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ON THE WAVE PROPAGATION IN DUCTS WITH TWO-PHASE FLOW

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Abstract.*There are few studies about elastic wave propagation in ducts with two-phase flow, which are extremely relevant in applications such as oil and gas production and nuclear engineering. Typically, two-phase flows and structure interaction have been described under a modal based approach, i.e. taking into account natural frequencies and mode shapes. This approach requires previous information about boundary conditions and pipe length span, for instance, which makes it harder to generalize to a broader set of operational conditions. In this work, a simple fluid-structure coupling approach is described and it is used to find the velocity of propagating waves along the duct due to the fluid influence. The wave dispersion relation is calculated for some typical cases considering both empty pipe and pipe transporting two-phase flows. A dispersed bubbles flow pattern is considered and the homogeneous model is used to describe the two-phase flow properties. It is shown that this approach can be potentially used to describe the fluid-structure interaction under a broader set of operational conditions.*

Keywords: *two-phase flow, wave number, fluid-structure interaction, dispersion curve*

1. INTRODUCTION

Two-phase flow induced vibration is an important phenomenon in industry. Hydrodynamic forces acts on the structure and it can obstruct smooth operation of engineering devices and could potentially cause serious consequences such as system failures. Different mechanisms induce vibration on the structures where the main ones are turbulence induced vibration and the two-phase flow excitation mechanism (Miwa *et al.*, 2014). The former is associated with the presence of eddies of different sizes that decay near the pipe wall region dissipating energy and generating random pressure fluctuations. The latter is associated with the instant reconfiguration of the phases inducing pertubations on the flow. This phenomenon has been studied extensively over the past years. Experimental studies verified the influence of two-phase flow parameters such as void fraction and flow-pattern on the pipe vibration (Ortiz-Vidal *et al.*, 2017). Furthermore, Hibiki and Ishii (1998) studied the effect of flow-induced vibration on local flow parameters in two-phase flow and concluded that both the distribution parameter and the void fraction are influenced by the vibrating structure when compared to a pipe with no vibration.

Most of the work avaiable in the literature focus on vibration induced by cross-flows (Pettigrew and Knowles, 1997),(Pettigrew *et al.*, 2005),(Feenstra *et al.*, 2003),(Khushnood *et al.*, 2004), whilst Miwa *et al.* (2014) and Hibiki and Ishii (1998) are the only among those who researched on internal flow. Even so, the studies were perfomed focusing on stability of the system and the excitation mechanisms involved. Few authors such as Ortiz-Vidal *et al.* (2017), Evans *et al.* (2004) and Albrecht (1982) investigated the relationship between dynamic response of the structure and void fraction.

Various engineering application require a prediction of two-phase flow pattern to adjust operation conditions to obtain the maximum efficiency in processes such as oil extraction. Ortiz-Vidal *et al.* (2017) found a relationship between the peak-frequency of the vibration response of the structure and the void fraction of the flow in order to predict the flow pattern. Although, this approach can be limited since it depends on boundary conditions and pipe geometry. Evans *et al.* (2004) investigated a technique to measure flow rate using a correlation between standard deviation of the frequency-averaged acceleration of the pipe and mixture velocity. It was concluded that the changes in natural frequency are very small in relation to the changes in the flow velocity, therefore the method was poor for slow flow rates but had a good estimate for higher flow velocities.

Alternative methods to predict the two-phase flow pattern have been proposed by several authors. The main early methods used in literature involved the use of an electrical contact probe to detect void fraction (Bergles *et al.*, 1968) aided by a statistical analysis of the void signal through probability density function (PDF) (Jones and Zuber, 1975) and power spectral density (PSD) (Albrecht, 1982). Although, the same flow pattern could deliver different results. Also, the signal processing needed to determine the results is long and difficult making it impossible to apply those methods in engineering applications.

This paper propose an investigation on the wave propagation on elastic pipes conveying two-phase flow. A homogeneous model is used to describe a dispersed bubbles pattern behaviour and the classical Euler-Bernoulli beam model to describe the structure behaviour, since its an axissymetric flow pattern. A relationship is found between the mixture velocity, void fraction and the wavenumber and the effects of the two-phase flow on the wave propagation are discussed.

2. GOVERNING EQUATIONS

The classic approach to deduce the governing equation for two-phase flow induced vibration consist in developing separate models for fluid and structural dynamics and they are coupled with hydrodynamic and structural force terms. The models available for the structure are near-linear and can be modelled as linear-oscillator. For the two-phase flow model it can be more complicated due to complex motions and interactions at the phase interfaces nonlinearities have to be taken into account (Miwa *et al.*, 2014).

Different two-phase flow models can be used to simplify the fluid balance equations; For instance, the two-fluid model, the homogeneous model and the drift-flux model. Each one of this models has a better fit depending on the flow pattern observed, i.e. the dispersed bubble pattern fits better with the homogeneous model while the slug flow pattern fits better with the drift-flux model where slippage exists between phases (Carrizales *et al.*, 2015). Thus, since in this paper the dispersed bubbles pattern is considered, the homogeneous model is used to model the flow. For simplicity, the flow is assumed to be in steady state, fully developed, horizontal and one-dimensional. Also, it is assumed that there is no phase change through out the pipe.

The equation of motion can be derived from the analysis of the forces acting on a small element of the pipe and the two-phase flow, as shown in Fig. (1).

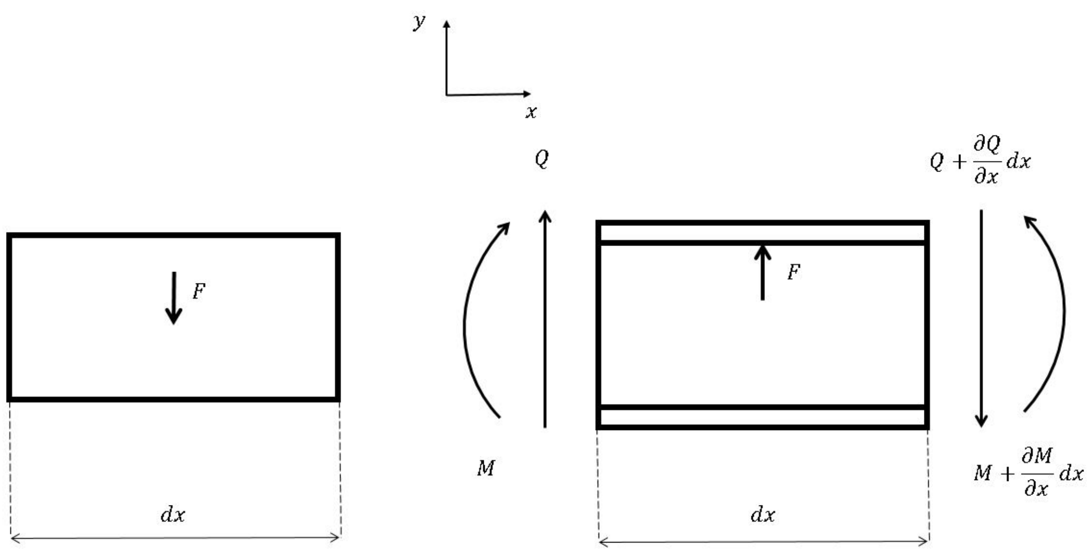


Figure 1: Free body diagram on both pipe (right) and fluid (left) elements.

Here, E is the elasticity modulus, I is the second moment of area, ρA is the mass per unit length of the pipe and ρ is the pipe material density, m is the mass per unit length of the two-phase fluid and J is the mixture velocity.

It is assumed that the displacement $w(x,t)$ in the y direction occurs only in the plane that contains the x axis. The force balance for the pipe element is given by

$$\frac{\partial Q}{\partial x}dx - Fdx = -\rho A dx \frac{\partial^2 w}{\partial t^2}, \tag{1}$$

and for the fluid

$$Fdx = -\left(\frac{\partial}{\partial t} + J\frac{\partial}{\partial x}\right)\left[m\left(\frac{\partial w}{\partial t} + J\frac{\partial w}{\partial x}\right)\right]dx. \tag{2}$$

The momentum balance for the pipe element in relation to the left side of the pipe element at the pipe axis gives

$$-\frac{\partial Q}{\partial x}dx^2 - Qdx + \frac{\partial M}{\partial x}dx = 0. \tag{3}$$

Ignoring high order terms the following relation arises

$$Qdx = \frac{\partial M}{\partial x}dx. \tag{4}$$

Since the pipe deformation is considered to be small enough, it behaves linearly so the flexural moment is related to the pipe deflection through

$$M = EI\frac{\partial^2 w}{\partial x^2}. \tag{5}$$

It follows from substituting Eqs. (2) and (5) in Eq. (1) that

$$EI\frac{\partial^4 w}{\partial x^4}dx = -\rho A\frac{\partial^2 w}{\partial t^2}dx - \left(\frac{\partial}{\partial t} + J\frac{\partial}{\partial x}\right)\left[m\left(\frac{\partial w}{\partial t} + J\frac{\partial w}{\partial x}\right)\right]dx. \tag{6}$$

Thus, expanding the last term of the right hand side of Eq. (6) gives the equation of motion as

$$EI\frac{\partial^4 w}{\partial x^4} + (\rho A + m)\frac{\partial^2 w}{\partial t^2} + 2mJ\frac{\partial^2 w}{\partial t \partial x} + mJ^2\frac{\partial^2 w}{\partial x^2} + \left(\frac{\partial m}{\partial t} + J\frac{\partial m}{\partial x}\right)\left(\frac{\partial w}{\partial t} + J\frac{\partial w}{\partial x}\right) = 0, \tag{7}$$

where the mass per unit length of the two-phase flow may be a function of time and space depending on the flow pattern. Since the dispersed bubble flow pattern is being considered in this paper, it is assumed to be constant through out the pipe. Both mass per unit length and mixture velocity are defined in the next section with the homogeneous model description for the dispersed bubble flow pattern. The first term of Eq. (7) expresses the restoring force caused by the pipe's elasticity, the second term is the sum of the force of inertia acting on the pipe and the fluid, the third term is the Coriolis' force due to the flow velocity J and the angular velocity $\frac{\partial}{\partial t}\left(\frac{\partial y}{\partial x}\right)$ due to the pipe displacement y, the fourth term is the centrifugal force caused by the velocity J and the curvature rate $\frac{\partial^2 y}{\partial x^2}$ and the last term gives the momentum change of two-phase flow due to the change of mass distribution in time and space. This equation was deduced by Hara (1975), and it has been used for a modal approach of the problem.

3. DISPERSED BUBBLE FLOW

A dispersed bubble flow develops at high liquid rates and low gas rates where the gas phase tends to mix with the liquid. Therefore, values for the superficial gas and liquid velocities were chosen to simulate the dispersed bubble pattern according to Taitel and Dukler (1976) for horizontal gas-liquid flows. Furthermore, values for the gas and liquid flow properties were used based on oil and natural gas properties. A flow pattern map based on Taitel and Dukler (1976) semi-theoretical formulation is shown in Fig. (2).

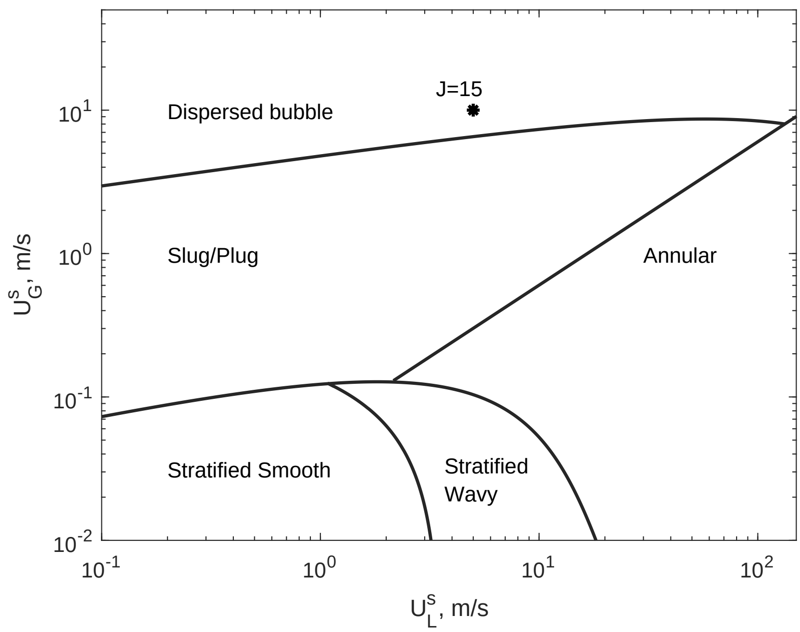


Figure 2: Dispersed bubbles pattern for the velocity values chosen.

For a dispersed two-phase flow pattern, it is suitable to use a homogeneous model, which consider the same mean velocity for both phases. Moreover, the homogeneous model consider both phases flowing together as a mixture with equivalent properties. Therefore, there is no slip between phases and the mixture properties are defined by

$$\rho_m = \rho_g \alpha + \rho_l (1-\alpha), \tag{8}$$

$$J = \frac{j_g}{\alpha} = j_g + j_l. \tag{9}$$

where α is the void fraction, j_g is the gas phase superficial velocity and j_l is the liquid superfical velocity. Thus, considering A_i the internal area of the pipe, the mass per unit length for the two-phase fluid is given by

$$m = \rho_m A_i \tag{10}$$

4. DISPERSION CURVE

A dispersion curve gives the relation between the wavelength $\frac{2\pi}{k}$ of a mechanical wave propagating through a structure and the frequency ω of a given excitation. The expression that describes this relation is obtained when a harmonic solution is assumed for a equation of motion for any continuum media. Each term in the equation of motion is related to a force acting on the structure and often is derived from a force balance analysis in a small element of the structure. Moreover, the phase velocity of the wave may be defined as

$$c = \frac{\omega}{k}, \tag{11}$$

Eq. (11) shows the clear dependence on the phase velocity and the wavenumber, which is dependent on the flow properties. The dispersion curve can be easily estimated experimentally through the relation between the vibration amplitude in points along the structure.

The disperson relation is obtained from Eq. (7) when a harmonic solution is assumed in time and space in the form $y(x,t) = e^{i(\omega t - kx)}$. Moreover, since the mass per unit length for the fluid is given by Eq. (10), the fluid mass is constant and Eq. (7) becomes simply

$$EI\frac{\partial^4 w}{\partial x^4} + (\rho A + m)\frac{\partial^2 w}{\partial t^2} + 2mJ\frac{\partial^2 w}{\partial t \partial x} + mJ^2\frac{\partial^2 w}{\partial x^2} = 0. \tag{12}$$

Substituting the harmonic solution in Eq. (12), the dispersion relation is expressed as

$$EIk^4 - mJ^2k^2 + 2mJ\omega k - (m + \rho A)\omega^2 = 0. \tag{13}$$

Equation (13) is a fourth order polynomial equation with four roots. Each root is associated with a wave mode. For a lossless pipe, there are two complex roots, with a large imaginary part which is associated with near field waves that decay exponencially with space and two purely real roots that are associated with propagating wave through the structure, one on the positive x direction and the other on the opposite direction. Since the two-phase flow is in one direction, the real wavenumbers have a slight difference between them, where the wavenumber associated with the wave velocity going in the direction of the flow is higher than the wavenumber associated with the wave velocity going in the opposite direction.

5. NUMERICAL RESULTS

A numerical investigation was carried out to detect the mixture velocity and void fraction given a dispersion curve. The values used were $E = 210\ GPa$, $\rho = 7560\ kg/m^3$,$D = 50.8\ mm$ and $h = 3.35\ mm$ for the pipe and $\rho_l = 870\ kg/m^3$, $\rho_g = 2.28\ kg/m^3$, μ_l = $0.01 Pa.s$, $\mu_g = 2.10^-6\ Pa.s$ for the fluids, where h is the pipe wall thickness and μ_l and μ_g are the oil and gas dynamic viscosities.

The dispersion curve of the wave propagating in the positive direction for different values of J is shown in Fig. (3). The value for the superficial gas velocity was fixed in $j_g = 5\ m/s$, thus for the curve where mixture velocity value is $5\ m/s$ only the gas phase exist. It can be noticed that the difference between the dispersion curves are small for a fair change in the mixture velocity, which could be a problem when measurement noise is present.

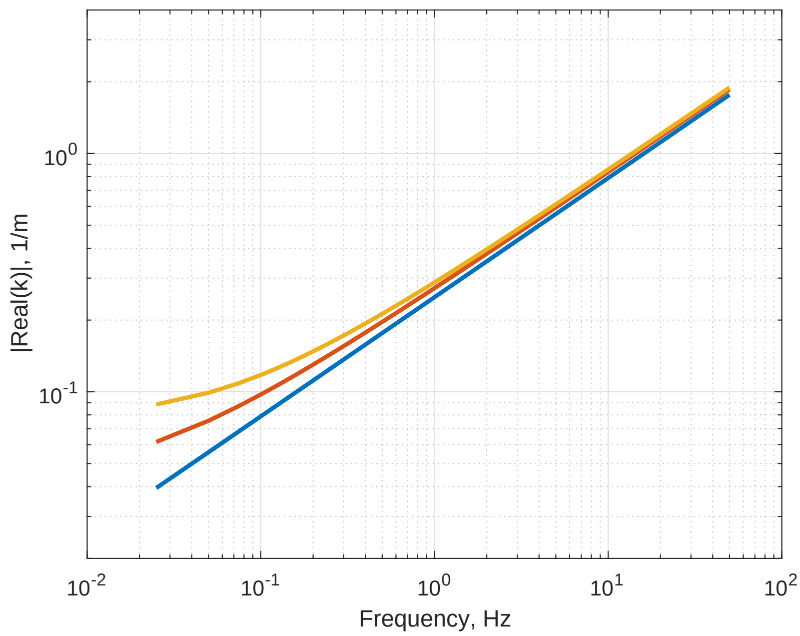


Figure 3: Dispersion curve for the dispersed bubble pattern for different values of mixture flow velocity (5 m/s ,10 m/s ,15 m/s).

The dispersion curve can be determined experimentally based on vibration frequency response measurements (e.g Hinke *et al.* (2004),Kalkowski *et al.* (2017)). From Eq. (13), the mixture velocity can be determined from the experimentally estimated wavenumber and some simple parameters from the pipe and fluid as

$$J = \frac{\omega}{k} - \frac{\sqrt{\frac{EIk^4 - M\omega^2}{m}}}{k}. \tag{14}$$

which gives a set of linearly independent equations for each known ω and k in the dispersion curve. Choosing any two points in the curve arises a system of two equations and two variables, α and J, which can be used to determine those two variables given a dispersed bubble flow.

The effect of the two-phase flow velocity on the dispersion curve is small. For a high variation of the flow velocity there is a small variation in the wavenumber, and as the frequency increases the wavenumber variation becomes larger. Another notable effect of the two-phase flow velocity on the dispersion curve is the wave assymetry for the positive and negative going waves. Figure (4) shows the diference between them, where k_1 and k_2 are the propagating waves going to the positive and negative direction, respectively, and k_3 and k_4 are the near-field waves. This effect may be explained since the flow excites the structure in a preferential direction, i.e., the flow direction.

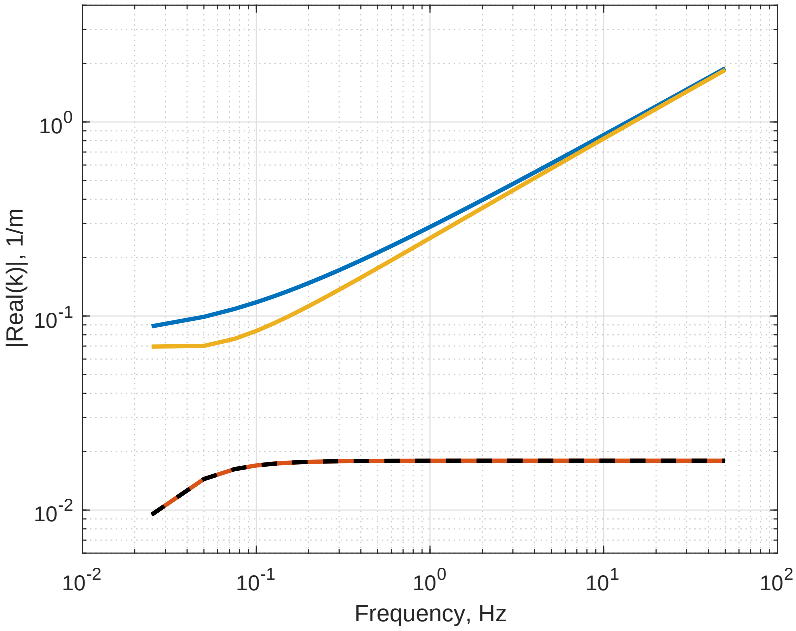
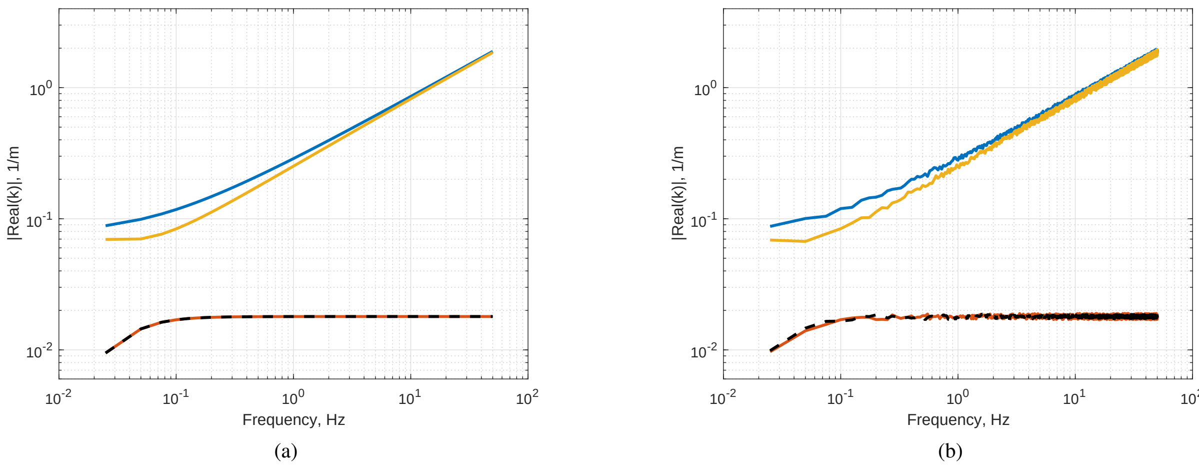


Figure 4: Comparison between the waves arised by the two-phase flow (k_1 ▬▬, k_2 ▬▬, k_3 ▬▬, k_4 ■■■■■).

For the other two wave modes, it is possible to verify that they have very small real part at all the frequency range and increasing imaginary part. This means that they decay really fast in space therefore do not propagate and are called near-field waves. This type of wave is only important near the pipe conections and discontinuities.

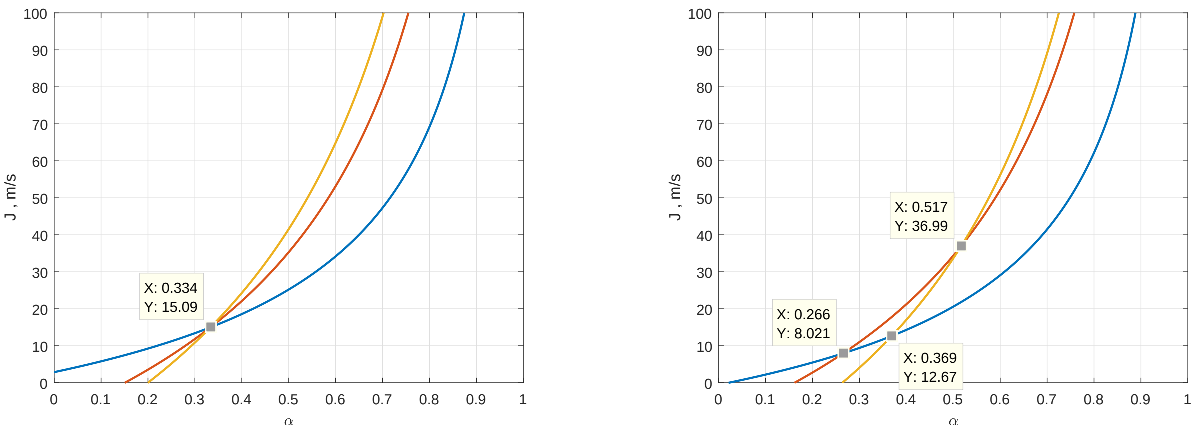
Moreover, with Eq. (14) it is possible to determine both flow velocity and void fraction for a dispersed bubble flow. Although, the variation in the wavenumber is small when compared to the variaton of the flow velocity, which can be a challenge to use this equation experimentally since the difference between the wavenumber may be in the same order of the signal noise and thus giving rise to a broad range of possible flow velocities and void fraction.

In order to investigate the effect of the noise on the results, an arbitrary value of void fraction and mixture velocity were defined as $\alpha = 0.33$ and $J = 15\ m/s$. Then, the relation defined in Eq. (14) was used to determine those values through the dispersion curve. A random synthetic signal noise in the order of 3% of the original curve was added to the wavenumber to verify the effect of the uncertainity of the experiment on the determination of the void fraction and mixture velocity. Figure (5) compares the dispersion curves and Fig. (6) compares the results with and without noise. Three values for the frequency and the corresponding wavenumber were arbitrary chosen to plot the relation given by Eq. (14): $f_1 = 5Hz$, $f_2 = 25Hz$ and $f_3 = 45Hz$.



(a) (b)

Figure 5: Dispersion curves with (b) and without (a) noise, obtained from the given values for J and α.



(a) (b)

Figure 6: Mixture velocity and void fraction with (b) and without(a) noise, determined through Eq. (14). Each curve is obtained using different points along the dispersion curve. (f_1 ▬, f_2 ▬, f_3 ▬)

The method is sensitive to signal noise though, since a large variation on the flow velocity induces a small variation on the dispersion curve. Figure (6) shows this behaviour, where random noise was added to the wavenumber and different values for the mixture velocity and void fraction were found. The mean value were taken using the values obtained in Fig.6(b) and the result was $\alpha = 0.38$ for the void fraction and $J = 18.9\ m/s$ for the mixture velocity. Since different values through out the dispersion curve may be used, it might be useful to determine the mixture velocity and void fraction using a large set of values and applying a statistical analysis to get a more precise result.

6. CONCLUSION

Flow-induced vibration due to two-phase flow is a recurrent phenomenon in industry. Often, it is needed to determine two-phase flow characteristics such as flow mixture velocity and void fraction, since with both quantities it is possible to estimate the flow pattern with a certain precision. The modal analysis is the commonly used approach to do such analysis (Ortiz-Vidal *et al.*, 2017), (Hara, 1975) which not only needs well-defined boundary conditions but also information about the excitation mechanism such as pressure fluctuation. In this matter, the wave approach gives a more general view on the problem, evaluating the behaviour of the wave propagation through the structure related only to the flow velocity and void fraction. Through this relation, it is possible to determine those quantities solving a linear system of equations. Even though, for a experimental application noise will be present and the noise effect on the results seems to cause a certain error in the estimated quantities. With an appropriate use of signal processing to clean the noise and a statistical analysis to minimize error, a good estimate could be achieved for the dispersed bubble flow. Next steps include an experimental investigation with a controlled dispersed bubble flow pattern to validate the method for this flow pattern. In sequence, functions to describe the variation in time and space for the mass per unit length could be derived to input in Eq. (7) for other flow patterns in order to determine a new relation between wave propagation in the pipe and flow properties.

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