at all candidate particles.

3.4 FEM

According to Cook et. al. (2002) the Finite Element Method is a mathematic technique to numerically solve engineering problems expressed by partial differential equations in the domain. The software used at this paper is ANSYS Workbench 19 student version to solve solid mechanics problems.

4. METHODOLOGY

The load cell geometry (Fig. 2) was modelled by ANSYS Workbench FEM software. The parameters of the HBM load cell (2018a) was assumed as initial design. The commercial load cell has a load capacity of 400N (40 kgf). This load cell weights (m_o) 0.141 kg, and has a stress safety factor of 1.58. The first bending natural frequency (f_{1_o}) is 443 Hz and the load cell sensitivity ($\left|\left(\varepsilon_{1_o} - \varepsilon_{2_o} - \varepsilon_{3_o} + \varepsilon_{4_o}\right)\right|$) is 0.007899. This data were evaluated based on the FEM software ANSYS Workbench using the nominal values for dimensions and material properties.

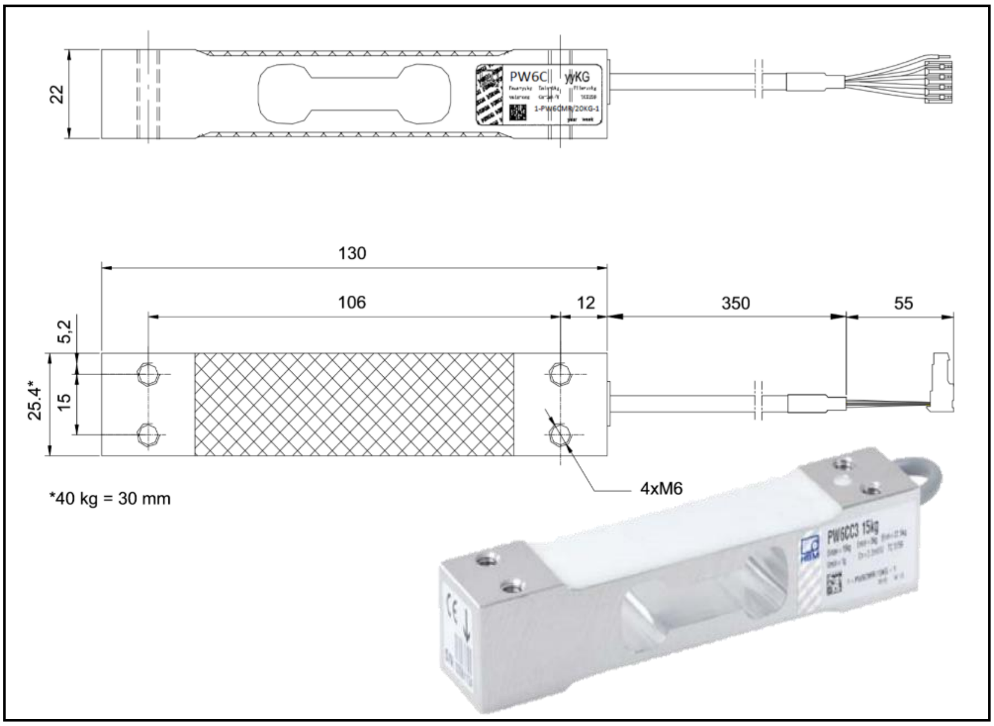

Figure 2. Original Load Cell geometry. Source: HBM (2018a).

The design variables elected to be optimized were: the height of the load cell (x_1); the width of the load cell (x_2); the main binocular hole-diameter to height ratio (x_3) and the height of the binocular gap to diameter of the main hole ratio (x_4). These parameters are depicted in Fig. 3. So, the objective function that takes into account mass, sensitivity and natural frequency can be defined by Eq.(9).

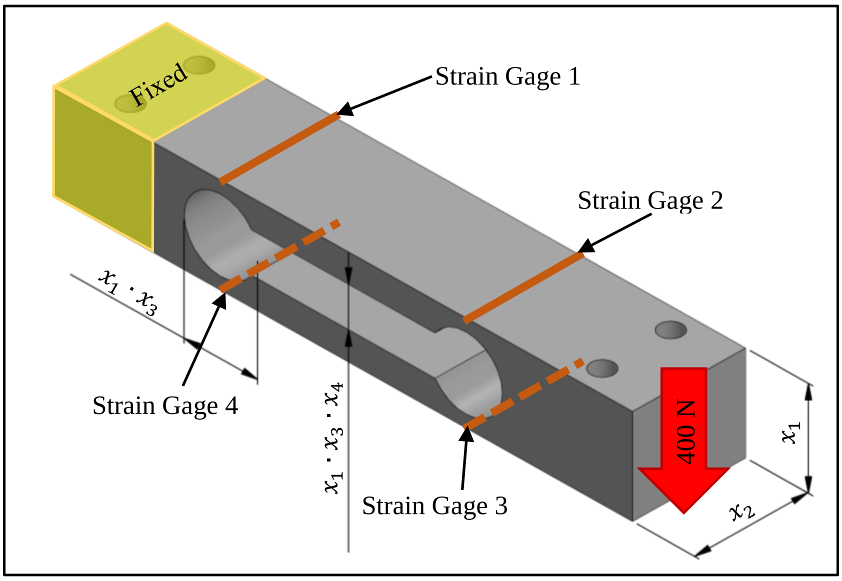

Figure 3. Load Cell design variables and boundary conditions.

One can notice that the optimization parameters x_3 and x_4 are proportions of the load cell's height. This means that the products ($x_1 \cdot x_3$ and $x_1 \cdot x_3 \cdot x_4$) should be evaluated prior the analysis, in order to obtain the respective height of binocular gap and main hole diameter to perform later the finite element mesh generation.

$$minimize\ f(\boldsymbol{x}) = \frac{m}{m_o} \cdot \frac{f_{1_o}}{f_1} \cdot \frac{|(\varepsilon_{1_o}-\varepsilon_{2_o}-\varepsilon_{3_o}+\varepsilon_{4_o})|}{|(\varepsilon_1-\varepsilon_2-\varepsilon_3+\varepsilon_4)|}$$
$$s.t. \begin{cases} SF_\sigma \geq 1{,}58 \\ SF_{\varepsilon_i} \geq 1{,}58 \quad for\ i = 1\ to\ 4 \\ \boldsymbol{x}_{min} \leq \boldsymbol{x} \leq \boldsymbol{x}_{max} \end{cases}, \quad (9)$$

where m is the mass of the load cell, f_1 is the first natural frequency, ε_1, ε_2, ε_3 e ε_4 are de strains at the strain gage fixing points. The parameters m_o, f_{1_o}, ε_{1_o}, ε_{2_o}, ε_{3_o} e ε_{4_o} are the nominal values of the described variables, which, in turn, render a dimensionless objective function.

The SF_σ is the stress safety factor obtained by the FEM software, SF_{ε_i} is the strain safety factor at the interface between strain gage and the load cell surfaces at points i indicated at Fig. 3 and obtained by the Eq.(10). Both safety factor for stresses or strains are constraints that assures some degree of safety to the load cell usage.

$$SF_{\varepsilon_i} = \frac{\varepsilon_{\max\_strain\_gauge}}{\varepsilon_i} \quad (10)$$

$\varepsilon_{\max\_strain\_gauge}$ is the maximum allowable elongation of the used strain gage, when occurs the adhesive detachment. In this case, it was chosen Y series strain gage with 0,050 m/m for maximum elongation (HBM, 2018b).

And $\boldsymbol{x}$ is the vector of optimization variables, $\boldsymbol{x}_{max}$ and $\boldsymbol{x}_{min}$ are, respectively, the maximum and minimum allowable values to the optimization variables. The upper and lower bounds values are in Tab. 1.

Table 1: Maximum and minimum values to the optimization variables.

Optimization Variables	**Minimum**	**Maximum**
x_1 [m]	0.01	0.05
x_2 [m]	0.02	0.05
x_3 [-]	0.10	0.90
x_4 [-]	0.10	0.90

The optimization process was performed using the Particle Swarm Optimization (PSO) algorithm with a total of 100 iterations and 15 particles, and to convert the problem of Eq. (9) into a non-constrained problem, the penalty method (Bhatti, 2000) was applied as previously presented, the Fig. 4 shows the pseudo-cod. The Python Language at Script Console of Ansys Workbench (ANSYS, 2018) was used for implementation. Fig. 5 shows the ANSYS Workbench configuration.

```
Initialize the optimization variables
Loop over the iterations t
    Loop over all particles j
        Update the FEM analysis.
        Evaluate the penalty objective function using Eqs. (5) and (6).
        If actual objective function is best than all others evaluations for particle j
            Update pbest_j as the actual objective function
        If actual objective function is best than all others evaluations of the analysis
            Update the gbest as the actual objective function
        Update the optimization variables of particle j to the next iteration with Eqs. (7) and (8).
Check convergence and if stopping criteria is met, write the best solution so far and exit
```

Figure 4. Pseudo-code of the optimization algorithm implemented in Python Script.

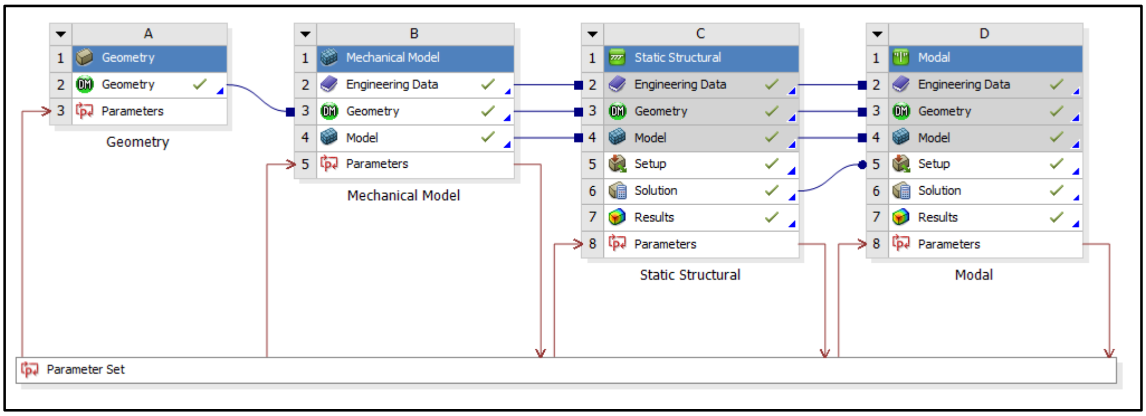

Figure 5. Ansys Workbench flowchart for the FE Analysis.

The analysis was performed with 100 iterations as limit and 10 particles.

5. RESULTS

As the result of the optimization process, the obtained optimized design variables in the last iteration were 0.010 m for height, 0.020m for width, 0.35 for hole diameter to height ratio and 0.90 for gap height to hole diameter ratio. The resulted optimized load cell mass was 0.063 kg, with a stress safety factor of 1.54 and sensitivity of 0.0066. The first natural frequency resulted in 509 Hz. The stress safety factor along the load cell for the original and optimized geometry is presented with the Safety Factor distribution in Fig. 6 and with the First Natural Frequency in Fig. 7 and the Penalized Objective Function history along iterations in Fig. 8.

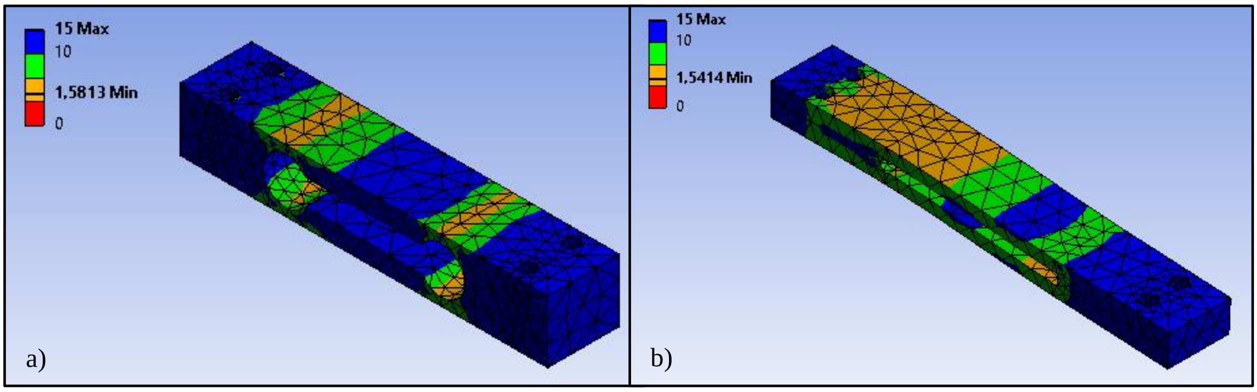

Figure 6. a) Nominal and b) Optimized Load Cell Safety Factor distribution by Ansys FEM analysis.

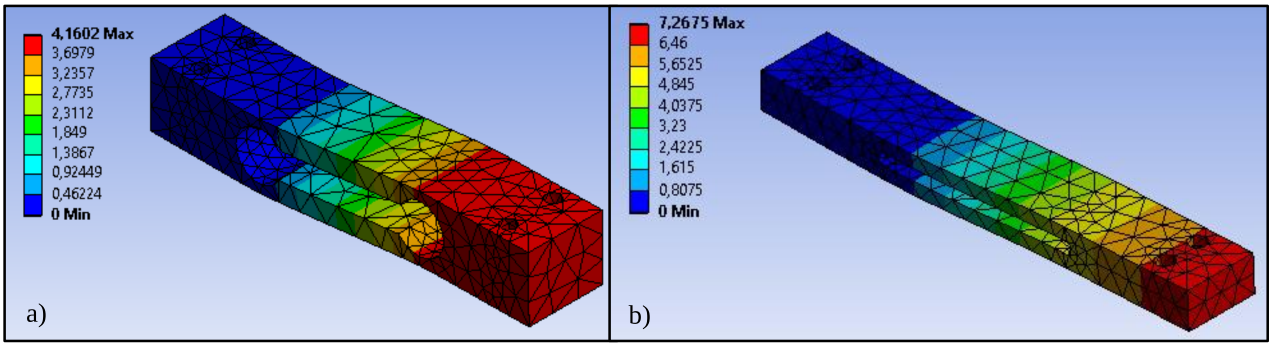

Figure 7. a) Nominal and b) Optimized Load Cell First Natural Frequency by Ansys FEM analysis.

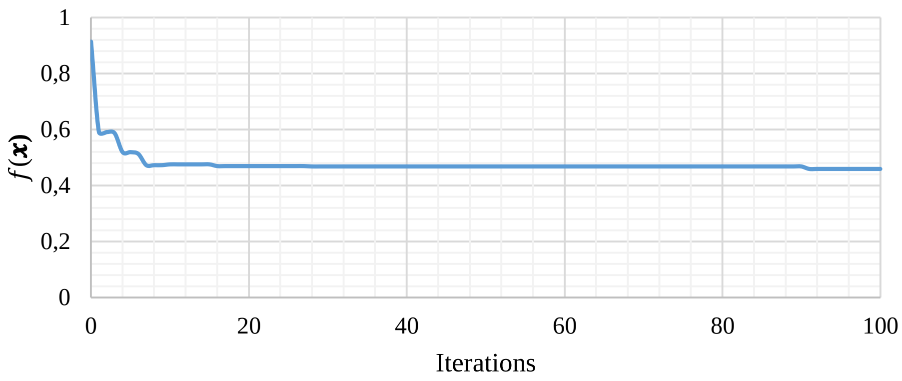

Figure 8. Penalized Objective Function values along iterations.

A comparison for the main parameter of the nominal load cell and the optimized one are showed at the Tab. 2, and the corresponding optimization variables at depicted in Tab. 3.

Table 2: Nominal and optimized load cell main parameters comparison.

Parameter	**Original Load Cell**	**Optimized Load Cell**
Mass [kg]	0.141	0.063
First Natural Frequency [Hz]	443	509
Objective Function	1.00	0.4593
Sensitivity evaluated by the Eq. (4) [m/m]	0.0076899	0.006654

Table 3: Nominal and optimized load cell design variable comparison.

Design Variables	**Nominal Load Cell**	**Optimized Load Cell**
x_1 [m]	0.022	0.01
x_2 [m]	0.0254	0.02
x_3 [-]	0.75	0.316
x_4 [-]	0.55	0.607

6. CONCLUSIONS

The result obtained with the optimization process represents 54% of reduction in the objective function value, despite of the reduction in the load cell sensitivity of 17%. The obtained reduction on mass was 66% and the gain on first natural frequency about 15%. The actual gain with the optimization process was at the final mass of the load cell and at the first natural frequency, which increases the range of application.

7. REFERENCES

Ansys. Workbench *scripting guide*. 2018. Available on: <http://itrss.mst.edu/media/informationtechnology/itrss/documents/ansysdocs/Workbench%20Scripting%20Guide.pdf>. Accessed in August, 1st, 2018.

Bhatti, M.A. 2000. *Practical optimization methods: With Mathematica applications*. Iowa: Springer.

Beckwith, T.G., Marangoni, R.D., Lienhard, V. J.H. 2011. *Mechanical measurements*. 6th Ed. Boston: Pearson.

Cook, R. D. Malkus, D.S., Plesha, M.E., Witt, R.J. 2002. *Concepts and applications of finite element analysis*. Danvers: John Wiley & Sons, 4th. Edition.

Esposito, A., Gomes, H.M. 2013. “Evolutionary parallel and serial programming algorithm for structural optimization.” In *Proceedings of the 14th WSEAS International Conference on Evolutionary Computing* (6-8 August), 2013, Valencia, Spain. Proceedings of the 14th WSEAS International Conference on Evolutionary Computing. Valencia: WSEAS. Vol. 1. pp. 82-87.

HBM. PW6C: Single point load cells. 2018. Available on: <https://www.hbm.com/fileadmin/mediapool/hbmdoc/technical/B01994.pdf>. Accessed in October, 2nd, 2018.

HBM. Strain gauges: First choice for strain measurements. Available on: <https://www.hbm.com/fileadmin/mediapool/hbmdoc/technical/S01265.pdf>. Accessed in November, 20th, 2018.

Kennedy, J., Eberhart, R. 1995. "Particle swarm optimization". In *Proceedings of Icnn'95 - International Conference on Neural Networks*. [s.l.], Vol. 4, pp.1942-1948.

Soni, A., Priyadarshi, P. 2019. "Finite element analysis and optimization of a beam type load cell for an external balance design". Internal report. Available in <https://www.academia.edu/6462303/Finite_Element_Analysis_and_Optimization_of_a_Beam_Type_Load_Cell_for_an_External_Balance_Design>. Accessed in April, 4th, 2019.

Varne, V., Shinde, V., Kamble, V. 2016. "Parametric optimization of Pancake type load cell sing response surface". In *International Journal of Current Engineering and Technology*, MMIT, Lohgaon, Pune, Maharashtra, India, NCPSM-2016, INPRESSCO IJCET Special Issue-6 (Oct 2016).

Vijay, A.K., Gore, P.N. 2012. Use of FEM and photo elasticity for shape optimization of S type load cell". *Indian Journal of Science and Technology*, Vol. 5, No. 3.

Zheng, Y.-L., Ma, L.-H., Zhang, L.-Y., Qian, J.-X. l. 2003. "Empirical study of particle swarm optimizer with an Increasing Inertia Weight". In Conference: Evolutionary Computation, 8-12 Dec. 2003. CEC '03. Canberra, ACT, Australia, Australia The 2003 Congress onVolume: 1. DOI: 10.1109/CEC.2003.1299578

8. RESPONSIBILITY NOTICE

The authors are the only responsible for the printed material included in this paper.