25th ABCM International Congress of Mechanical Engineering
October 20-25, 2019, Uberlândia, MG, Brazil

COB-2019-0774
MODELING AND OPTIMIZING A VEHICLE SUSPENSION CONSIDERING USER SAFETY AND COMFORT IN AN IRREGULAR ROAD

Julio Cesar de Castro Basilio
José Geraldo Telles Ribeiro
Francisco José da Cunha Pires Soeiro
State University of Rio de Janeiro, Department of Mechanical Engineering, Rio de Janeiro, Brazil
juliobasilio@ymail.com
jose.gt.ribeiro@gmail.com
soeiro@uerj.br

Abstract. *Passive vehicle suspension should reconcile parameters that result in comfort and safety for vehicle users, however, as presented in this study, these characteristics may be conflicting. Considering the initial phase of a suspension design, this study aims to obtain reliable analysis and find optimal values of suspension parameters: spring stiffness and damper damping coefficient. According to ISO 2631, the comfort criterion is related to the human perception of vibration resulting from oscillations in the vehicle, with sprung mass acceleration being the measure of comfort. Scientific literature indicates that road irregularities may hinder the driver's ability to control the vehicle, because larger swings decrease the contact force between the tire and the ground, and consequently, reduce vehicle handling and ride safety. To better understand the behavior of a suspension system, simulations were performed in the Matlab® software through the mathematical development of a quarter car model (QCM), with two-degree-of-freedom (DOF). Using as input to the system, a mathematical model representing classes of road profiles was implemented, aiming to approximate the analysis of real situations. Suspension stiffness and damping were analyzed using two system parameters known as: undamped natural frequency and damping ratio. Thus, the influence of system parameters on comfort and safety criteria are facilitated and allow greater generalization. Defining masses (sprung and unsprung) and tire stiffness, Matlab® simulations confirmed the conflict between user comfort and safety criteria, highlighting the need to use optimization methods to find the ideal values of suspension parameters. Whereas the goals of suspension optimization are to minimize vertical acceleration of the sprung mass to improve comfort, while minimizing wheel dynamic force for improved safety, this is a multiobjective optimization problem. Thus, the method to be used was the genetic algorithms (GA), more specifically the NSGA-II algorithm, as it is widely used in other studies and is available in the Matlab® toolbox.*

Keywords: *Suspension, road profile, quarter car model (QCM), safety and comfort criteria, vehicular dynamics*

1. INTRODUCTION

1.1 Suspensions and Vertical Dynamics

A vehicle's suspension system is a set of mechanical components that connect the wheels to the vehicle's frame. When in motion, it is the suspension system that keeps occupants comfortable and isolated from road noise, bumps and vibrations. He is also responsible for improving driveability by allowing the driver to maintain control of the vehicle on irregular terrain or at sudden stops (Yucheng, 2008).

Vertical dynamics comprises the analysis of the vertical translation movement on z-axis (bounce), as well as the rotations around the x-axis (roll) and the y-axis (pitch), due to the irregularities of the terrain. In this area of vehicle dynamics the characteristics of vehicle components are analyzed, such as structural components of the chassis and suspension, always aiming at safety and comfort. Among the portrayed movements, Its highlight the quantities of interest of the bounce related vertical dynamics, the purely vertical movement, such as the variation of the contact force between the tire and the ground and the acceleration of the sprung and unsprung masses, as well as the natural frequencies of the vehicle. To know and measure these parameters, it is necessary to understand the vibration, phenomenon of vertical dynamics resulting from vertical oscillations, and the dynamic models that are used as tools for the mathematical development of systems. According to (Diniz, 2014), the vibration caused by exciting sources is the dynamic response of the vehicle, being a way of characterizing the dynamic behavior of the vehicle.

1.2 Vibrations, natural frequency and damping ratio

According to (Jazar, 2008), in general, mechanical vibrations are the result of the continuous transformation of kinetic energy to potential energy and potential energy to kinetic energy, alternately. Thus, the mechanical element that stores the kinetic energy is called mass, and the element that stores potential energy is called the spring. If the total value of mechanical energy, sum of kinetic and potential energies, decreases during vibration, there is a mechanical element that dissipates energy, which is called a damper. According to (Rao, 2009), if a system, after an initial excitation, is left to vibrate on its own, the frequency with which it will oscillate without external forces will be its natural frequency. A vibration system with n degrees of freedom (DOF) will generally have distinct natural vibration frequencies. From analysis and solution of an undamped free vibration problem with only one DOF, the natural frequency of the system can be characterized by:

$$f_n = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \tag{1}$$

The variable f_n measured in cycles per second (Hertz) is referred to as the undamped natural frequency since damping is ignored. According to (Popp and Schiehlen, 2010), if the natural frequency of chassis f_{n_s} (sprung mass) is too low, while the natural frequency of wheels f_{n_p} (unsprung mass) is too high, frequency decoupling may be used, where each frequency will be calculated by its mass and the stiffness of the elastic element supporting it. When the problem of free vibration is no longer just a mass-spring system and has a damping, as in vehicle suspensions, its solution depends on the dimensionless parameter called damping ratio ξ, given by the ratio between the actual damping b and its critical damping $\bar{b}$:

$$\xi = \frac{b}{\bar{b}} = \frac{b}{2\sqrt{mk}} \tag{2}$$

2. METHODOLOGY

2.1 Safety Criteria

According to (Sarami, 2009), handling is one of the characteristics of the vehicle that provides stability and safe driving. This occurs when there is the constant contact of the tire with the road, ie the handling capacity of the vehicle by the driver is related to the contact force between the tire-ground. Loss of vehicle control due to reduced or varying contact force will affect passenger safety, particularly during maneuvers such as cornering, braking or acceleration. The purely vertical movements of the chassis and wheels are mainly affected by the roughness of the road surface, producing bounce movements that reduce the contact force of the tire, and consequently control of the vehicle's steering and handling system. There is no standard for quantifying or formulating the driveability of the vehicle, but from what has been seen, the contact force between the tire and the ground is a way to know the handling ability. Thus, for safe driving, this force should vary as little as possible because high variation reduces the ability of steering control by the driver. (Popp and Schiehlen, 2010) also state that the contact force of the tire can be used to quantify safety. For him, the load on the wheel, given by the variable F_{wheel}, is a criterion for driving safety, and that this force is composed of two other forces: static and dynamic. The static force is original weight of the vehicle, and this analysis, remains constant over time. Dynamic force is generated by vehicle vibration due to track irregularities. The graph presented by Fig. 1 represents the variation of the force on the wheel according to time, and the static and dynamic forces, represented by the variables F_{stat} and F_{dyn}, respectively.

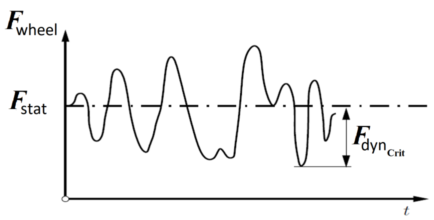

Figure 1. Graph of the force on the wheel as a function of time, and its static and dynamic components.

Where $F_{dyn_{crit}}$ is the critical dynamic force on the wheel. Thus, the lower the critical dynamic wheel force means less variation in the wheel force, and consequently, better handling. Already a greater dynamic force, increases the variation and reduces the contact of the tire with the ground. For roads with a randomly disturbed surface, dynamic wheel load variations can be determined from numerical simulations. In this study, root mean square (RMS) will be used. From this, (Popp and Schiehlen, 2010) proposed a dimensionless factor called safety margin, given by:

$$R = \frac{F_{stat} - F_{dyn_{crit}}}{F_{stat}} . \tag{3}$$

Variable R represents the safety margin, which is the rate of change of wheel force relative to static force. In the same study, (Popp and Schiehlen, 2010) indicate that for safe travel, even in the worst case (rough road, front wheel and high speed), the safety margin should be close to $R = 1 - \frac{F_{dyn_{crit}}}{F_{stat}} \approx 0.75$.

(Chi *et al.*, 2008), (Rill, 2007) and (Sun *et al.*, 2012) use tire deflection force to represent the dynamic force of the wheel, ie the dynamic force will be equal to the tire stiffness k_p multiplied by the difference between wheel displacement and roughness from the road (system input). Can be written as:

$$F_{dyn} = k_p(z_p - h), \tag{4}$$

where z_p is the wheel offset in the vertical direction, and h is the offset applied by the road roughness. Meanwhile, the static force is related to the vehicle's weight on the suspension.

2.2 Comfort Criteria

(Popp and Schiehlen, 2010) state that user comfort during the ride is linked to the human perception of vibrations originating from track irregularities, where experiments related to ergonomics (occupational vibrations) show that this perception of vibrations depends on vertical acceleration. To quantify comfort, some measurement methods are proposed by different literature's. The most widely used measurement method is through the acceleration effective value or root mean square (RMS). According to (Rao, 2011), the use of root mean square (RMS), vibration levels of specification, is due to the number of frequencies transmitted to a human, that is rarely exposed to a single frequency. This allows assessing the average energy in the oscillatory motion, showing the potential for damage by vibration. To understand comfort, it is necessary to know the sensitivity of the human body to vibrations. Its complex biomechanical structure (bones, joints, muscles and other organs) allows each part of the body to be able to dampen or amplify vibrations, depending on the frequency range to which it is exposed. Amplifications happen due to a phenomenon called resonance, when parts of the body vibrate at frequencies close to their respective natural frequencies. In 1998, an engineer named Sven-Olof Emanuelsson presented Fig. 2 to show the various resonances in the human body. For him the human body is considered a complex mechanical system, with multiple degrees of freedom, and can be represented by spring-damping systems (Pereira, 2005) (Lopes, 2012).

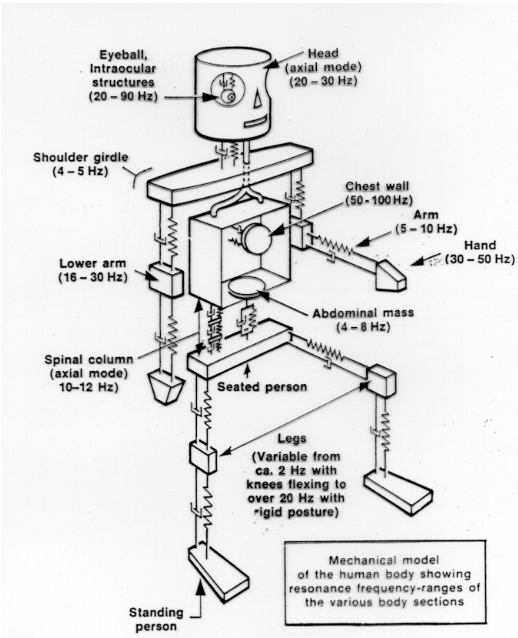

Figure 2. Mechanical model of the human body showing the resonant frequency ranges of the various body parts.

Corroborating the resonant frequency ranges presented, (Popp and Schiehlen, 2010) and (Rao, 2011) state that a vertical excitation between 4 and 8 Hz is perceived as very unpleasant, since in this frequency range the stomach resonance occurs. For Rao, in addition to the resonances of each part of the human body, the following effects can be observed at different frequencies: motion sickness (0.1 - 1 Hz), blurred vision (2 - 20 Hz), speech disorder (1 - 20 Hz), task interference (0.5 - 20 Hz) and fatigue (0.2 - 15 Hz). (Drehmer, 2010), in his study, uses only RMS acceleration to compare and obtain comfort indices. Thus, Tab.1, obtained by the British Standard (BS6841, 1987) and (ISO2631, 1997), uses the root mean square of vertical acceleration as an indicator of the person's level of perception of comfort.

Table 1. Comfort scale according to ISO 2631-1 and BS 6841.

Acceleration (m/s^2)	Comfort Level
< 0.315	Comfortable
0.315 - 0.63	Slightly comfortable
0.5 - 1	A little comfortable
0.8 - 1.6	Uncomfortable
1.25 - 2.5	Very uncomfortable
> 2	Extremely uncomfortable

2.3 Mathematical Model of the Suspension System

According to (Verros *et al.*, 2005), the simplicity of the model coupled with qualitatively correct information contributes to its being widely used in the automotive industry, especially in the initial conception phase. (Sarami, 2009) used the Quarter Car Model (QCM), representing one of the four vehicle suspension units, to define the fundamental parameters of a suspension system. (Sousa and Avila, 2017) also used the simplified model of the $1/4$ vehicle with two degrees of freedom (DOF) to increase the suspension system efficiency by optimizing the suspension parameters. As it is widely used in several studies, mainly in the suspension system analysis related to vehicle comfort and safety, this study uses the quarter-car model, with 2 DOF's, presented in Fig. 3. In this model the car is divided into four parts, ie each part is associated with a wheel and suspension. The fundamental parameters of a suspension system are defined by this model and presented in Tab. 2. It is noteworthy that just like (Sousa and Avila, 2017) and (Chi *et al.*, 2008), this model represents only the elasticity of the tire through its stiffness, neglecting tire damping, as explained by (Freitas Jr, 2006).

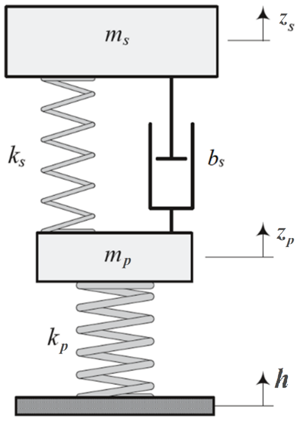

Figure 3. Scheme quarter car model (QCM) with 2 DOFs used for mathematical development of the study.

Table 2. Parameters and variables used in the simplified model.

Variable	Units	Description
m_s	kg	Sprung mass (1/4 vehicle excluding axles and wheels)
m_p	kg	Unsprung mass (wheel and tire mass)
k_s	N/m	Suspension stiffness
b_s	Ns/m	Suspension damping
k_p	N/m	Tire stiffness
z_s	m	Vertical displacement of sprung mass
z_p	m	Vertical displacement of unsprung mass
h	m	Road displacement (input)

(Rao, 2011) proposes to use Newton's second law to obtain the equations of motion of the system. Newton's second law of motion can be stated according to Eq. 5:

$$m_i \ddot{z}_i = \sum_j F_{ij} \ , \tag{5}$$

where $\sum_j F_{ij}$ denotes the sum of all forces acting on mass m_i.

Considering the vertical movement of the masses as positive when upwards, and assuming that the model used is not in static equilibrium, ie the input h is positive and nonzero, the free body diagram is represented by Fig. 4 below.

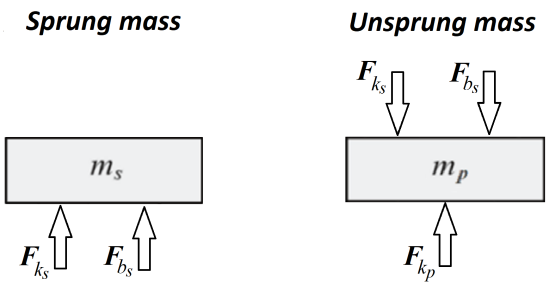

Figure 4. Free body diagram indicating all forces acting on the sprung and unsprung mass.

Therefore, by applying Newton's second law for sprung mass and unsprung mass, the differential equations of the dynamic behavior of the vehicle are obtained.

$$m_s\ddot{z}_s = F_{ks} + F_{bs} \tag{6}$$

$$m_p\ddot{z}_p = F_{kp} - F_{ks} - F_{bs} \tag{7}$$

Thus,

$$m_s\ddot{z}_s = k_s(z_p - z_s) + b_s(\dot{z}_p - \dot{z}_s)\ , \tag{8}$$

$$m_p\ddot{z}_p = k_p(h - z_p) - k_s(z_p - z_s) - b_s(\dot{z}_p - \dot{z}_s)\ . \tag{9}$$

Once the equations of motion are obtained, the main responses (output) of the system to road excitation h over time t are defined. Based on the objectives of this study, it is necessary that the outputs to be observed are the vertical acceleration of the sprung mass $\ddot{z}_s$ to analyze vehicle user comfort and the dynamic force of the wheel F_{dyn} to analyze driving safety and contact ground-tire. As presented throughout this paper, the dynamic wheel strength is obtained by Eq. 4. To simplify the resolution of the equations that characterize the behavior of the suspension system, Eqs. 8 and 9 were put in matrix form, ie in the form of State Space, which equations are given by:

$$\dot{X} = AX + BU \qquad and \qquad Y = CX + DU\ . \tag{10}$$

At where, A is state (or system) matrix, B is the input matrix, C is the output matrix, D is the feedthrough (or feedforward) matrix, X is the state vector, $\dot{X}$ is the derived from the state vector, Y is the output (or response) vector and U is the input vector. Defining the state vector and the output vector as:

$$X = \begin{vmatrix} z_p \\ \dot{z}_p \\ z_s \\ \dot{z}_s \end{vmatrix} \qquad and \qquad Y = \begin{vmatrix} \ddot{z}_s \\ F_{dyn} \end{vmatrix}\ . \tag{11}$$

The matrix form can be developed by reorganizing Eqs. 8 and 9, isolating the state variables and obtaining the matrices of the State Space form. Thus, the mathematical model of the system will be given by:

$$\ddot{X} = \begin{vmatrix} \dot{z}_p \\ \ddot{z}_p \\ \dot{z}_s \\ \ddot{z}_s \end{vmatrix} = \begin{vmatrix} 0 & 1 & 0 & 0 \\ -\frac{k_p+k_s}{m_p} & -\frac{b_s}{m_p} & \frac{k_s}{m_p} & \frac{b_s}{m_p} \\ 0 & 0 & 0 & 1 \\ \frac{k_s}{m_s} & \frac{b_s}{m_s} & -\frac{k_s}{m_s} & -\frac{b_s}{m_s} \end{vmatrix} \begin{vmatrix} z_p \\ \dot{z}_p \\ z_s \\ \dot{z}_s \end{vmatrix} + \begin{vmatrix} 0 \\ \frac{k_p}{m_p} \\ 0 \\ 0 \end{vmatrix} h(t) \tag{12}$$

$$Y = \begin{vmatrix} \ddot{z}_s \\ F_{dyn} \end{vmatrix} = \begin{vmatrix} \frac{k_s}{m_s} & \frac{b_s}{m_s} & -\frac{k_s}{m_s} & -\frac{b_s}{m_s} \\ -k_p & 0 & 0 & 0 \end{vmatrix} \begin{vmatrix} z_p \\ \dot{z}_p \\ z_s \\ \dot{z}_s \end{vmatrix} + \begin{vmatrix} 0 \\ k_p \end{vmatrix} h(t) \tag{13}$$

It is noteworthy that as (Popp and Schiehlen, 2010) and (Rajamani, 2006), simulations and optimizations of this study using the decoupling frequencies to analyze the suspension parameters through its natural frequency and damping ratio. Thus, the undamped natural frequency and damping ratio of the sprung mass are represented by:

$$f_{n_s} = \frac{1}{2\pi}\sqrt{\frac{k_s}{m_s}} \qquad and \qquad \xi_s = \frac{b_s}{2\sqrt{m_s k_s}}\ . \tag{14}$$

2.4 Road Profile Mathematical Model (Input)

According (Rill, 2007), the driving performance of the vehicle, ie a trip with comfort and safety for the user, are mainly influenced by the roughness and friction properties of the highway. Thus, it is necessary to use a realistic road model for closer analysis of the real dynamic behavior of the suspension. He claims that for a road model to be realistic it must at least provide a road profile, described as a vertical elevation along the longitudinal axis of the track.

(Cunha *et al.*, 2017) used a stochastic uncertainty road model to address the nonlinear dynamics of an orchard tower sprayer, because due to ground irregularities oscillations are caused, negatively affecting spraying. Using the Monte Carlo method to calculate the propagation of uncertainties through the stochastic model, simulations revealed large lateral vibrations in vehicle operation. This brings the model closer to the real problem, enabling analysis for possible solutions.

(Dharankar *et al.*, 2016) used the power spectral density (PSD) in the numerical generation of the road profile to simulate the vehicle suspension, comparing the white noise filtering and the harmonic overlapping methods, where they are used to generate the profiles. elevation values corresponding to ISO 8608 road types by numerical simulation. This study suggests that it is appropriate to generate the road profile first in the space domain and then simulate the vehicle at a certain speed over that profile to obtain the time domain solution. This results in advantages such as reduced simulation time and the use of the same profile to compare suspension simulation results with different parameters.

(Agostinacchio *et al.*, 2013), in their study of vibrations induced by road surface irregularities, describe the surface of a road as the combination of a large number of periodic bumps, some longer and some shorter, with different amplitudes. Assuming that a given road has equal statistical properties throughout the analyzed section, the international standard ISO 8608, which describes methodologies to be used for road surface profile generation, is used to develop a representative mathematical model for different road roughness profiles characterized by different levels of irregularity. (ISO8608, 2016) is based on the comparison between the measured PSD and the profile classes established by it, defined by a series of artificial PSD's. It has fundamental concepts such as spatial frequency n, also called wavelength, which is defined in cycles per meter, as opposed to the unit Hertz (cycles/second), road profile h, which is the variation in road surface height, and the power spectral density (PSD), which is the function used to stochastically describe the roughness of the track. Thus, (Agostinacchio *et al.*, 2013) propose an artificial road profile through the ISO classification described as:

$$h(x) = \sum_{i=0}^{N} \sqrt{\Delta n}\, 2^{K}\, 0.001(\frac{n_0}{i\Delta n})\, cos(2\,\pi\, i\, \Delta n\, x + \varphi_i), \tag{15}$$

where i ranges from 0 to $N = \frac{n_{max}}{\Delta n}$, L is the length of road profile, B is the sampling interval, n_{max} is the maximum spatial frequency of theoretical sampling $(1/B)$, Δn is the range of n_i equally spaced ($\Delta n = 1/L$), x is the longitudinal axis variable from 0 to L, K is the constant value according to ISO road profile classification (see Tab. 3) and φ_i is the random phase angle following a uniform probability distribution within the range 0 to 2π.

Table 3. Classification of road roughness by K values.

Road Class		
Upper limit	Lower limit	K
A (Very Good)	B (Good)	2
B (Good)	C (Regular)	3
C (Regular)	D (Poor)	4
D (Poor)	E (Very Poor)	5

Using the K values from 2 to 5, according to the classification in Tab. 3, the graphs were obtained in Matlab® software with the behavior of the road profile of each class as a function of roughness height $h(x)$ and length from the road L. Figure 5 makes it possible to compare the behavior of each road class, from best to worst quality. The road length of 250 meters, vehicle speed of 30 m/s and a spatial frequency with a range between 0.004 and 4 m/s^{-1} were considered.

2.5 Considerations and Hypotheses

To apply the two degrees of freedom model, representing a quarter of the vehicle, some hypotheses and simplifications were made, precisely because it is a pre-design tool. Thus, the following considerations were adopted:

a) the model and its parameters are linear;

b) the model does not fully predict the dynamics of a vehicle as it is not able to represent pitch, yaw and roll movements;

c) the model does not consider the damping in the tire, by being very small compared with the suspension damping;

d) the vehicle travels along a straight path;

e) the following suspension system (QCM model) and road profile (input) data were considered for simulation and optimization using Matlab® software:

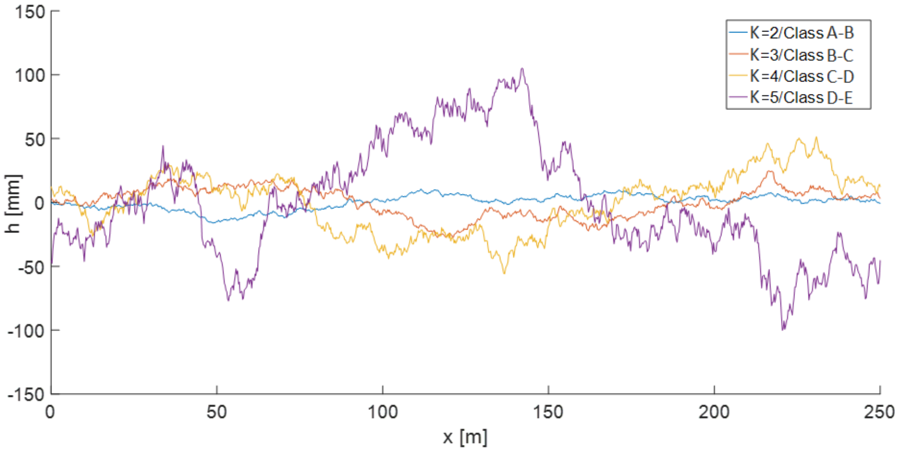

Figure 5. Graph of the road profile classes obtained with the mathematical model proposed by Agostinacchio et al (2013).

Table 4. Important data for simulation and optimization.

Total vehicle mass	1000 kg
Unsprung mass - 1/4 vehicle m_s	250 kg
Wheel/Tire mass m_p	40 kg
Tire stiffness k_p	200000 N/m
Road profile class	$K = 2$ (Class A-B)
Runway length	250 m
Travel speed	30 m/s

3. RESULTS

3.1 Simulations

Once the system and input mathematical models are defined, numerical simulations are performed to evaluate their behavior in various parameter configurations, trying to understand the influence of the parameters, undamped natural frequency and the damping ratio, for comfort and in vehicle safety. For this, evaluations must be performed, as a function of time, where the behavior of the system is analyzed over the entire travel range, and as a function of the value in RMS. Thus, it is possible to establish a restriction of the suspension parameters, as well as understanding the need for optimization tools. Initially, the simulations in the Matlab® software were performed by varying the frequency from 0 to 10 Hz, using the damping rates: 0.1, 0.3, 0.5, 0.7 and 0.9. Figures 6 and 7 show the simulations results of the chassis vertical acceleration and wheel force dynamics, respectively.

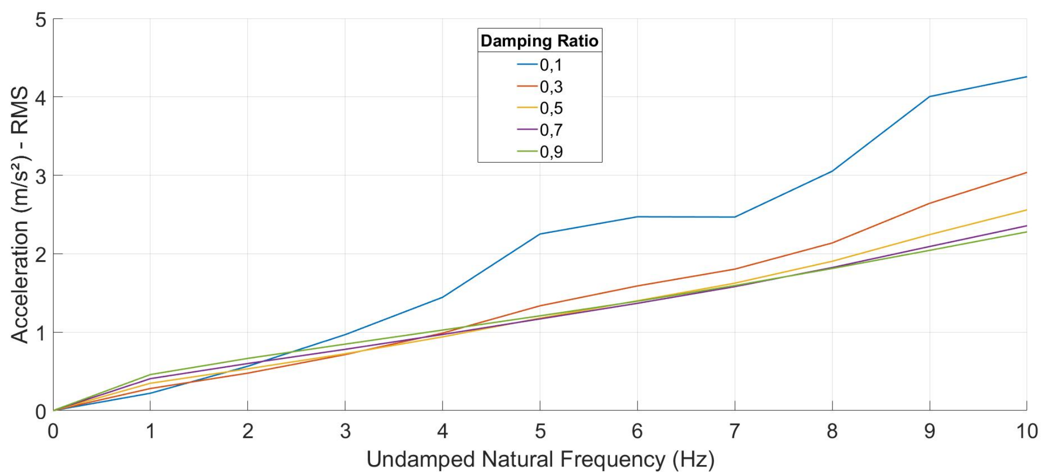

Figure 6. Graph with the RMS acceleration resulting from the road profile input to the suspension system.

The acceleration graph (Fig. 6) shows the influence of frequency on acceleration growth, ie the lower the frequency, the lower the acceleration and the better the comfort. The same happens with the damping ratio that, practically up to 4 Hz, the lower the rate, the lower the acceleration. However from 4 Hz the exact opposite occurs.

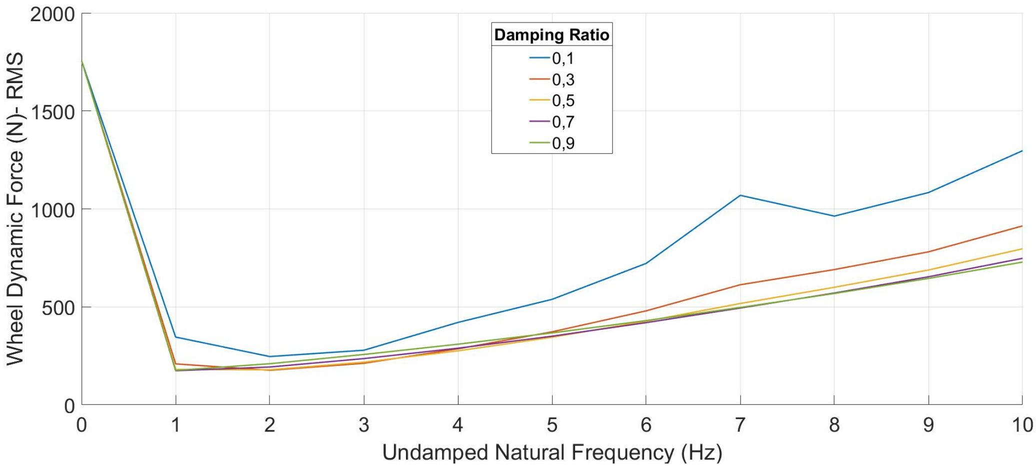

Figure 7. Graph with the RMS wheel dynamic force resulting from the road profile input to the suspension system.

In the wheel dynamic force graph (Fig. 7) its behavior can be divided into before and after 1 Hz. Frequencies below 1 Hz result in very high wheel dynamic force values, which is bad for driving safety. When the frequency approaches 1 Hz, the force decreases. For the frequency above 1 Hz, initially up to 3 Hz there is not much variation in force when compared to the growth that occurs later with increasing frequency. Except for the damping ratio close to 0.1, which has higher force values compared to other rates.

3.2 Optimization

Since simulation solutions only describe vehicle behavior at different system parameter configurations, it is necessary to use an optimization method that defines between the various undamped natural frequency and damping ratio settings, which are the parameter pairs able to indicate the best level of comfort and safety for the vehicle, and hence the optimum values for stiffness and suspension damping coefficient.

(Mitra *et al.*, 2015) uses the genetic algorithm (GA) as an optimization technique to improve and resolve the conflict between ride comfort (RC) and road holding (HR). The model used has 4 degrees of freedom, representing a quarter of the vehicle together with the driver's seat and body. Through the genetic algorithm optimization of the Matlab® toolbox, the optimal system parameters were obtained, presented through the Pareto graph. With a subsequent analysis, comparing and evidencing the increased comfort in the optimized versus the non-optimized model.

(Drehmer *et al.*, 2016) optimizes the suspension system parameters in different road irregularity profiles to ensure driver comfort and safety. The vehicle model has 8 DOF and the biodynamic driver model 4 DOF, totaling 20 DOF in the full vehicle model used. The multiobjective optimization of the suspension parameters, in the face of the conflict between comfort (measured by seat acceleration and suspension travel) and safety (measured by ground grip), used the particle swarm algorithm (PSO), and compared it with the deterministic quadratic sequential programming algorithm (SQP).

(Nagarkar *et al.*, 2016) used an 8 degrees of freedom model for the optimization and analysis of comfort and safety criteria. The 1/4 car model, with 4 DOF, consists of the seat structure and the driver's seat cushion, coupled with a biomechanical model, which has 4 DOF, representing the driver. Using as a criterion to be optimized the RMS accelerations in the biomechanical model, together with the suspension deflection and the dynamic force in the tire, the non-dominated ordering genetic algorithm (NSGA-II) and the multi-objective particle swarm optimization (MOPSO-CD) were used. Due to their multiple degrees of freedom and, consequently, several design criteria to be optimized, the use of these multi-objective methods was successful in the study, with the detail that the MOPSO-CD algorithm took less computation time compared to NSGA-II for optimization.

(Sousa and Avila, 2017) improved the efficiency of the suspension system by analyzing the vertical dynamics of a two-degree model of a quarter-of-freedom vehicle subjected to optimization by genetic algorithms (Matlab®). Observing the RMS values of chassis acceleration and the suspension travel displacement, used to measure comfort, the optimization applying the genetic algorithm proved to be satisfactory, assisting in the choice of suspension design parameters, the stiffness associated with the suspension spring, and the damping coefficient associated with the damper.

In this study, the optimization method to be used must be able to treat multiobjective problems, since there are two objectives, comfort and safety, observed through the RMS values of the vertical acceleration of the chassis and the dynamic force of the wheel. Thus, the multiobjective optimization method that will be applied in the quarter car model is the genetic algorithm (GA) method. Since it was observed its constant use in studies and scientific articles about suspension system, besides being among the methods available in Matlab® toolbox, same software that was used for numerical simulations. The method in the Matlab® software is represented by the *gamultiobj* code that uses a controlled elitist genetic algorithm, a variant of the NSGA-II (Non-dominated Sort Genetic Algorithm). For this analysis and optimization, the design variables will be represented by the suspension system parameters: damping ratio ξ_s and undamped natural frequency f_{n_s}, both related to the sprung mass. Thus, by setting the values of the sprung and unsprung masses, and the tire stiffness, it is possible to find the damping ratio and natural undamped frequency values of the system that minimize chassis vertical acceleration and contact loss between tire and ground, through wheel dynamic force, and consequently, design the ideal suspension that combines user comfort with driving safety. Analyzing the user comfort information and the numerical simulations performed, it is evident that the undamped natural frequency of the sprung mass must be above 1 Hz, due to sickness caused by resonance and also by high values of the wheel dynamic force, and below 4 Hz, since the human organism is very sensitive to vibrations in the 4 to 8 Hz range. Furthermore, the system treated in the study is underdamped, i.e., with ξ less than 1. The definition of the objective function is essential for the optimization method, since it represents the criterion that should be used to compare different projects. Thus, there are two goals to be achieved: comfort and safety. As seen above, how to achieve the first objective is to minimize the RMS acceleration of the chassis (sprung mass), so as to get the best comfort for the user. The second objective will be achieved by minimizing the RMS wheel dynamic force, thus ensuring smaller variations in contact between the tire and the road, resulting in good driving ability and driving safety. Thus, objective functions are given by the response vector of the mathematical model in the form of the state space:

- $\ddot{z}_s(f_{n_s}, \xi_s)$: Chassis vertical acceleration function;
- $F_{dyn}(f_{n_s}, \xi_s)$: Wheel dynamic force function.

Thus, the multiobjective optimization problem will be described as:

$$\text{Find } \vec{x^*} = \left\{ \begin{array}{c} f_{n_s} \\ \xi s \end{array} \right\} . \tag{16}$$

That minimizes $\ddot{z}_s(f_{n_s}, \xi_s)$ and $F_{dyn}(f_{n_s}, \xi_s)$, subject to:

$$1Hz \leq f_{n_s} \leq 4Hz \quad and \tag{17}$$

$$0.1 \leq \xi_s \leq 0.9 \ . \tag{18}$$

The NSGA-II algorithm will generate a set of optimal Pareto solutions. Then some selection criteria or additional rules should be used to select values of closer parameters. The discomfort scale table established by ISO 2631 defines a selection criterion related to comfort. Ideally, the acceleration is below 0.315 m/s^2 for the result to be considered as comfortable. Another selection criterion is the safety margin R, which relates the wheel dynamic wheel force and the sprung weight (chassis). As noted earlier, for a trip to be considered safe, the safe margin value must be greater than 0.75 or 75%. This means that the dynamic force of the wheel must be less than 25% of the weight of the sprung mass. If the sprung mass is 250 kg, then it has the following criteria:

$$F_{dyn} \leq 625N \tag{19}$$

From the knowledge of design variables and their constraints, as well as additional criteria in the selection of optimal results, was used a Matlab® optimization method called gamultiobj, referring to the Multiobjective Genetic Algorithms method. The results obtained are shown in Fig. 8.

In order to reduce the number of viable results from the Pareto Front, some procedures were performed taking into consideration the selection criteria listed above, prioritizing the results below 0.315 m/s^2 and 625 N and applying the design variables obtained in the results of optimization, but now in more severe conditions to obtain the results of optimal design variables under other conditions. Firstly, the acceleration values above 0.315 m/s^2 were highlighted in the original optimization results (n^o. 19, 20 and 21). It is noteworthy that all the results presented safe force values, that is, below 625 N. Then, increasing the travel speed to 40 m/s^2, were also highlighted those with acceleration above 0.315 m/s^2 (n^o. 14, 15, 16, 17 and 18). Finally, while maintaining the travel speed at 40 m/s, a more critical road profile ($K = 3$) was used to highlight forces values above 625 N (n^o. 1, 2, 3, 4, 5, 6 and 7). Moreover, this last condition did not present any acceleration value below 0.315 m/s^2, but those closest to it are among the results that were not highlighted in any of the previous conditions. Table 5 explains exactly the analyzes described above.

The results of optimization that better behaved in more severe conditions, either a higher speed or a road of poor quality were those with undamped natural frequency around 1.00 Hz and damping ratio around 0.20. Consequently, suspension stiffness values close to 10000 N/m and damping coefficient between 550 to 750 Ns/m.

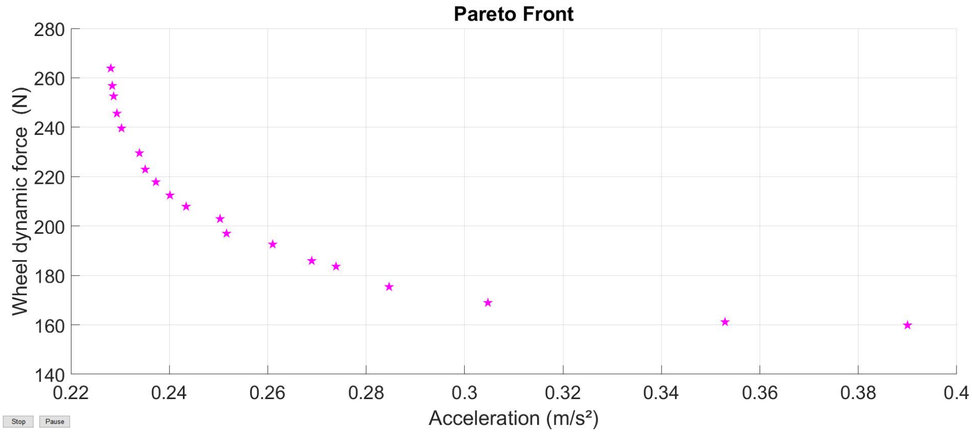

Figure 8. Graph resulting from optimization of acceleration and force with Pareto Front generation.

Table 5. List of results applied to speed and road conditions more severes, highlighting those with better behavior.

N^o	Optimization				Normal Condition		Severe Conditions			
	Design Variables				(Results)		K=2 ; v=40 m/s		K=3 ; v=40 m/s	
	f_{n_s} (Hz)	ξ_s	k$_s(N/m)$	b$_s(Ns/m)$	$\ddot{z}_s(m/s^2)$	F$_{dyn}(N)$	$\ddot{z}_s(m/s^2)$	F$_{dyn}(N)$	$\ddot{z}_s(m/s^2)$	F$_{dyn}(N)$
1	1.00	0.12	9870	369	0.23	263.84	0.26	347.27	0.65**	812.42*
2	1.00	0.12	9870	369	0.23	263.84	0.26	347.27	0.65**	812.42*
3	1.00	0.13	9884	394	0.23	256.77	0.27	341.39	0.61*	735.07*
4	1.00	0.13	9887	409	0.23	252.54	0.26	331.96	0.55*	773.10*
5	1.00	0.14	9880	437	0.23	245.56	0.27	322.19	0.58*	703.24*
6	1.00	0.15	9879	464	0.23	239.56	0.28	315.02	0.63**	721.56*
7	1.01	0.16	9995	514	0.23	229.53	0.28	302.68	0.58*	629.32*
8	1.00	0.18	9893	552	0.24	222.90	0.28	290.76	0.56*	579.81
9	1.00	0.19	9892	585	0.24	217.80	0.28	280.45	0.56*	556.59
10	1.00	0.20	9900	623	0.24	212.42	0.30	297.70	0.58*	554.61
11	1.00	0.21	9953	659	0.24	207.87	0.30	279.72	0.64*	587.38
12	1.02	0.22	10326	702	0.25	202.89	0.30	267.24	0.66*	576.16
13	1.00	0.24	9888	761	0.25	196.96	0.30	249.97	0.62*	519.29
14	1.03	0.25	10508	810	0.26	192.57	0.31*	245.08	0.64**	506,39
15	1.03	0.28	10493	900	0.27	185.90	0.33*	241.39	0.65**	475,52
16	1.04	0.29	10739	935	0.27	183.61	0.33*	233.13	0.76**	540.77
17	1.00	0.35	9924	1098	0.28	175.36	0.37*	239.61	0.69**	443.29
18	1.02	0.40	10313	1279	0.30	168.93	0.38*	218.52	0.74**	425.10
19	1.08	0.51	11526	1739	0.35*	161.14	0.45*	209.62	0.89***	415.40
20	1.20	0.55	14234	2072	0.39*	159.83	0.47*	194.31	0.93***	388.76
21	1.20	0.55	14234	2072	0.39*	159.83	0.47*	199.01	0.93***	388.76

Comfort index (m/s^2)	Scale	Indicator	Safety index (N)	Scale	Indicator
Smaller than 0.315	Comfortable	None	Smaller than 625 N	Safe	None
0.315 - 0.63	Slightly comfortable	*	Bigger than 625 N	Unsafe	*
0.5 - 1	Little comfortable	**			
0.8 - 1.6	Uncomfortable	***			

4. CONCLUSION

As shown in the results, the quarter-car model was representative for preliminary suspension design studies. The model perfectly reproduced the conflict between comfort and safety criteria. In addition, more detailed analysis of such criteria allows to further restrict the region in which to work with design variables, undamped natural frequency, and damping ratio, within acceptable values of comfort and safety. The ideal range of natural frequency and damping ratio, considering the vehicle mass of 1000 kg, tire stiffness at 200 kN/m and 40 kg unsprung weight, was close to 1 Hz and damping ratio of 0.2. This represents suspension stiffness and damping values close to 10000 N/m to 650 Ns/m, respectively. In addition, it was possible to use the mathematical model that reproduce the different road profiles for application in simulations and optimizations performed in Matlab®. Making the results obtained closer to reality.

5. REFERENCES

Agostinacchio, M., Ciampa, D. and Olita, S., 2013. “The vibrations induced by surface irregularities in road pavements - a matlab approach”. *European Transport Research Review*, Vol. 1, No. 1, pp. 267–275.

BS6841, 1987. “Measurement and evaluation of human exposure to whole-body mechanical vibration and repeated shock.” London: British Standard Guide, 1987.

Chi, Z., He, Y. and Naterer, G.F., 2008. “Design optimization of vehicle suspensions with a quarter-car vehicle model”. *Transactions of the CSME*, Vol. 32, No. 2, pp. 297–312.

Cunha, A., Felix, J.L.P. and Balthazar, J.M., 2017. “Quantification of parametric uncertainties induced by irregular soil loading in orchard tower sprayer nonlinear dynamics”. *Journal of Sound and Vibration*, Vol. 408, pp. 252–269.

Dharankar, C.S., Hada, M.K. and Chandel, S., 2016. “Numerical generation of road profile through spectral description for simulation of vehicle suspension”. *Journal of the Brazilian Society of Mechanical Sciences and Engineering*, Vol. 39, No. 6, pp. 1957–1967.

Diniz, D.D.S., 2014. *Estudo da dinâmica vertical em suspensão duplo A de um veiculo off-road tipo baja*. Master’s thesis, Universidade Federal de Campina Grande, Campina Grande, Brasil.

Drehmer, L.R.C., 2010. *Otimização de Parâmetros Concentrados de Suspensão para Conforto e Segurança Veicular*. Master’s thesis, Universidade Federal do Rio Grande do Sul, Porto Alegre, Brasil.

Drehmer, L.R.C., Casas, W.J.P. and Gomes, H.M., 2016. “Otimização multiobjetivo de parâmetros concentrados de suspensão para conforto e segurança veicular em dinâmica vertical”. *Asociacion Argentina de Mecanica Computacional*, Vol. 34, No. 39, pp. 2729–2754.

Freitas Jr, L.M.P., 2006. *Estudo da dinâmica vertical de uma suspensão veicular do tipo Macpherson*. Master’s thesis, Universidade de São Paulo, São Paulo, Brasil.

ISO2631, 1997. “Mechanical vibration and shock _ evolution of human exposure of whole _ body vibration: General requirements.” Beuth, Berlin, 1997.

ISO8608, 2016. “Mechanical vibration _ road surface profiles _ reporting of measured data.” Geneva, Switzerland, 2016.

Jazar, R.N., 2008. *Vehicle Dynamics _ Theory and Application*. Springer, New York, 1st edition.

Lopes, J.L., 2012. “Análise de vibração ocupacional de corpo inteiro em maquinas colhedoras de cana-de-açúcar”. *Revista ABHO (Associação Brasileira de Higienistas Ocupacionais)*, Vol. 26, pp. 6–16.

Mitra, A.C., Desai, G.J., Patwardhan, S.R., Shirke, P.H., Kurne, W.M.H. and Banerjee, N., 2015. “Optimization of passive vehicle suspension system by genetic algorithm”. *Procedia Engineering - 12th International Conference on Vibration Problems*, , No. 12, pp. 1158–1166.

Nagarkar, M.P., Patil, G.J.V. and Patil, R.N.Z., 2016. “Optimization of nonlinear quarter car suspension seat driver model”. *Journal of Advanced Research*, Vol. 7, No. 6, pp. 991–1007.

Pereira, C.C.G., 2005. *Curvas de percepção e conforto humano para vibrações verticais*. Master’s thesis, Universidade Federal de Ouro Preto, Ouro Preto, Brasil.

Popp, K. and Schiehlen, W., 2010. *Ground Vehicle Dynamics*. Springer, New York, english edition edition.

Rajamani, R., 2006. *Vehicle dynamics and control*. Springer, New York, 1st edition.

Rao, S.S., 2009. *Engineering Optimization: Theory and Practic*. John Wiley & Sons, New Jersey, 4th edition.

Rao, S.S., 2011. *Mechanical Vibrations*. Prentice Hall, New Jersey, 5th edition.

Rill, G., 2007. *Road Vehicle Dynamics: Fundamentals and Modeling*. CRC Press, New York, 1st edition.

Sarami, S., 2009. *Development and Evaluation of a Semi-active Suspension System for Full Suspension Tractors*. Ph.D. thesis, Technical University of Berlin, Berlin, German.

Sousa, D. and Avila, S., 2017. “Otimização dos parâmetros da suspensão para um modelo de um quarto de veiculo”. *Revista Interdisciplinar de Pesquisa Em Engenharia*, Vol. 2, No. 13, pp. 178–194.

Sun, X., Chu, Y., Fan, J. and Yang, Q., 2012. “Research of simulation on the effect of suspension damping on vehicle ride”. *Energy Procedia - International Conference on Future Electrical Power and Energy Systems*, Vol. 17, No. Part A, pp. 145–151.

Verros, G., Natsiavas, S. and Papadimitriou, C., 2005. “Design optimization of quarter-car models with passive and semi-active suspensions under random road excitation”. *JVC/Journal Vibration and Control*, Vol. 11, No. 5, pp. 581–606.

Yucheng, L., 2008. “Recent innovations in vehicle suspension systems”. *Recent Patents on Mechanical Engineering*, Vol. 1, No. 3, pp. 206–210.

6. RESPONSIBILITY NOTICE

The authors are the only responsible for the printed material included in this paper.