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PHYSICAL, MATHEMATICAL AND COMPUTATIONAL MODELING OF NATURAL CONVECTION IN A TWO-DIMENSIONAL POROUS CAVITY

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Abstract. *In this paper the authors present the physical, mathematical and computational modeling of the natural convection in a square cavity filled with a porous medium. The horizontal walls of the cavity are adiabatic, while the vertical walls are maintained at constant and different temperatures. The modeling equations are solved in a two-dimensional fixed Eulerian and Cartesian domain by the finite-difference method. The results are in accordance with those encountered in literature.*

Keywords: *computational fluid dynamics, numerical simulation, porous media, natural convection*

1. INTRODUCTION

The dynamic and thermal phenomena associated with flows through porous media have been a subject of great interest of scientists and engineers because of its wide application in materials, mechanical, chemical and biomedical engineering. Problems involving absorption, ion exchange, distillation, grain cooling, packed-beds reactors and thermal energy exchangers are some examples. The differentially heated cavity case (DHC) is one of the classical problems in porous media. The temperatures imposed in the vertical walls induce a flow due to the small variations in the specific mass of the fluid phase. Its solution indicates the correct modeling of both mass and energy transport phenomena.

The flow through porous media is traditionally modeled according to Darcy's law. Such model describes the main effects of the porous media over the fluid disregarding advective non-linear therms in the linear momentum equation, which represents the specific case of low Reynolds flows, i.e., creeping flow condition, convenient for the studies of philters, which was the main objective of Darcy's work (Darcy, 1856). Most of engineering practices, however, require the modeling of higher Reynolds number flows. A non-linear therm was added to Darcy's model, due to the experimental analysis conducted by Forchheimer, to correct the advection inertia effect (Forchheimer, 1901). This adjusted model, that conciliates both the inertial and viscous effects, is present in a number of studies such as Guo and Zhao (2002), Wang *et al.* (2015) and et al (2018).

The energy transport in porous media is based on the volumetric-average of the properties of fluid and solid phases, as presented by Vafai (2005). Results on the free convective flows using Darcy-Forchheimer model for porous media can be found in the works of Lauriat and Prasad (1989), Goyeau and Songbe (1996), Al-Amiri (2000) and Khanafer *et al.* (2003).

An in-house code was developed for the modeling of the natural convection in a cavity filled with a homogeneous porous medium. The modeling equations are solved in a two-dimensional fixed Eulerian and Cartesian domain through the finite-difference method. The results are analyzed and compared with those available in the literature.

2. PHYSICAL AND DIFFERENTIAL MODELING

For the modeling of the natural convection in a porous cavity, it is possible to model the fluid as Newtonian and the flow as incompressible. The viscous transformation will be disregarded. The solid phase physical properties are modeled as homogeneous and isotropic through the domain. A graphic representation of the domain and of the boundary conditions is presented at Fig. (1), in which T_l and T_r represent the temperatures imposed for the left and the right walls of the cavity, respectively.

For the differential modeling of fluid mechanics, the application of Reynolds Transport Theorem, Newton's second law and energy balance in a differential volume, provide the relations required to describe the movement of the fluid and evaluate the energy transport and transformations. The first, when employed to the analysis of the mass flux through a control volume (REV), provides the continuity equation, which ensures that the mass balance will be respected. The

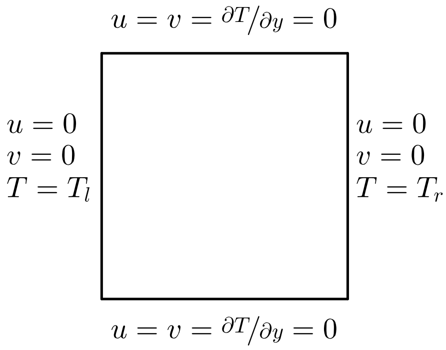

Figure 1: Graphic representation of the boundary conditions.

linear momentum equations, known as the Navier-Stokes equations, are obtained from Newton's second law, that presents the relation between the acceleration perceived by the system and the external forces acting upon it. Finally, the solution of the temperature field is possible through the differential energy equation, obtained from the thermal energy balance applied to a control volume.

For the modeling of flows through isotropic porous media, Darcy-Forchheimer empirical model proposes the addition of a source therm for the linear momentum equation, described as follows.

$$-\boldsymbol{\nabla} P_{pore} = \frac{\mu \mathbf{V}}{K} + \frac{F\rho|\mathbf{V}|\mathbf{V}}{\sqrt{K}}, \tag{1}$$

where $\boldsymbol{\nabla} P_{pore}$ is the pressure drop due to the interaction with the porous medium, μ the fluid dynamic viscosity, K the permeability, **V** the velocity vector, ρ the specific mass of the fluid and F the Forchheimer coefficient (function of porosity and microscopic solid geometry).

For the modeling of the thermal phenomena, the thermal energy equation must account for the conduction, by both fluid and solid phases, and the advection, which occurs only for the fluid phase. Physical properties of the porous medium may be estimated by their volumetric average, function of the porosity (ratio between fluid and total volume in the system).

$$(\rho C)_m = \varepsilon (\rho C)_f + (1-\varepsilon)(\rho C)_s, \tag{2}$$

$$k_m = \varepsilon k_f + (1-\varepsilon)k_s, \tag{3}$$

where the subscripts f, s and m refer to the properties of fluid, solid and porous medium, respectively. The specific thermal capacity is given by C, the thermal conductivity coefficient by k, and the porosity by ε.

The continuity, linear momentum and energy equations for the flow through an isotropic porous media are presented bellow:

$$\boldsymbol{\nabla} \cdot \mathbf{V} = 0, \tag{4}$$

$$\frac{1}{\varepsilon}\frac{\partial \mathbf{V}}{\partial t} + \frac{1}{\varepsilon^2}(\mathbf{V}\cdot\boldsymbol{\nabla})\mathbf{V} = -\frac{1}{\rho}\boldsymbol{\nabla} p - \mathbf{g}\beta(T - T_o) + \frac{\nu}{\varepsilon}\boldsymbol{\nabla}^2\mathbf{V} - \frac{\nu}{K}\mathbf{V} - \frac{F|\mathbf{V}|\mathbf{V}}{\sqrt{K}}, \tag{5}$$

$$(\rho C)_m \frac{\partial T}{\partial t} + (\rho C)_f \mathbf{V}\cdot\boldsymbol{\nabla} T = k_m \boldsymbol{\nabla}^2 T, \tag{6}$$

where p represents the pressure, **g** the gravitational field, β the thermal expansion coefficient, ν the kinematic viscosity (ratio of the viscosity to the specific mass), T the volume averaged temperature (fluid and solid phases jointly) and T_o the reference temperature. This set of equations models the incompressible flow of newtonian fluids through an homogeneous porous medium in transient regime.

The modeling parameters for the buoyancy-driven flow in porous media are usually defined as the conductivity ratio ($\lambda = k_f/k_m$), Prandtl number ($Pr = \nu/\alpha$), Rayleigh number ($Ra = g\beta L^3 \Delta T/\nu\alpha$), Darcy number ($Da = K/L^2$) and the porosity ($\varepsilon$). The symbols α and L represent the medium's thermal diffusivity ($\alpha = k_m/\rho C_m$) and the cavity edge length, respectively.

3. NUMERICAL MODEL

With the physical and differential models defined, a numerical model can be used to obtain an approximate solution to this problem. The domain is discretized evenly and Taylor's expansion is used to the approximation of both first (Central Difference Scheme) and second order derivatives. For a function in a two-dimensional space, the domain may be written as indicated in Eq. (7).

$$\mathcal{M} = \{(t^N, x_I, y_J);\ t^N = N\Delta t,\ x_I = I\Delta x,\ y_J = J\Delta y,\ N = 0,1,\ldots,K,\ I = 0,1,\ldots,L,\ J = 0,1,\ldots,M\}. \quad (7)$$

The Eqs. (4), (5) and (6) can only be applied to a continuous domain and must be rewritten. The modified value for the horizontal and vertical velocity components ($\hat{u}$ and $\hat{v}$, respectively) are obtained from the following equations.

$$\hat{u}_{I,J} = -\varepsilon\Delta t\beta\left[\frac{T^N_{I-1,J}+T^N_{I,J}}{2T_o}-1\right]g_x - \varepsilon\frac{\Delta t}{\rho}\,\frac{p^N_{I,J}-p^N_{I-1,J}}{\Delta x} + \frac{\Delta t\,\mu}{\rho}\,\frac{u^N_{I+1,J}-2u^N_{I,J}+u^N_{I-1,J}}{\Delta x^2} - \varepsilon\frac{\mu}{\rho K}u^N_{I,J} + \frac{\Delta t\,\mu}{\rho}\,\frac{u^N_{I,J+1}-2u^N_{I,J}+u^N_{I,J-1}}{\Delta y^2} - \frac{\Delta t}{\varepsilon}\,u^N_{I,J}\,\frac{u^N_{I+1,J}-u^N_{I-1,J}}{2\Delta x} - \frac{\Delta t}{\varepsilon}\,v^N_{I,J}\,\frac{u^N_{I,J+1}-u^N_{I,J-1}}{2\Delta x} - \varepsilon\frac{F|V^N_{I,J}|u^N_{I,J}}{\sqrt{K}} + u^N_{I,J}, \quad (8)$$

$$\hat{v}_{I,J} = -\varepsilon\Delta t\beta\left[\frac{T^N_{I,J-1}+T^N_{I,J}}{2T_o}-1\right]g_y - \varepsilon\frac{\Delta t}{\rho}\,\frac{p^N_{I,J}-p^N_{I,J-1}}{\Delta y} + \frac{\Delta t\,\mu}{\rho}\,\frac{v^N_{I+1,J}-2v^N_{I,J}+v^N_{I-1,J}}{\Delta x^2} - \varepsilon\frac{\mu}{\rho K}v^N_{I,J} + \frac{\Delta t\,\mu}{\rho}\,\frac{v^N_{I,J+1}-2v^N_{I,J}+v^N_{I,J-1}}{\Delta y^2} - \frac{\Delta t}{\varepsilon}\,v^N_{I,J}\,\frac{v^N_{I,J+1}-v^N_{I,J-1}}{2\Delta y} - \frac{\Delta t}{\varepsilon}\,u^N_{I,J}\,\frac{v^N_{I+1,J}-v^N_{I-1,J}}{2\Delta x} - \varepsilon\frac{F|V^N_{I,J}|v^N_{I,J}}{\sqrt{K}} + v^N_{I,J}. \quad (9)$$

The correction for the pressure (p^o) is obtained from the continuity equation, as indicated in Eq. (10), and the value of the velocity components must be corrected from its modified value through this correction. The pressure field is logged at each time step as presented in Eq. (12).

$$\boldsymbol{\nabla}^2 p^o = \boldsymbol{\nabla}\cdot\mathbf{V} \approx \frac{\hat{u}_{I+1,J}-\hat{u}_{I,J}}{\Delta x} + \frac{\hat{v}_{I,J+1}-\hat{v}_{I,J}}{\Delta y}, \quad (10)$$

$$u^{N+1}_{I,J} = \hat{u}_{I,J} - \frac{p^o_{I,J}-p^o_{I-1,J}}{\Delta x}, \quad (11)$$

$$p_{I,J} = p_{I,J} + p^o_{I,J}. \quad (12)$$

The energy equation, in its turn, can be rewritten as indicated bellow. In Eq. (13), the parameters $\check{u}$ and $\check{v}$ represent the interpolated velocity components.

$$T^{N+1}_{I,J} = \Delta t\,\alpha\left[\frac{T^N_{I+1,J}-2T^N_{I,J}+T^N_{I-1,J}}{\Delta x^2} + \frac{T^N_{I,J+1}-2T^N_{I,J}+T^N_{I,J-1}}{\Delta y^2}\right] - \frac{\Delta t\,(\rho C)_f}{(\rho C)_m}\left[\check{u}^N_{I,J}\,\frac{T^N_{I+1,J}-T^N_{I-1,J}}{2\Delta x} - \check{v}^N_{I,J}\,\frac{T^N_{I,J+1}-T^N_{I,J-1}}{2\Delta x}\right] + T^N_{I,J}. \quad (13)$$

4. RESULTS

To present properties that are exclusive of porous media, this work evaluates the deviation of the flow in function of the medium's permeability, or Darcy number. Both aspect and conductivity ratios are unitary, the Rayleigh number will be maintained at 10^6. The simulations will be conducted for a homogeneous porous medium composed mostly of fluid ($\varepsilon = 0.9$). The transient solution at 20 minutes will be compared with the permanent results presented by Lauriat and Prasad (1989).

The influence of the Darcy number on the temperature field is, as it is possible to observe in Fig. (2) and (3), directly related to the advection of thermal energy. The increase in permeability reduces the drag imposed by the solid phase and allows the fluid phase to transport energy more efficiently than the medium's conduction. The spatial averaged Nusselt number on the cold wall for $Da = 10^{-5}$ is presented in Tab.(1). The results of the present work are in agreement with those proposed by Lauriat and Prasad (1989).

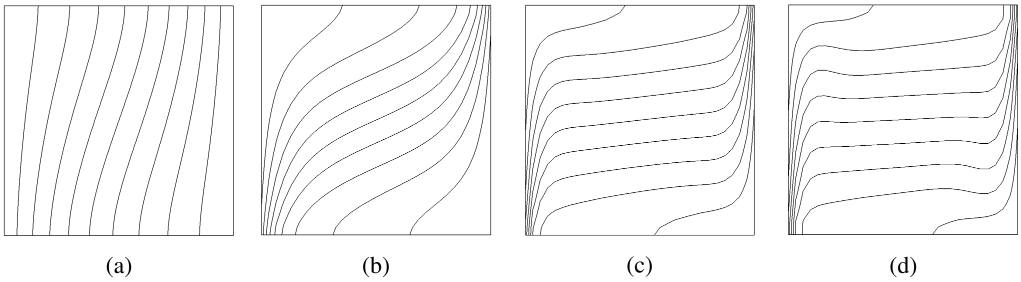

Figure 2: Isotherms for free convection at $t = 20$ minutes in a square cavity for $\lambda = 1$, $Pr = 0.71$, $Ra = 10^6$, $\varepsilon = 0.9$: (a) $Da = 1 \cdot 10^{-5}$, (b) $Da = 1 \cdot 10^{-4}$, (c) $Da = 1 \cdot 10^{-3}$, (d) $Da = 3 \cdot 10^{-3}$.

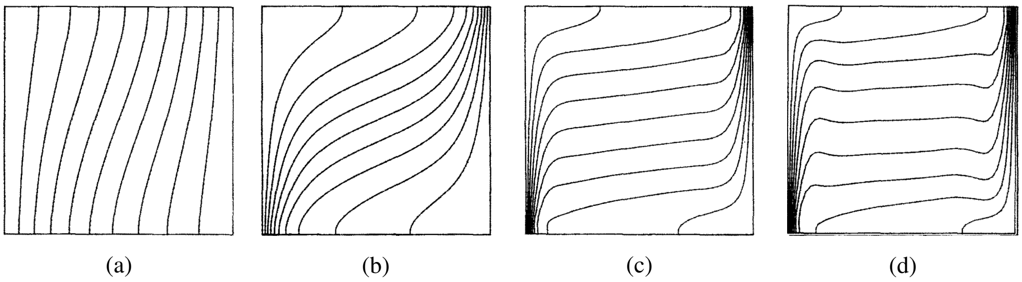

Figure 3: Isotherms for free convection at permanent regime in a square cavity presented by Lauriat and Prasad (1989) for $\lambda = 1$, $Pr = 0.71$, $Ra = 10^6$, $\varepsilon = 0.9$: (a) $Da = 1 \cdot 10^{-5}$, (b) $Da = 1 \cdot 10^{-4}$, (c) $Da = 1 \cdot 10^{-3}$, (d) $Da = 3 \cdot 10^{-3}$.

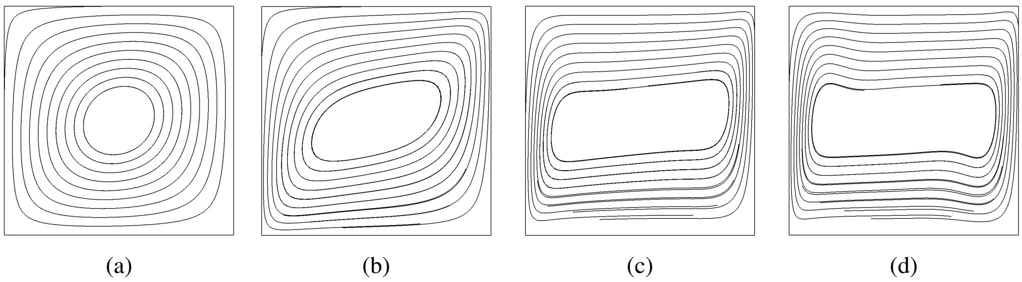

Figure 4: Streamlines for free convection at $t = 20$ minutes in a square cavity for $\lambda = 1$, $Pr = 0.71$, $Ra = 10^6$, $\varepsilon = 0.9$: (a) $Da = 1 \cdot 10^{-5}$, (b) $Da = 1 \cdot 10^{-4}$, (c) $Da = 1 \cdot 10^{-3}$, (d) $Da = 3 \cdot 10^{-3}$.

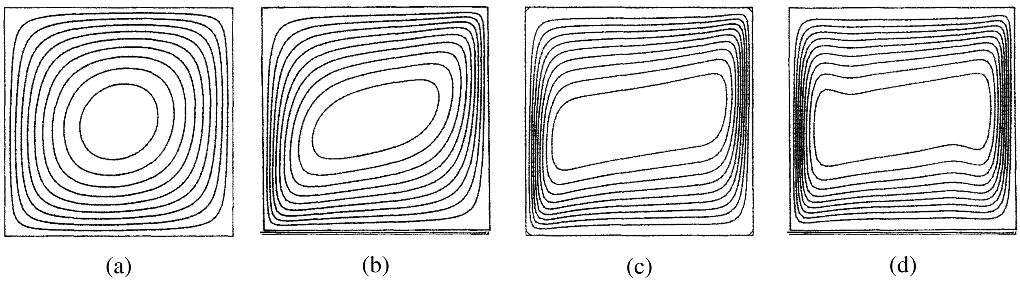

Figure 5: Streamlines for free convection at permanent regime in a square cavity presented by Lauriat and Prasad (1989) for $\lambda = 1$, $Pr = 0.71$, $Ra = 10^6$, $\varepsilon = 0.9$: (a) $Da = 1 \cdot 10^{-5}$, (b) $Da = 1 \cdot 10^{-4}$, (c) $Da = 1 \cdot 10^{-3}$, (d) $Da = 3 \cdot 10^{-3}$.

Table 1: Average Nusselt number on the cold wall for different Rayleigh numbers.

	Ra = 10^6	Ra = 10^7	Ra = 10^8
Present work	1.07	3.14	12.59
Lauriat and Prasad (1989)	1.07	3.02	12.42

5. CONCLUSION

The modeling of mass, energy and linear momentum transport phenomena in a homogeneous porous medium was presented. An in-house code was developed and simulations were conducted aiming the solution of the classic differentially heated cavity case. The results were compared with those presented in the literature and their analysis allow the evaluation of the flow's characteristics in function of the medium's permeability. An inverse relation was established between the magnitude of advective effects and the Darcy number, since the latter is related to the medium's resistance to the flow.

6. ACKNOWLEDGEMENTS

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8. RESPONSIBILITY NOTICE