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# DYNAMIC SIMULATION OF A PROSTHETIC FOOT CONSIDERING ROLLING AND CONTACT DEFORMATION

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**Abstract:** *The present work intends to develop an analysis for an orthopedic prosthesis for a transtibial amputee, focusing on the prosthetic foot. With this, it proposes a geometry for the prosthetic foot considering the effect of the contact and the bearing that the foot does during the walk. The proposed model is composed of two masses, the individual and the prosthetic foot, and by the respective vertical displacements, at first it is considered the degree of freedom regarding the prosthetic foot rotation, in a second stage the prosthetic foot rotation and consequent relative displacement of the ankle are considered as auxiliary variables calculated from geometric parameters and materials. The effective mass, the vertical and rotational ankle rigidity, the stiffness of the sole and the displacement of the contact point during the rolling are considered as parameters of the simulation. Damping factors can also be analyzed as non-linear. The simulation is performed from the parameter survey and the integration of subdomains of the time of the gait using integration of Runge Kutta of 4th order for a multibody model. Results of these simulations are presented as angles, contact force and vertical displacements.*

**Keywords:** *dynamic system, contact, bearing, prosthetic*

## 1. INTRODUCTION

The gait is a periodic movement and contains two phases: support and balance. The gait is relatively symmetrical in relation to the angular movements of the main joints, patterns of muscle activation and discharge of weight in the lower limbs. As a consequence, it is efficient moving the center of mass in the direction of locomotion. In (Winter et al, 1990) the full gait cycle can be described starting at the moment when the heel touches the ground and is terminating when the same heel touches the ground again, thus defining the boundaries of the gait. For (Rose et al, 2006) the support phase involves 60% of the stride and is summarized in two moments of double support of the limb, when the contralateral foot is in contact with the ground, and an intermediate period of unipodal support, when the limb is in the balance phase. In order to consider the advantages of foot bearing during human walk (Adamczyk et al, 2006) performed an experimental analysis and numerical simulation in which he relates the work performed in the collision (impact + contact) of the heel at each step of the walk with the foot bearing. (Moreira et al, 2009) instead of using a single arc with radius of constant curvature, represented the sole of the foot discretized by spheres of different radii of curvature. (Flores et al, 2016) considered the impact contact problems for biomechanical systems, among the main is the spine, meniscus and foot. An aspect of high relevance in the prosthetic foot is the rigidity of the material, which function as wedges, since it will absorb the impact caused by the heel, and energy loss of the energy expended for the total or partial deformation of the material. Considering the impact velocity, material properties, collision of bodies and geometric characteristics of the contact surface, all this for the calculation of the contact force.

## 2. METHODOLOGY

From a kinematic analysis of an individual's gait, a geometry is proposed for the prosthetic foot, considering the effect of the contact and the bearing that the foot does. The contact generates a force that causes deformation, when the whole system is represented in rigid and flexible elements for a dynamic analysis.

It is important to emphasize that, when the contact prosthetic foot touches the ground, much of its mass is supported on the intact foot and therefore does not participate in the vibration caused by the impact of this device. This leads to the need to consider fractions of the total mass during the course, starting from zero to the total value, when the individual rests entirely on the prosthetic foot. This behavior is represented by an effective mass that presents a variation associated to the ground reaction force, obtained from the kinematic analysis and specific for each individual. The variation of the body mass implies that the inertia matrix varies along the course.

The proposed model for the prosthetic foot only considers the support phase, that is, the prosthetic foot is in contact with the ground all the time of the simulation. According to the proposed geometry, the shape of the sole of the prosthetic foot corresponds to a convex curve, formed by circles with different radii of curvature, as shown in Fig. 1, and this curve is discretized by spheres (exemplified in Fig. 1 on the left, by blue spheres) representing the flexible stiffness elements of the model, which contact the ground and suffer impacts and rolling. There is also flexible element that functions as a mechanical ankle, in Fig. 1 to the right, this ankle consisting of a standard pin that rotates during the individual's walk.

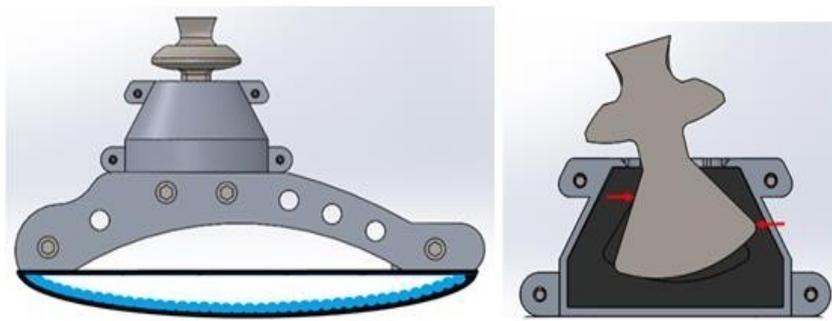


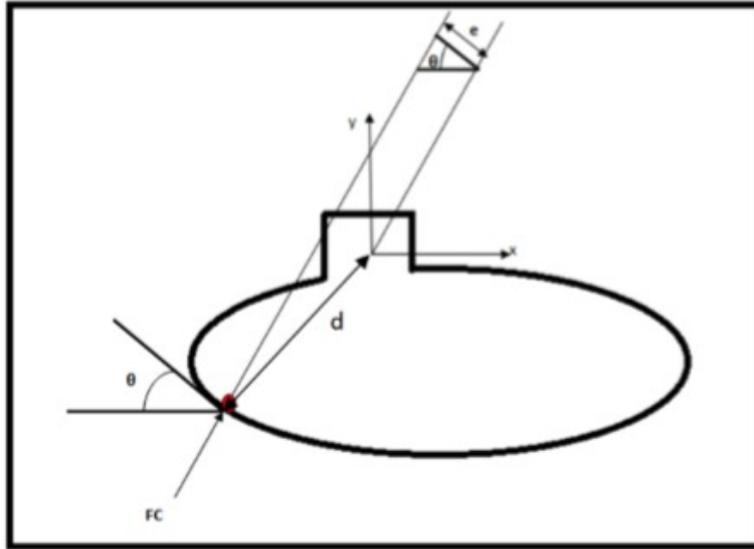
Figure 1 - The proposed physical model of the prosthetic foot.

The kinematic study of prosthetic foot movement depends on determining the points of contact along the movement. For the contact condition, a geometric constraint is established corresponding to the point at which the representative line of the soil tangents the convex curve referring to each arc of the circle. The mathematical determination is presented below. Whenever a point of contact is found a sphere becomes associated with it, then this sphere is defined as 'activated'. An 'activated' sphere suffers a non-linear contact force which in the proposed model corresponds to the Hertz contact force (FH). It is important to emphasize that this force is admitted as perpendicular to the straight line representing the ground throughout the bearing, because this model does not consider the effect of friction. For the calculation of equivalent contact stiffness, the hypothesis is considered, that the contact area between the prosthetic foot plant and the ground is shaped like a circle. The Hertz theory of contact between two spheres is applied, considering one of them with infinite radius. It is proposed that the sphere with finite radius is made of the mechanical property of modulus of elasticity and the sphere of infinite radius is the ground, which has a much greater modulus of elasticity than that of the polymeric material, (Johnson *et al*, 1985).

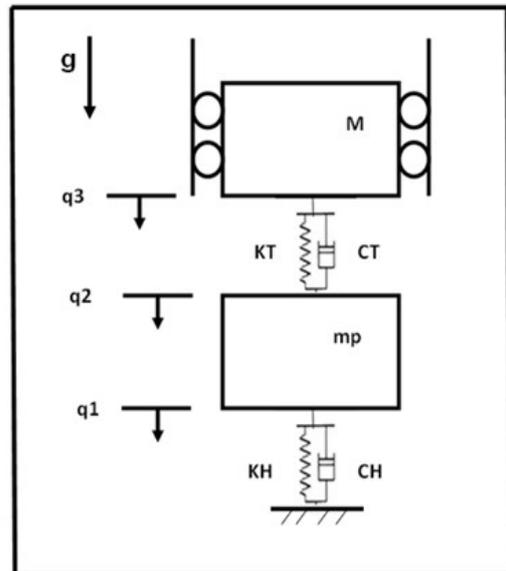
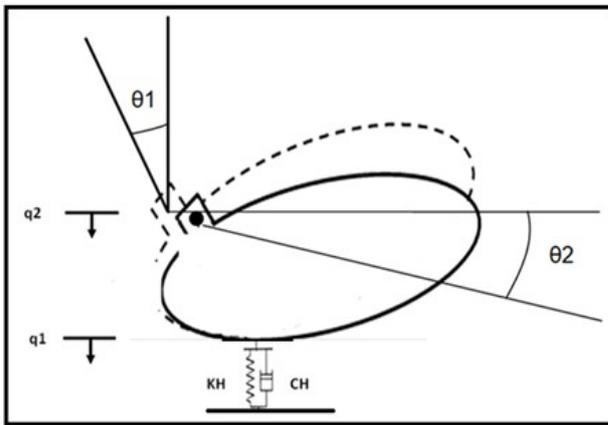
It is also necessary to know the angle of the leg  $\theta_1$  and the angle of rotation of the ankle  $\theta_2$ . For the positioning  $\theta$  (sum  $\theta_1$  and  $\theta_2$ ) of the prosthetic foot in relation to the ground. From the centers of circle arcs it is possible to find the coordinates  $(x_c, y_c)$  of the points of contact of the convex curve, by geometry the expressions for the coordinates of contact point is found. From the schematic of the geometric shape of the prosthetic foot, as shown in Fig. 2a, and considering the angle  $\theta$  and the coordinates of the contact point (red dot), one can find the arm of the lever ( $e$ ) that generates moment provoked by FC that corresponds to the contact force perpendicular to the tangent line and ( $d$ ) the distance between the point of contact and the origin of the reference system.

Both the rotation of the prosthetic foot and the lowering or elevation of the ankle due to this rotation are not associated with terms of inertia and are not considered as generalized displacements in the construction of the governing equations. However, these displacements, vertical and angular, influence the equilibrium and are used in the assembly of the system of equations as terms of forces. It is important to make an observation at this point in the description of the model. The angular movement of the leg is performed by the individual through a muscular action, as previously said. It is an active drive that must be evaluated through kinematics. In turn, the angular displacement of the ankle is passive, caused by the moment action performed by the contact force and the lever arm relative to the point of contact. Although it is a passive movement it can be perfectly determined by the geometry and parameters involved.

a)



b)



c)

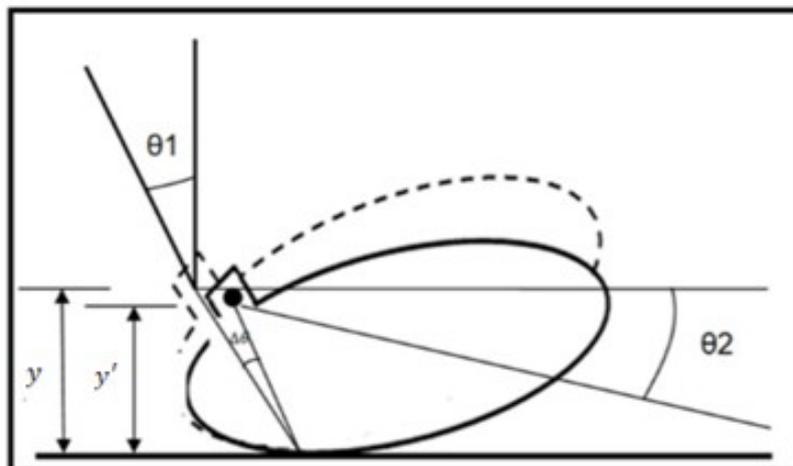


Figure 2- a) Prosthetic foot schematic. b) Inertia terms, flexible elements and degrees of freedom of the proposed model. c) Angular rotation of the prosthesis due to the force of contact.

### 3. EQUATION OF THE DYNAMIC MODEL

The mechanical solution given to the analyzed device allows prosthesis angular rotation in sagittal plane as its sole shape and stiffness characteristics. Initially, the dynamics behavior must be described by four degrees of freedom: body position, ankle position, ankle rotation and contact point position. Through contact force it is possible to determine contact point deformation and corresponding ankle moment. These values vary with ankle rotation and, on other words, with time. Then, known the contact forces, the displacement of sole and ankle rotation are determined and can be eliminated of global equation system. Thus, the model considers only two masses one corresponding to the prosthetic foot ( $m_p$ ), with constant value throughout the analysis, and the other representing the rest of the body of the individual, called effective mass ( $M$ ). Two degrees of freedom of vertical displacement are related to these masses, as the inertial effect of rotation of the prosthetic foot is small relative to the masses of the body and the prosthetic foot, it is disregarded. Both the individual's body and the prosthetic foot structure are considered to be rigid in the constructed model. The flexible elements used consider the mechanical ankle stiffness (vertical displacement and rotation) and the rigidity of the lower part of the prosthetic foot, formed by polymer materials of non-linear behavior. It is mounted a mass damping spring system with movement only in the vertical direction, as shown in Fig. 2b. Flexible elements are used,  $K_T$  the ankle compression spring stiffness,  $C_T$  the ankle damping coefficient,  $K_H$  the Hertz stiffness,  $C_H$  the Hertz contact damping coefficient and the generalized displacements  $q_1$ ,  $q_2$  and  $q_3$ . In addition,  $q_1$  represents the displacement of the base of the prosthetic foot,  $q_2$  represents the displacement of the ankle and  $q_3$  represents the generalized displacement of the body. The structure is considered rigid and its rotational inertia is very low, therefore rotation is considered as a rigid body movement, so the equation of motion has a correction factor  $\Delta q_2$ , which corresponds to that relative rotation of the prosthetic foot in relation to the ankle, since the equation of motion will now contemplate only two degrees of freedom.

$$q_2 = q_1 + \Delta q_2 \quad (1)$$

$$K_T(q_3 - q_2) - K_H q_1 + m_p g = m_p \ddot{q}_1 \quad (2)$$

$$M g - K_T(q_3 - q_2) = M \ddot{q}_3 \quad (3)$$

Substituting Eq. 1 into Eq.2 and Eq.3 respectively.

$$\begin{bmatrix} m_p & 0 \\ 0 & M \end{bmatrix} \begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_3 \end{bmatrix} + \begin{bmatrix} (K_H + K_T) & -K_T \\ -K_T & K_T \end{bmatrix} \begin{bmatrix} q_1 \\ q_3 \end{bmatrix} = \begin{bmatrix} m_p g - K_T \Delta q_2 \\ M g + K_T \Delta q_2 \end{bmatrix} \quad (4)$$

Again the equation including damping considers the damping coefficient matrix as a function of the inertia matrix ( $m$ ), the stiffness matrix ( $k$ ) and a damping factor  $\zeta$ .

$$c = 2\zeta\sqrt{mk} \quad (5)$$

The moment at the ankle ( $m_t$ ) is resisted by a bending spring of the ankle. Where  $K_\theta$  is the stiffness coefficient and  $\theta_2$  is the relative angle of the prosthetic foot with the ankle,

$$\theta_2 = \frac{K_H q_1 e}{K_\theta} \quad (6)$$

By geometry we seek to find the value of the arm of the lever ( $e$ ), besides finding a distance ( $d$ ) in relation to the origin of the axes, as shown in Fig. 2a.

$$e = (x_c - y_c \tan(\theta)) \cos(\theta) \quad (7)$$

Taking an increment of this correction motion, from the schematic in Fig. 2c, we find the values of  $y$  and  $y'$ , which are geometric variables of the prosthetic foot, ie the radii of the circles that make up the convex curve corresponding to the sole of the foot, referenced in Fig. 2c.

$$\Delta\theta = \Delta\theta_1 + \Delta\theta_2 \quad (8)$$

$$\gamma = \text{atan}\left(\frac{x_c}{y_c}\right) \quad (9)$$

$$y = d \cos(\gamma) \quad (10)$$

$$y' = d \cos(\gamma + \Delta\theta) \quad (11)$$

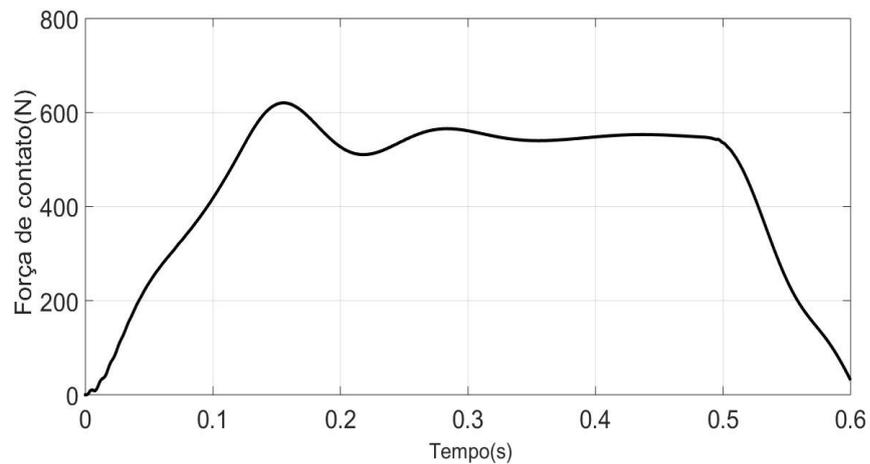
Finally, we find  $\Delta q_2$ .

$$\Delta q_2 = y - y' \quad (12)$$

#### 4. RESULTS

All parameters contained in the simulation are (Silva, 2018), the curve shown in Fig. 3a corresponds to the contact force after the simulation, and its maximum value is 620 N, by the calculation of the impact factor (peak value). The value of  $\Phi$  is equal to 113%, that is, the impact generates a 13% increase in body weight. According to (Perry, 2005) the normal pattern of the vertical forces generated in the support phase presents the peak value approximately 110%. It is possible to find the angle  $\theta_2$ , in the graph shown in Fig. 3b, in relation to the simulation time, it is possible to evaluate the variation of this angle, which corresponds to the rotation of the prosthetic foot in relation to the leg. Note, there is a variation around  $-1.5^\circ$  to  $4^\circ$ , the negative value corresponds to plantar flexion, the positive value corresponds to dorsiflexion and when it has a null value of  $\theta_2$  means that the foot is in the neutral position. In (Perry, 2005) the ankle amplitude obtained experimentally for an integral foot, analyzing up to 60% of the cycle, the angular variation of the ankle has the limit of  $7^\circ$  for plantar flexion (plantar flexion) and  $10^\circ$  for dorsiflexion.

a)



b)

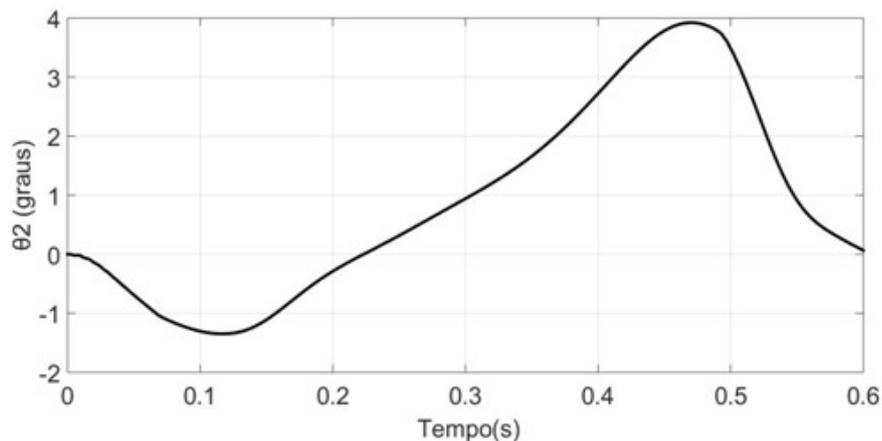
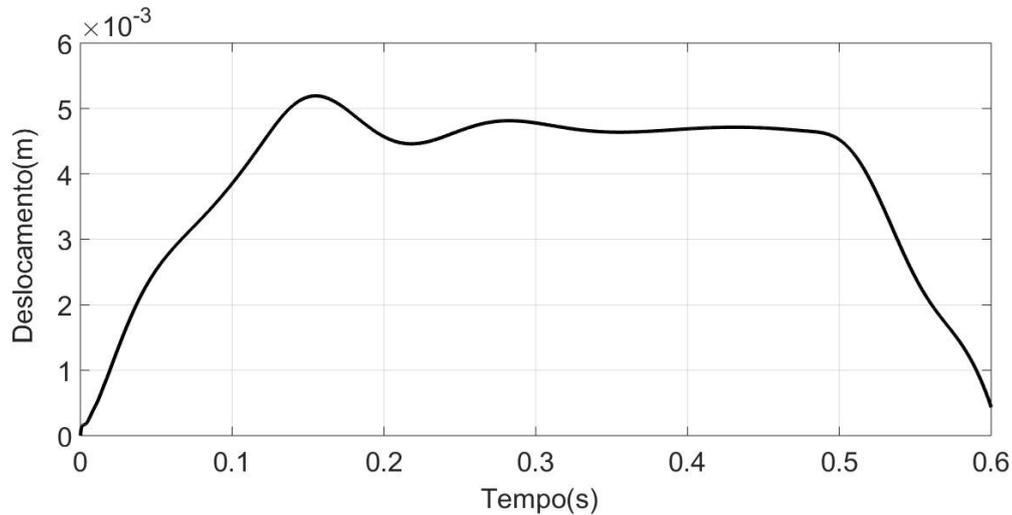


Figure 3 - a) Contact force. b) Angle  $\theta_2$ .

It is possible to make a dynamic simulation from the correction factor  $\Delta q_2$ , the response of the vertical displacement of the body and of the prosthetic foot, shown in Fig. 4a and 4b, respectively. In both, as the contact force increases, the vertical displacement of the body also increases. Achieve a peak value of approximately 5 mm in the first simulation and approximately 3.5 mm in the second simulation.

a)



b)

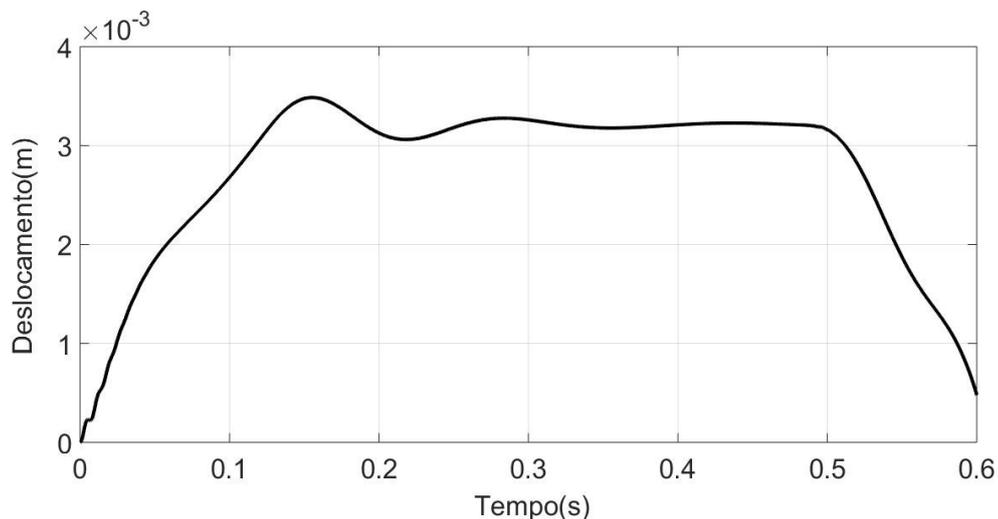


Figure 4 - a) Displacement of the body. b) Displacement of the prosthetic foot.

## 5. STUDY CASE

In this section two case studies are performed for contact force analysis. In the first study we consider the moment when the prosthetic foot is impacted. Next, a run is performed with the use of the prosthetic foot.

In the first case considers all the parameters described in first simulation, less the initial velocity value of the prosthetic foot, it is considered an increase of five times the velocity, in a physical situation would be the same to impact the foot with greater speed in the ground. Contact force is used for analysis, Figure 5 has a maximum value of 624 N, generating a impact factor of approximately 114%, that is, for an increase of five times the model responded with a 1% increase in impact.

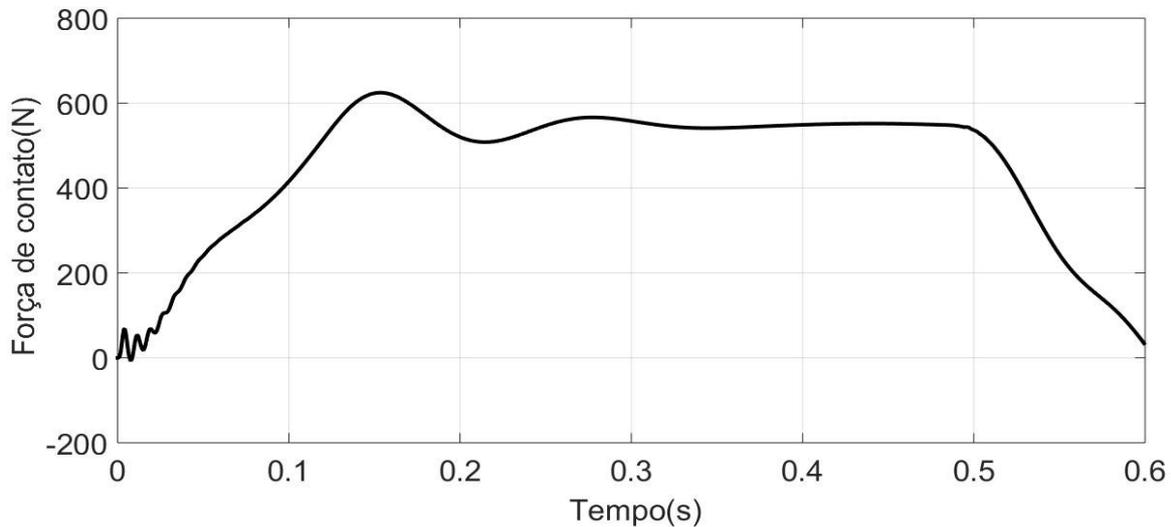


Figure 5 – First case, contact force.

The second case considers the running, whose main characteristic is the fact that the effective mass is constant all the time, because a single foot is in contact with the ground, there is practically no double phase of support. Considering all the parameters of the first simulation, except effective mass, which is now constant equal to 54 kg. Again, the contact force is used for analysis by Figure 6 has a maximum value of 1206 N, generating an impact factor of 220%. According to (Perry, 2005) for the running impact factor is 225%.

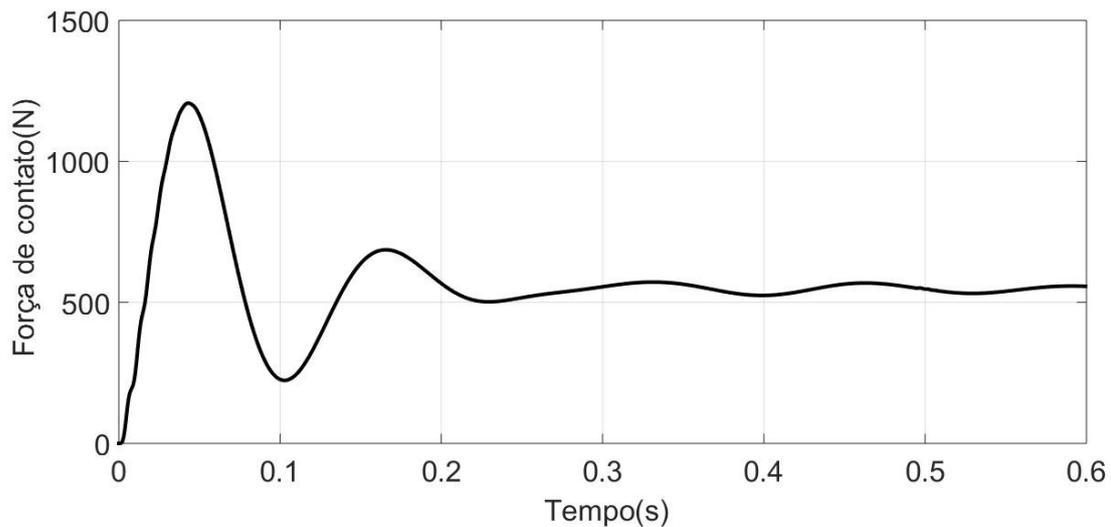


Figure 6 – Second case, contact force.

## 6. CONCLUSION

A methodology is developed that includes, besides the vertical movement of the bodies, the rotation of the prosthetic foot and the contact force that varies its point of application along the movement. Thus, both the impact factor and the bearing dynamics are obtained. Also, is compared the data obtained in the simulation with those presented in the literature obtained experimentally, which are proven to be realistic. In particular, the proposed geometric configuration for the sole allows a coherent kinematic response for the hypothesis performed for the contact points, validating the bearing determination procedure.

## 7. ACKNOWLEDGMENTS

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