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ENERGY TRANSFER IN COMPOUND FOUNDATIONS FOR ARBITRARY BONDING CONDITIONS

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Abstract. This work discusses energy transfer in a piled raft foundation problem for different bonding configurations found in practice. The surface raft is modeled as circular rigid plate, the pile is modeled as an elastic bar embedded in perfectly bonded contact within the soil, which in turn is modeled as a viscoelastic, transversely isotropic, homogeneous half-space. Traction distribution between plate and soil and between pile and soil is written as piece-wise constant boundary elements. Coupling between the foundation and the soil is obtained via direct equilibrium and kinematic compatibility at the interfaces. The system is under time-harmonic, external vertical loads uniformly distributed on the surface of the plate, or vertically- propagating, time-harmonic seismic pressure waves. A Winkler layer of stiffness K_w is added throughout the plate-soil interface, and an additional spring K_p is added at the plate-pile contact point. Arbitrary bonding conditions between plate-soil and plate-pile can be obtained by varying K_w and K_p . A parameter α varying from 0 to 1 is defined in order to make the bonding conditions easier to interpret. The limiting condition in which all energy is transferred from the plate directly to the pile, and then to the soil, is represented by $\alpha=1$, and arbitrary values of K_w and K_p . The other limiting condition in which all energy is transferred from the plate directly to the soil is represented by $\alpha=0$, and different values of K_w and K_p . The article discusses the appropriate description of α in terms of K_w and K_p to represent all bonding cases in between. A parameterized foundation model is also used in this study. This model is obtained by directly adding together the complex spring constant of plate-soil and pile-soil interaction. This parameterized model disregards energy transfer from the plate to the pile through the soil. The present model contributes to our understanding of energy transfer between different parts of compound foundations with different bonding conditions.

Keywords: *Dynamic Soil-Foundation Interaction, Raft Foundations, Pile vibration, Coupled Foundations*

1. INTRODUCTION

The dynamics of soil-structure interaction is an important consideration in the design of structures subject to loads such as earthquakes, winds and machine vibrations. According to Di Laora (2009), the response of the structure and of the soil in a soil-structure problem are coupled, and the process by which structure vibration influences soil response and vice versa is called dynamic soil-structure interaction (DSSI).

The literature shows that there are two main categories of models of DSSI. The first is based on finite element discretization approach that provides sophisticated representation of the foundation and pile, but lacks accuracy in modelling the soil component. This includes modeling the soil as nonlinear springs, or with infinite elements, which may fail to satisfy Sommerfeld's radiation condition for the unbounded medium, or model the propagation of waves in the medium as guided waves. Dutta and Roy (2002) list a comprehensive summary of soil models and their limitations. The second broad category of DSSI models is based on boundary element discretizations (Labaki, Mesquita and Rajapakse, 2013 and 2014).

The soil is an unbounded domain presenting outgoing and non-reflected waves that withdraw energy from the excitation source. This effect is known as Sommerfeld radiation condition (Sommerfeld, 1949) and is also called radiation damping. The modeling of unbounded domains presenting radiation damping requires special techniques that incorporate this damping effect, such as a semi-analytical method based on a Green's function approach (GF) or the

Boundary Element Method (BEM). The stationary dynamic response of the soil has been successfully obtained by the BEM and GF strategies (Carrion, Sousa and Mesquita, 2007; Labaki, Mesquita and Rajapakse, 2014).

Rajapakse and Wang (1993) presented the solution of a boundary value problem describing the dynamic interaction of concentrated and distributed loads applied at the surface or embedded in the soil. As for the pile problem, Rajapakse and Shah (1987) presented an extensive, detailed model of an elastic embedded bar and discussed various discretization schemes for the problem. These are the main models used in this work.

Lima, Labaki and Mesquita (2016, 2019) studied the problem of a piled-raft foundation under external time-harmonic excitations or under an incident vertically-propagating pressure wave and investigated how the continuous change from one support mechanism (soil) to the other (pile) influences the dynamic response of the foundation. That study showed that the response of the coupled system depends significantly on the amount of load that is transferred from the foundation to the pile and/or to the soil. The continuous change from one support mechanism to the other is described through the dimensionless parameter α . The limiting condition in which all energy is transferred from the plate to the pile, and then to the soil, is represented by $\alpha=1$, and the limiting condition in which all energy is transferred from the plate directly to the soil is represented by $\alpha=0$.

Labaki, Barros and Mesquita (2019) presented the corresponding response in the case in which a Winkler layer is added at the plate-soil interface. Their model enables arbitrary bonding conditions between the soil and the foundation to be considered. The results showed that different bonding conditions affect the dynamic response of the system significantly. Figure 1 shows a comparison between the results presented by Lima, Labaki and Mesquita (2016, 2019) in terms of α and the results presented by Labaki, Barros and Mesquita (2019) in terms of K_w . These cases consider a piled raft subjected to a unit external force. The results are shown in terms of the normalized vertical compliance of the foundation. In these cases, $E_{hs}=2.5\text{Pa}$, $\eta_{hs}=0.01$, $\nu_{hs}=0.25$, $\rho_{hs}=1\text{kg/m}^3$, $l_p=10\text{m}$, and $E_p=2500\text{Pa}$.

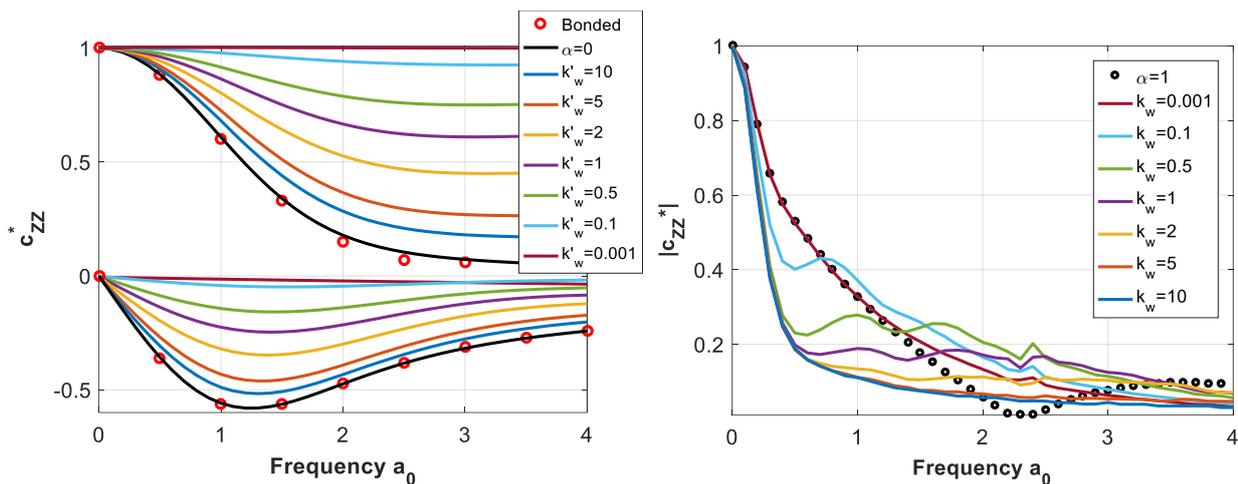


Figure 1. Comparison between the response obtained by Lima, Labaki and Mesquita (2016) and Barros, Labaki and Mesquita (2019) for a piled raft foundation under to a vertical force.

Figure 1 shows that both formulations agree with regard to the response of the piled plate. The difference in the response of Fig. 1b is that the manner in which Lima, Labaki, and Mesquita (2016, 2019) modeled the pile does not consider the inertia of the pile for higher frequencies.

2. STATEMENT OF THE PROBLEM

Consider the piled-raft foundation problem illustrated in Fig. 2. The soil is modeled as a 3D isotropic, elastic half-space, with shear modulus G_{hs} , Poisson's ratio ν_{hs} , mass density ρ_{hs} and material damping coefficient η_{hs} . The pile is modeled as a series of connected one-dimensional, two-noded finite beam elements, with Young's modulus E_p , mass density ρ_p , radius a_p and length l_p . The foundation is modeled as a rigid circular plate with mass m_f and radius a_b , and is centered at the origin of the coordinates system and connect to the pile head at its center. The coordinate system is placed so that the x-y plane is aligned with the surface of the half-space, the pile is aligned along the z-axis and the center of the foundation coincides with the origin of the coordinate system (Fig.1). A Winkler layer of stiffness K_w is added at the continuous plate-soil interface. An additional spring K_p is added at the plate-pile interface. These additional connecting layers may represent arbitrary bonding conditions between these systems in engineering practice. The goal of this article is to study the influence of different bonding conditions in the vertical time-harmonic response of the present foundation.

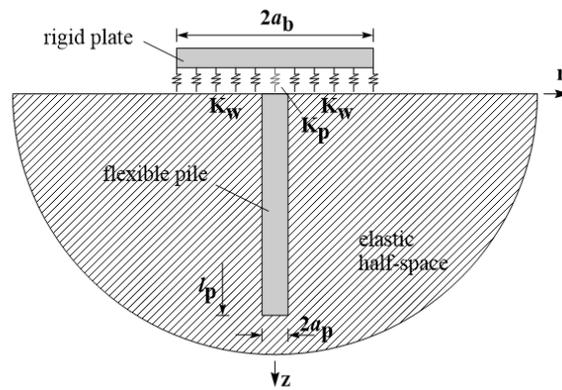


Figure 2. Piled raft connected to the soil through a Winkler layer (K_w) and a spring (K_p).

3. NUMERICAL MODEL

Consider an external, time-harmonic, vertical point load $Q(t) = Q_0 e^{i\omega t}$, applied at the center of the rigid foundation, where Q_0 is the amplitude of the loading, ω is the circular frequency, and $i = \sqrt{-1}$. The dynamic equilibrium, in the frequency domain, of the rigid foundation under the action of external force is given by Eq.1:

$$Q_0 - 2p \int_{a_p}^{a_b} t_b r dr - f_0 = -m_b \omega^2 u_0 \quad (1)$$

in which, t_b is the vertical stress along the soil-foundation contact, f_0 is the force exerted by the pile at the base of the plate and u_0 is the vertical displacement of the foundation.

For the numerical solution of the problem, the soil-foundation contact is divided into n_b concentric rings and the tractions t_b is assumed constant in each of these concentric rings. Thus, Eq. (1) can be rewritten as:

$$\{s_b\}^T + \{t_b\} + f_0 - m_b \omega^2 u_0 = Q_0 \quad (2)$$

in which $\{t_b\}$ is the column vector with the values of t_b in each ring and $\{s_b\}$ is the vector column with the soil-foundation contact areas of each ring.

The pile is modeled using the Finite Element Method (FEM), using bar elements of length l_e , with two nodes and one degree of freedom per node. The pile is divided into n_p elements, so that $n_n = n_p + 1$ is the number of nodes in the pile model.

By assuming a linear interpolation function along the bar elements, the stiffness and mass matrices of the pile elements are given respectively by Eq. (3) and Eq. (4).

$$[K_e] = \frac{\pi a_p^2 E_p}{l_e} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \quad (3)$$

$$[M_e] = \frac{\pi a_p^2 \rho_p l_e}{6} \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \quad (4)$$

The global stiffness $[K]$ and mass $[M]$ matrices are obtained from the matrices of elements by the standard FEM assembly process.

The soil-pile interface is divided into n_p cylindrical segments of length l_e and radius a_p , plus one disc element at the pile base. The dynamic response of the pile can be described by Eq. (5):

$$\underbrace{([K] - \omega^2 [M])}_{[\bar{K}]} u_f + [A] \{t_p\} - \{i_0\} f_0 = \{0\} \quad (5)$$

in which $i_0 = \{1 \ 0 \ 0 \ \dots \ 0\}^T$, $t_p = \{t_{p1} \ t_{p2} \ \dots \ t_{pn_p}\}^T$ is the vector of contact tractions at the discretized pile-half-space interface, u_f is the vector of nodal displacements of the discretized interface, $[A]$ is a transformation matrix which transform the tractions at the half-space-pile in nodal equivalent loads.

The response of the elastic half-space to the actions applied by both the rigid plate and the pile are obtained by the indirect formulation of the boundary element method (I-BEM). The tractions and displacements along the soil-pile and soil-foundation interfaces are expressed by:

$$\begin{aligned}\{u_p\} &= [U_{pp}]\{q_p\} + [U_{pb}]\{q_b\}, \\ \{u_b\} &= [U_{bp}]\{q_p\} + [U_{bb}]\{q_b\}, \\ \{t_p\} &= [T_{pp}]\{q_p\} + [T_{pb}]\{q_b\}, \\ \{t_b\} &= [T_{pp}]\{q_p\} + [T_{pb}]\{q_b\},\end{aligned}\tag{6}$$

in which u_p and u_b are respectively the vertical displacement of the center of the soil-pile and soil-foundation elements. The matrices $[U_{ij}]$ and $[T_{ij}]$ contain respectively the displacement and the tractions in the center of the interface element i due to a uniform unit load applied on element j . The vectors q_p and q_b are fictitious load applied along the soil-pile and soil-foundation interfaces. Since the plate is supported on the surface of the half-space, $[T_{bp}] = [0]$ and $[T_{bb}] = [I]$, where $[I]$ is the identity matrix. Therefore, $\{t_b\} = \{q_b\}$.

Substituting the third of Eqs. (6) into Eq. (5) gives:

$$[\bar{K}]\{u_f\} + [A][T_{pp}]\{q_p\} + [A][T_{pb}]\{q_b\} - \{i_0\}f_0 = \{0\}\tag{7}$$

The kinematic compatibility between the displacement $\{u_p\}$ and $\{u_f\}$ imposes that $\{u_p\} = [D]\{u_f\}$, in which:

$$[D] = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 & \dots & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & \dots & 0 & 1 \end{bmatrix}\tag{8}$$

The displacement of the rigid foundation is $\{u_b\} = \{1\}u_0$. The kinematic compatibility between the foundation and the pile tip is $\{u_0\} = \{i_0\}^T\{u_f\}$. Therefore, the displacement $\{u_b\}$ is:

$$\{u_b\} = \{1\}\{i_0\}^T\{u_f\}\tag{9}$$

Equations (2), (6), (7), (9), the kinematic compatibility between the foundation and the pile tip and the kinematic compatibility between the displacement $\{u_p\}$ and $\{u_f\}$ form a system of equations, which in matrix form is:

$$\begin{bmatrix} [\bar{K}] & [A][T_{pp}] & [A][T_{pb}] & -\{i_0\} \\ -[D] & [U_{pp}] & [U_{pb}] & \{0\} \\ -\{1\}\{i_0\}^T & [U_{bp}] & [U_{bb}] & \{0\} \\ \{i_0\}^T & \{s_b\}^T [T_{pb}] & \{s_b\}^T [T_{bb}] & 1 \end{bmatrix} \begin{Bmatrix} \{u_f\} \\ \{q_p\} \\ \{q_b\} \\ f_0 \end{Bmatrix} = \begin{Bmatrix} \{0\} \\ \{0\} \\ \{0\} \\ Q_0 \end{Bmatrix}\tag{10}$$

The solution of the system of Eq. (10) gives the nodal displacements $\{u_f\}$, the fictitious loads $\{q_p\}$ and $\{q_b\}$, and the force f_0 between the foundation and the pile. The foundation displacement is obtained from the relation between the u_0 and u_f . The tractions can be determined along the soil-pile and soil-foundation interfaces by substituting the values of the fictitious loads into Eqs (6). This embedded foundation model was first proposed by Barros, Labaki and Mesquita (2019), to which one may refer for further details.

3.1 Inclusion of Winkler layer K_w and spring K_p

This section extended the above formulation to address the problem of imperfect connection at the plate-soil and plate-pile interfaces, which are cases that one may encounter in geotechnical engineering practice. In the first case, a Winkler layer K_w is inserted between the foundation and the half-space. In the second case, a spring K_p is inserted to connect the top of the pile to the center of the rigid foundation.

When a Winkler layer K_w is added between the foundation and the soil, the displacements of the foundation and the displacements of the soil interface elements differ from each other. In this case, the force of contact between the foundation and the half-space will be given by:

$$\{t_b\} = K_w \left(\{u_b^*\} - \{u_b\} \right) \quad (11)$$

where u_b^* are the vector of displacements of the plate elements above the elements of the Winkler layer and K_w is the coefficient of the Winkler layer considered.

When a spring K_p is added between the center of the foundation and the pile head, the displacements of the foundation and the displacement of the pile head also differ from each other. The relationship between the displacements of the plate and of the pile head are then:

$$u_0 = u_{f0} + \frac{f_0}{K_p} \quad (12)$$

where u_0 are the displacements of the plate, u_{f0} is the displacement of the pile head and K_p is the coefficient of the spring considered.

When a spring K_p is present, the displacement of the plate elements will be given by:

$$\{u_b^*\} = \{I\} \left(\{i_0\}^T \{u_f\} + \frac{f_0}{K_p} \right) \quad (13)$$

Substituting Eqs. (13) and (11) in the second of Eq. (6) gives

$$\{t_b\} = K_w \{I\} \{i_0\}^T \{u_f\} - K_w [U_{bp}] \{q_p\} - K_w [U_{bb}] \{q_b\} + \frac{K_w}{K_p} \{I\} f_0 \quad (14)$$

Combining Eq. (14) with the fourth of Eq. (6) yields

$$-K_b \{I\} \{i_0\}^T \{u_f\} + (K_b [U_{bp}] + [T_{bp}]) \{q_p\} + (K_b [U_{bb}] + [T_{bb}]) \{q_b\} - \frac{K_b}{K_p} \{I\} f_0 = \{0\} \quad (15)$$

Changing the third equation of the system (10) by Eq. (15) gives the new equation of motion with the presence of K_w and K_p :

$$\begin{bmatrix} [\bar{K}] & [A][T_{pp}] & [A][T_{pb}] & -\{i_0\} \\ -[D] & [U_{pp}] & [U_{pb}] & \{0\} \\ -K_w \{I\} \{i_0\}^T & K_w [U_{bp}] + [T_{bp}] & K_w [U_{bb}] + [T_{bb}] & -\frac{K_w}{K_p} \{I\} \\ \{i_0\}^T & \{s_b\}^T [T_{pb}] & \{s_b\}^T [T_{bb}] & I \end{bmatrix} \begin{Bmatrix} \{u_f\} \\ \{q_p\} \\ \{q_b\} \\ f_0 \end{Bmatrix} = \begin{Bmatrix} \{0\} \\ \{0\} \\ \{0\} \\ Q_0 \end{Bmatrix} \quad (16)$$

The solution of Eq. (16) provides the displacement at the top of the pile u_{f0} . The displacement of the foundation u_0 is given by Eq. (12).

4. NUMERICAL RESULTS

Figure 3 considers the case of external excitation of a massless rigid foundation at the surface of the half-space with the presence of the pile. The results are presented in terms of the normalized vertical compliance of the foundation, $c_{zz}^* = c_{zz}/c_{zz}(a_0=0)$, in which $c_{zz} = u_0 a_p \mu_s / Q_0$, with respect to the normalized frequency $a_0 = \omega a \sqrt{\rho_s / \mu_s}$. In this section, the following parameters are considered: for the half-space, $E_{hs} = 2.5 \text{ Pa}$, $\eta_{hs} = 0.01$, $\nu_{hs} = 0.25$, $\rho_{hs} = 1 \text{ kg/m}^3$; for the foundation, $a_f = 1 \text{ m}$, and for the pile: $h_p/a_p = 10$, $\rho_p = 1 \text{ kg/m}^3$. The influence of the stiffness of the Winkler layer K_w and the spring K_p is shown in a comparison with the values of the formulation of α by Lima, Labaki and Mesquita (2016).

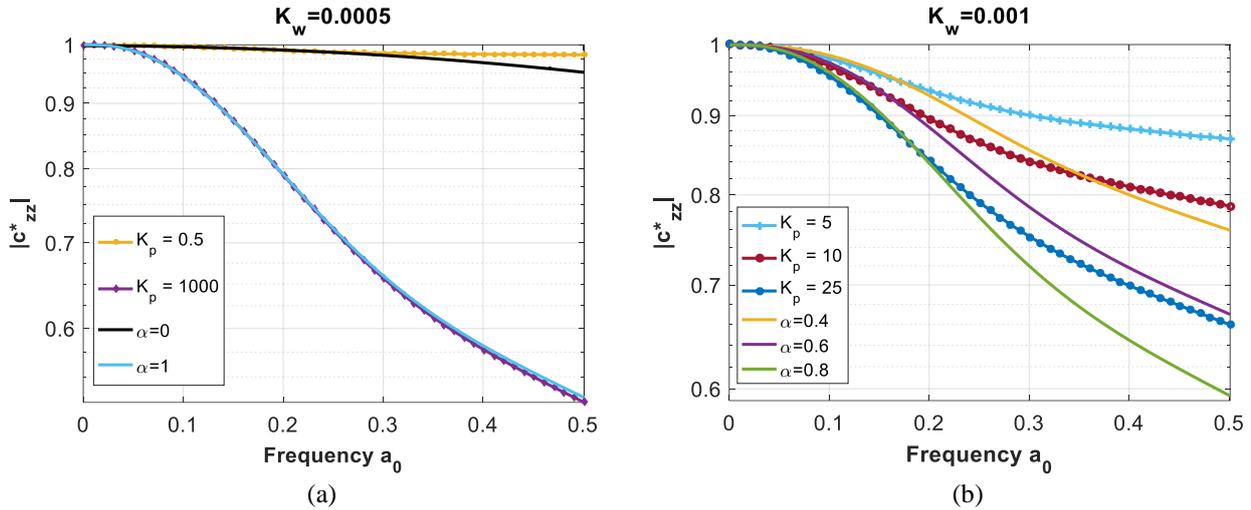


Figure 3. Comparison of the results for an external excitation of a piled-raft in values of K_w , K_p and (a) extreme and (b) intermediate values of α .

Figure 3a shows that by setting a value of K_w and changing only the value of K_p it is possible to reproduce the results of the α formulation by Lima, Labaki and Mesquita (2016). In the results of Fig. 3a, only the K_p value was changed. The properties of the pile, half-space and foundation were not altered. For the first frequencies a_0 , both results agree. As the value of a_0 increases, the results begin to show a small difference. Figure 3a shows only a comparison to the extreme values of α (0 and 1). Figure 3b shows a comparison with intermediate values of α . Like Fig. 3a, Fig. 3b shows that by setting a value of K_w and changing only the value of K_p it is possible to reproduce the results of the α formulation by Lima, Labaki and Mesquita (2016). For the first frequencies a_0 , both results agree. As the value of a_0 increases, the results begin to show a small difference. By changing the values of K_w and K_p in Fig. 3b, it is possible to get closer to the results obtained with the formulation of α for greater values of a_0 . In Figs. 3a and 3b, the value of K_w was fixed and only the value of K_p was modified. In Fig. 4, the opposite was done. The value of K_p was set and the value of K_w was varied.

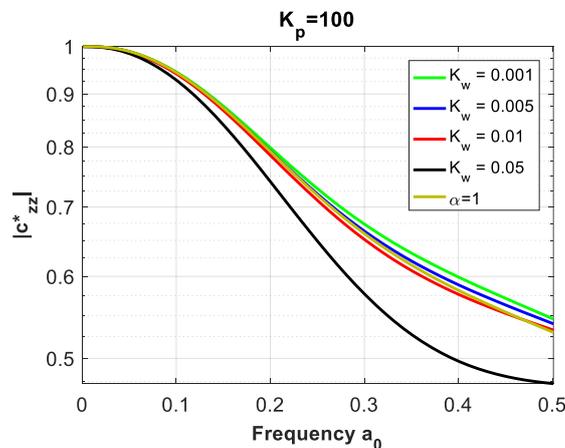


Figure 4. Influence of K_w , K_p in the response of the piled plate to an external excitation.

Figure 4 shows that by setting a value of K_p and changing only the value of K_w it is also possible to reproduce the results of the α formulation by Lima, Labaki and Mesquita (2016). In the results of Fig. 4, only the K_w value was changed. The properties of the pile, half-space and foundation were not altered.

Figures 3 and 4 show that the inclusion of imperfect bonding conditions at the plate-soil and plate-pile interfaces through Winkler and spring constants K_w and K_p agree with the limiting cases shown by Lima, Labaki and Mesquita (2016) for different ratios of energy transfer from plate to soil and from plate to pile to soil.

5. CONCLUSIONS

This article presented a model of the time-harmonic response of piled-raft foundations with elastic bonds between the surface raft and the soil and between the raft and its supporting pile. An IBEM model of the soil was coupled with the foundation and the pile at discrete points. Elastic continuity conditions were established at the foundation-soil and foundation-pile interfaces to represent imperfect bonding conditions. The results showed that different bonding conditions affect the dynamic response of the system significantly.

6. ACKNOWLEDGEMENTS

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