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STUDY OF THE ACOUSTIC REFLECTION AROUND A CYLINDER BY THE BOUNDARY ELEMENT METHOD

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Abstract. *The computational simulation of highly complex problems is of paramount importance for the development of complex mathematical models. Some of these problems are related to the acoustic radiation of the sound sources, whose modeling is of fundamental importance to understand the propagation of the acoustic waves and, consequently, to develop mechanisms for the reduction of acoustic noise. The propagation of acoustic waves involves different phenomena like radiation, absorption, transmission and reflection. In this work the numerical solution of the integral Helmholtz equation for a pulsating infinite cylinder in a free and homogeneous medium is obtained. The numerical formulation used for this problem is obtained using the Boundary Element Method (BEM). The results show that as the wavenumber increases the distribution of pressure exhibits a higher number of phase variation and the directivity graph presents a cardioid profile.*

Keywords: *Acoustic prediction, Numerical analysis, Boundary Element Method*

1. INTRODUCTION

With the continuous increase of air, rail and urban traffic over the years, studies focused on acoustic radiation are conveniently found. Thus, model surveys for the calculation of the generation and propagation of noise around simple surfaces have been growing exponentially in recent years. The characterization of the noise generation around these surfaces serves to map numerical methods as their effectiveness and speed.

Numerical methods can be used to study the effects of diffraction and reflection of acoustic waves on surfaces. Finite diffraction, finite volume and finite element methods require the use of computational meshes for the discretization of the fluid volume where the wave propagation occurs. Methods of contour elements and equivalent sources can also be employed to solve the problem of acoustic scattering since the equations that model this phenomenon are linear. This last class of methods solves a contour integral and, therefore, the mesh is not necessary to discretize the fluid volume, it is only necessary to discretize the surface of the interest. Thus, the Boundary Element Method (BEM) presents some advantages over the methods of differences, volumes and finite elements. For example, the generation of simulated meshes and ease of implementation of boundary conditions (Mendonça and Avelar, 2016).

Due to these advantages of numerical implementation, several researches related to application of BEM in acoustic spreading problems have been developed. Coox *et al.* (2017) introduced an indirect Boundary Element Method (BEM) within an isogeometric structure based on NURBS functions to solve three-dimensional acoustic problems in the frequency domain. The authors found that the proposed method is significantly more efficient and also more robust than its polynomial-based counterpart.

Śmigaj *et al.* (2015) presented a new open-source library discretized by the Boundary Element Method for the solution of integral contour equations for the Laplace, Helmholtz and Maxwell problems in three spatial dimensions. Trevelyan (1992), van Opstal *et al.* (2015) and Chen *et al.* (2016) studied the application of the Boundary Element Method in structural acoustics problems, and the latter applied the rapid multipoles method to solve the studied problem.

Based on the current importance that BEM represents for modern engineering, the objective of the present work is to evaluate how the generation and propagation of noise occurs in a cylinder submitted to a monopole type source. For the cylinder modelling, it was used 360 panels in the discretization and the numerical integration of the Helmholtz equation was performed by Gaussian quadrature using the MatLab software.

2. NUMERICAL IMPLEMENTATION

The Helmholtz equation, which describes a reflection of acoustic waves in a quiescent medium is given according to Kinsler *et al.* (1999) by:

$$\nabla^2 \hat{p} + k^2 \hat{p} = 0, \tag{1}$$

where $\hat{p}$ is the acoustic pressure in the frequency domain and $k = \omega/c_0$ is the acoustic wave number.

The Eq. (1) can be written for a point perturbation such as:

$$\nabla^2 G(\vec{x}, \vec{y}) + k^2 G(\vec{x}, \vec{y}) = \delta(\vec{x} - \vec{y}). \tag{2}$$

The solution of Eq. (2) represents the fundamental solution of the acoustic field for a point source of the monopole type. The BEM is developed from Green's second theorem allowing to solve the problem by integrating the contour surface instead of integrating into the entire volume of fluid. Assuming G as the function of Green and $\hat{p}$ as the acoustic pressure, the Green's second theory can be written according to Marburg and Nolte (2008) as:

$$\int_V [G(\vec{x}, \vec{y})\nabla^2 \hat{p} - \hat{p}\nabla^2 G(\vec{x}, \vec{y})] dV = \oint_S \left[G(\vec{x}, \vec{y}) \frac{\partial \hat{p}}{\partial n} - \hat{p} \frac{\partial G(\vec{x}, \vec{y})}{\partial n} \right] dS, \tag{3}$$

where $\partial \hat{p}/\partial n = 0$ in due to the boundary condition of the rigid wall (deduced from the linearized equation of Euler) and G is the Green function given as:

$$G(x, y) = \frac{i}{4} H_0^{(1)} \left(k\sqrt{(x_1 - y_1)^2 + (x_2 - y_2)^2} \right), \tag{4}$$

, where in this equation $H_0^{(1)}$ the Hankel function of the first type is zero order.

Replacing the Helmholtz equation, Eq. (1), on the equation of the second Green theorem, we have:

$$\int_V \hat{p}\delta(\vec{x}) dV = \oint_S \left[G(\vec{x}, \vec{y}) \frac{\partial \hat{p}}{\partial n} - \hat{p} \frac{\partial G(\vec{x}, \vec{y})}{\partial n} \right] dS. \tag{5}$$

As by definition the Dirac delta function is equal to one for integrals with infinite limits, we have that the formulation of the BEM for a surface is given by:

$$\hat{p}(\vec{x}) = \oint_S \left[G(\vec{x}, \vec{y}) \frac{\partial \hat{p}}{\partial n} - \hat{p} \frac{\partial G(\vec{x}, \vec{y})}{\partial n} \right] dS. \tag{6}$$

This equation represents the pressure at a point $\vec{x}$ due only to the effects of diffraction and reflection occurring on solid surfaces. To represent the incident pressure directly from the acoustic source we add a term relative to the source, which can be of any type. Generally, in acoustic problems, monopole, dipole or quadrupole type sources, in addition to plane waves, are used to represent the physical mechanisms of noise generation. Complex real-world sources can be constructed by combining the cited model sources through spatial distributions of them. In the present problem, a monopole type source was used in the BEM formulation for acoustic reflection problems. The surface of the domain should be discretized in elements (or panels) connected by dots describing its geometry. In each element, an acoustic source is placed in its centroid with the purpose of representing the reflection effects of the wave originated in the acoustic source that affects the geometry. However, the acoustic pressure is singular when the point of analysis is located on the surface. Thus, the formulation of the BEM for the analysis is given according Mendonça and Avelar (2016) by:

$$c(\vec{x})\hat{p}(\vec{x}) = \oint_S \left[G(\vec{x}, \vec{y}) \frac{\partial \hat{p}(\vec{y})}{\partial n_y} - \hat{p}(\vec{y}) \frac{\partial G(\vec{x}, \vec{y})}{\partial n_x} \right] dS - G(\vec{x}, \vec{z}), \tag{7}$$

where $c(\vec{x})$ is equal to 1/2 when $\vec{x}$ is located on the surface of cylinder S and $c(\vec{x})$ is unity when $\vec{x}$ is an observer away from the surface, in the fluid region. The derivatives about the normal direction to the surface are represented by $\partial()/\partial n$ and n is the normal vector that points into the rigid surface (outside the fluid region). The position of the monopole source is $\vec{z} = (L; 0)$.

After the discretization, the integral on the continuous surface becomes a sum of integrals in the discrete elements Γ_j. It can then write an equation such as:

$$\frac{1}{2}\hat{p} = \left[\sum_{j=1, j\neq 1}^{N} - \int_{\Gamma_j} \hat{p}_j \frac{\partial G_{i,j}}{\partial n} d\Gamma + G_{i,j} \right] i = 1, ...N. \tag{8}$$

Where the sub-indexes i and j refer to the observer and the source, respectively. In the matrix form, one has:

$$\begin{bmatrix} a_{11} & a_{12} & \dots & a_{1N} \\ a_{21} & a_{22} & \dots & a_{2N} \\ \vdots & \vdots & \dots & \vdots \\ a_{N1} & a_{N2} & \dots & a_{NN} \end{bmatrix} \begin{Bmatrix} p_1 \\ p_2 \\ \vdots \\ p_N \end{Bmatrix} = \begin{Bmatrix} -G(\vec{x}_1, \vec{z}) \\ -G(\vec{x}_2, \vec{z}) \\ \vdots \\ -G(\vec{x}_N, \vec{z}) \end{Bmatrix}. \tag{9}$$

In the above equation, the values $p1$, $p2$, etc, are the pressure values in each of the boundary elements of the cylinder. The coefficients of the matrix $[A]$ is calculated as:

$$a_{i,j} = \frac{1}{2}\delta_{i,j} + (1 - \delta_{i,j}) \oint_S \left[\frac{\partial G(\vec{x}_i, \vec{y}_j)}{\partial n_j} \right] dS, \tag{10}$$

at where $\delta_{i,j}$ is the Kronecker delta (sum of repeated indices does not occur here).

The analyzed problem is illustrated in Fig. 1. The cylinder has radius a = 0.5 and a monopole source is positioned at L = 3. The reflected noise will be calculated in 360 observers positioned at r = 5.

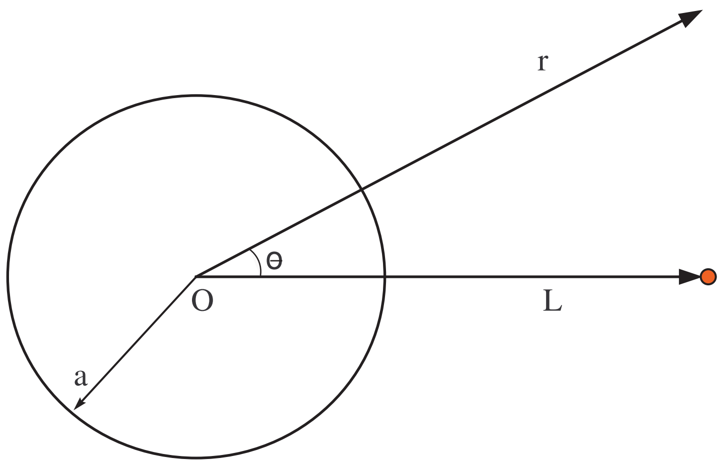


Figure 1: Geometry of the problem investigated

It was used 360 points on the cylinder in its discretization, that is, will have 360 boundary elements on the cylinder. To determine the pressures on the elements, one must be set as an observer and the others as acoustic sources. Thus, the elements can be sources or observers. The calculation of the integral in each discrete element was performed numerically by Gaussian quadrature. In the case of solving the problem on the surface, Amini and Wilton (1986) states which it is convenient to use four Gaussian points to avoid the centroid of the element corresponding to the point of singularity.

It is important to note that, although the analysis of the sources is evaluated from a point at the centroid of Γ_j, the integration makes a uniform source exist throughout the panel. That is, the integration represents the sum of infitastic point sources along any discrete element. Linear or parabolic distributions can be considered to obtain a high order solution to the BEM (the uniform distribution presents first order of precision). Details on this type of implementation can be obtained by Wrobel (2002) and Wolf and Lele (2011).

3. RESULTS

The acoustic pressure values were calculated for values of k = 1, 5 and 10 and are presented below. The pressure distribution is also presented when calculating a mesh with 240x240 observers uniformly scattered throughout the region of calculation

Figure 2 shows the pressure distribution for k = 1. It is noted that due to symmetry, the propagation of the acoustic waves occurs in a similar way for the upper and lower parts of the domain, that is, with the same phase. In Figs. 3a and 3b presents acoustic directivity graphs on the cylinder surface and on the observers, respectively. It is noticed that on the surface of the cylinder we have a directivity with monopole characteristic, which was expected due to the function of Green presented in the left side of Eq. (7). As for the observers, it is noted that the intensity of the acoustic pressure felt by them is of the order of 0.1 dB.

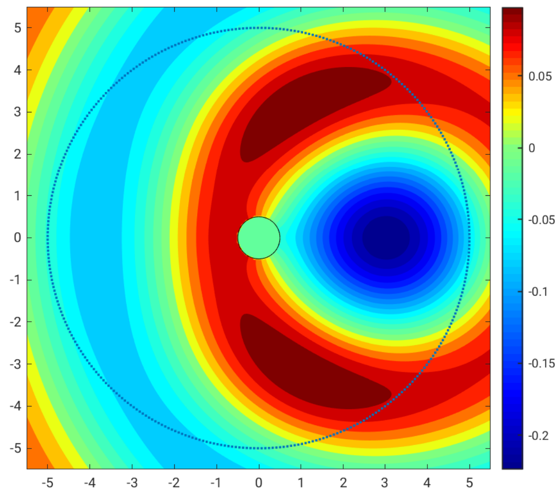
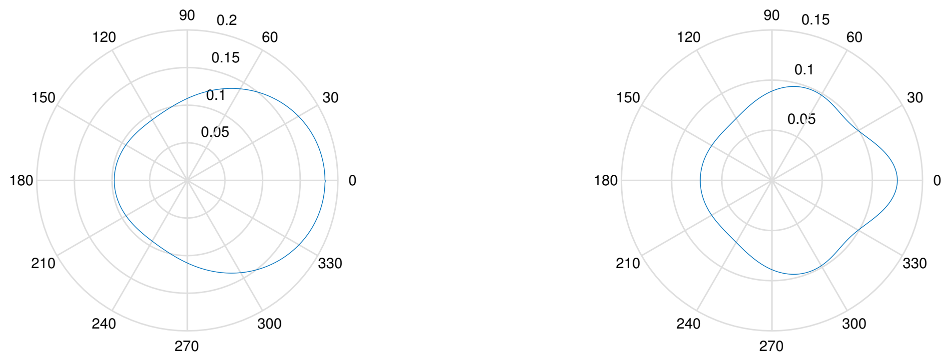


Figure 2: Distribution of the pressure for k = 1



(a) pressure on the cylinder surface

(b) pressure on observers

Figure 3: Acoustic directivity graphs for k =1

The Fig. (4) shows the case for wave number k = 5. As can be observed, the directivity graphs (Figs. 5a and 5b) that on the surface of the cylinder the characteristic of the source begins to be of the dipole type and in the observers begin to be generated lobes like a cardioid.

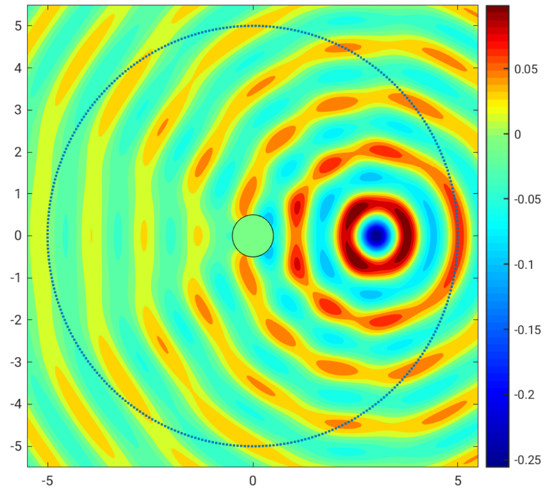
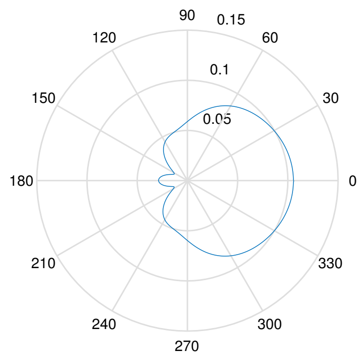
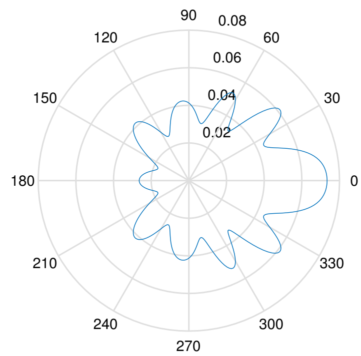


Figure 4: Distribution of the pressure for k = 5



(a) pressure on the cylinder surface



(b) pressure on observers

Figure 5: Acoustic directivity graphs for k = 5

Finally, the acoustic characteristics were evaluated for k = 10, these results are presented in the Figs. (6), and (7a, 7b). It is verified which for this wave number the characteristic of the source does not change in relation to the k = 5, however, the acoustic pressure intensity is higher. As for the observers, a similar characteristic is observed in the directivity, but with the appearance of more lobes in the cardioid.

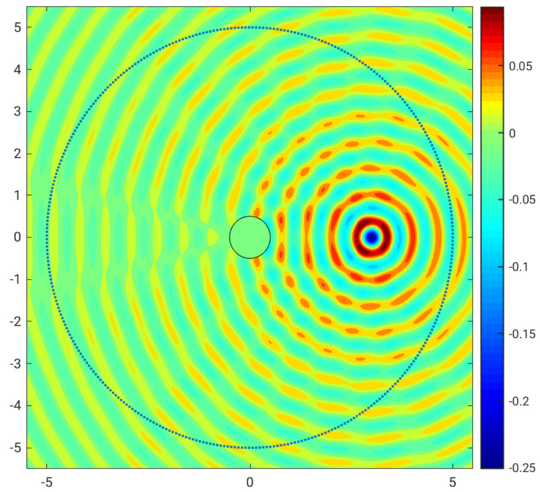
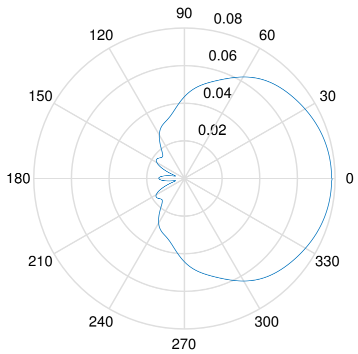
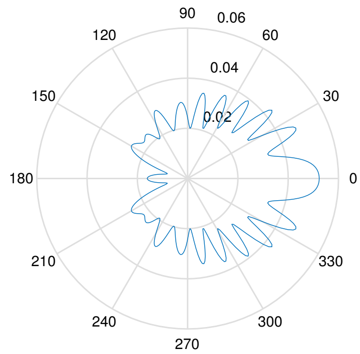


Figure 6: Distribution of the pressure for k = 10



(a) pressure on the cylinder surface



(b) pressure on observers

Figure 7: Acoustic directivity graphs for k = 10

4. CONCLUSIONS

The results of this work were achieved with numerical modeling and application of the Boundary Element Method related to the acoustical area. According to the numerical results, it could be observed which the wavelength decays as the wave number increases. On the other hand, in relation to the acoustic pressure on surface of the cylinder, it was noticed which for low number of the waves it presented a source with monopole type characteristics. And as the wave number increases the source tends to reflect noise with a dipole. In relation to the observers it was verified which for high numbers of wave, the sense noise has a lobulos form as a cardioid.

5. ACKNOWLEDGEMENTS

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