25th ABCM International Congress of Mechanical Engineering
October 20-25, 2019, Uberlândia, MG, Brazil

COBEM-2019-0930
NUMERICAL ANALYSIS OF A HEAT EXCHANGER FINNED USING FINITE DIFFERENCE AND FINITE ELEMENT METHODS

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Abstract. *The optimization of industrial processes for maximum utilization of available energy has been a line of scientific research quite widespread recently. Many industrial applications require the use of heat exchangers with piped fittings, whether or not finned, operating in air-conditioning, refrigeration, heating, radiator and other engineering applications. The present work carried out a numerical analysis of the temperature gradient on the fins of a heat exchanger, in which the fins have straight rectangular or rectangular pine geometry. This analysis was performed employing the finite difference and finite element methods in a code designed on the MATLAB® platform by manipulating the data provided from a heat exchanger to the classical boundary conditions on an extended surface. Finally, the results obtained show that the pin of rectangular profile presents greater efficiency when compared to the straight rectangular fin and that finite element method converges to the analytical solution with greater accuracy than the method of finite differences.*

Keywords: *Compact heat exchanger, Numerical analysis, Finite difference method, Finite element method.*

1. INTRODUCTION

In engineering there are numerous applications of thermal energy, these depend exclusively on the characterization of heat as the form of energy that can be transferred from one system to another as a consequence of the temperature difference between them. The most important are the thermal machine design and thermal systems, such as combustion engines, turbines, air conditioners, refrigerators and steam engines, which are designed based on heat transfer analysis (Bejan, 2013).

In this way, many researchers has been dedicated to the study of heat transfer in several areas of knowledge and practical applications in engineering, and have developed several methods to treat problems related to this form of energy. Among them the classical theory, which corresponds to the analytical solution of the problems of heat transfer in continuous systems through the mechanisms of heat transfer, that is, the use of the fundamental laws of thermodynamics together with the modes of heat transfer (conduction, convection and radiation), whose governing equations are partial differential equations (Wang, 2013). The analytical solution determined by classical theory is generally obtained through governing differential equations. However, it becomes impracticable to solve analytically the vast majority of the problems currently studied, mainly due to the complex geometries and the boundary conditions in which the problems are submitted.

Thus, Nithiarasu *et al.* (2016) commented that with the rapid evolution of the processing capacity of computers, several computational methods were developed to solve engineering problems. In this context, computational mechanics and numerical methods started to gain a prominent position in the different areas of research, being object of study of many researchers, as, Laloui *et al.* (2006), He *et al.* (2012), Bhutta *et al.* (2012) and Ozudogru *et al.* (2015), these studies being concentrated in several areas of heat transfer.

Among the numerical methods used to solve problems involving heat exchange, the finite difference method and the finite element method are highlighted. The finite difference method, presented in detail in Cuminato and Meneguette (2013) and Gilat and Subramaniam (2009), consists in a method of solving differential equations is based on the approximation derived by Taylor series expansion. In relation to the finite element method, it is defined according Chandrupatla *et al.* (2002) e Hutton (2017) as a numerical procedure that subdivides the domain of a problem into smaller parts to determine approximate solutions of problems with values on the boundary of the equations differentials.

In the engineering are studied several thermal devices, some of them already mentioned, but one that occupies prominent place is the heat exchanger. This device facilitates the exchange between two fluids that are at different temperatures, avoiding or not mixing with one another. Because it has this characteristic very acceptable to the principle of heat transfer, it is widely applied, and these applications range from domestic air-conditioning systems to chemical and production processes in large power plants.

Based on the assumptions presented above, the present work aims to numerically analyze the temperature gradient on the fins of a heat exchanger, in which the fins have straight rectangular or rectangular pine geometries. This analysis was performed employing the finite difference and finite element methods in a code designed on the MATLAB® platform by manipulating the data provided from a heat exchanger to the classical boundary conditions on an extended surface (fins).

2. NUMERICAL METHODOLOGY

For modeling and analysis of the temperature gradient in straight rectangular profile and rectangular pine profile fins, the corresponding analytical equation was applied for each contour condition at the tip of the fin in the computational code in MATLAB®. Afterwards, fin finite domain was divided into linear elements (1D) and linear and quadratic one-dimensional finite element modeling was performed using the theoretical finite element principles.

In agreement with (Belegundu and Chandrupatla, 2014), the finite-rate heat transfer field problem in fins is considered to be a special form of the general Helmholtz in Eq. (1) given by:

$$\frac{\partial}{\partial x}\left(k_x\frac{\partial \varphi}{\partial x}\right)+\frac{\partial}{\partial y}\left(k_y\frac{\partial \varphi}{\partial y}\right)+\frac{\partial}{\partial z}\left(k_z\frac{\partial \varphi}{\partial z}\right)+\lambda\varphi+\dot{Q}=0 \tag{1}$$

where φ is the field variable that must be determined. By setting φ = T, $k_x = k_y = k_z = k$ and λ = 0 and taking into account only x, which describes the one-dimensional heat conduction problem for temperature T, where k is the thermal conductivity and $\dot{Q}$ is the source or heat sink and The loss of heat by convection in the fin can be considered a source of negative heat. Thus, the governing equation is given by as in Eq. (2).

$$\int_0^L \left[\frac{d}{dx}\left(k\frac{dT}{dx}\right)-\frac{Ph_{ext}}{A_c}(T-T_\infty)\right]dx=0 \tag{2}$$

A lagrangian linear polynomial approximation form function for linear elements and a polynomial approximation form function for quadratic elements. Finally, the simultaneous algebraic equations were written in matrix form to determine all the unknown variables in a discretized number of points.

The general equation in matrix form for a linear element will be given in Eq. (3), as a function of the thermal conductivity of the copper (k_{cop}), of the particular cross-sectional area for each geometry (A_c), of perimeter of the fin (P), of the external convection coefficient (h_{ext}), of the ambient air temperature ($T_{C(in)}$), of the fin length divided by the number of discretizing elements (h_e), the heat rate on the current element ($\dot{Q}_e$) and the next element ($\dot{Q}_{e+1}$). It should be noted that, the heat rates($\dot{Q}_e$) and e ($\dot{Q}_{e+1}$) are given as a function of the other parameters and are modified accordingly with the contour condition at the end of the fin.

$$\left(\frac{kA_c}{h_e}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}+\frac{P(h_{ext})h_e}{6}\begin{bmatrix}2 & 1\\ 1 & 2\end{bmatrix}\right)\begin{bmatrix}T_e\\ T_{(e+1)}\end{bmatrix}=\begin{bmatrix}\dot{Q}_e\\ \dot{Q}_{(e+1)}\end{bmatrix}+\frac{h_eP(T_{C(in)})h_{ext}}{2}\begin{bmatrix}1\\ 1\end{bmatrix} \tag{3}$$

The general equation in the matrix form for a quadratic element is given as in Eq. (4), as a function of the same parameters, while for the other elements the internal contour condition applies.

$$\begin{aligned}\left(\frac{kA_c}{3h_e}\begin{bmatrix}7 & -8 & 1\\ -8 & 16 & -8\\ 1 & -8 & 7\end{bmatrix}+\frac{P(h_{ext})h_e}{30}\begin{bmatrix}4 & 2 & -1\\ 2 & 16 & 2\\ -1 & 2 & 4\end{bmatrix}\right)\begin{bmatrix}T_e\\ T_{(e+1)}\\ T_{(e+2)}\end{bmatrix}&=\\ =\begin{bmatrix}\dot{Q}_e\\ \dot{Q}_{(e+1)}\\ \dot{Q}_{(e+2)}\end{bmatrix}+\frac{h_eP(T_{C(in)})h_{ext}}{6}\begin{bmatrix}1\\ 4\\ 1\end{bmatrix}\end{aligned} \tag{4}$$

For the finite difference method, an approximation of the second-order linear ODE of the heat equation was made by the central difference method, then the division of the fin domain into subintervals equal to the amount of elements applied to the finite element method Δx, the discretized equation for the convection condition at the inner in Eq. (5) and the convective ends in Eq. (6), respectively, is given by:

$$T_{m-1}-2T_{m+1}+\frac{hP\Delta x^2}{kA_c}(T_\infty-T_m)=0 \tag{5}$$

$$T_{m-1}-T_{m+1}+\frac{h\Delta x}{kA_c}\left(\frac{P\Delta x}{2}+A_c\right)(T_\infty-T_m)=0 \tag{6}$$

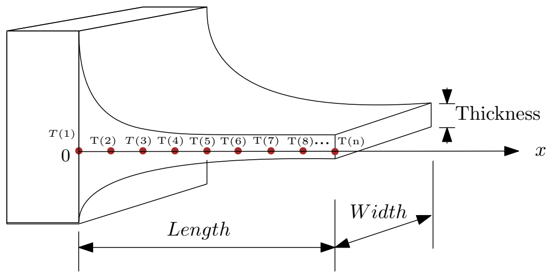
where T_{m-1} and T_{m+1} are respectively, the points before and after the point under study (T_m).

Finally, the data in tables and images with graphs were made to compare the fins with a straight rectangular profile and a rectangular profile pin.

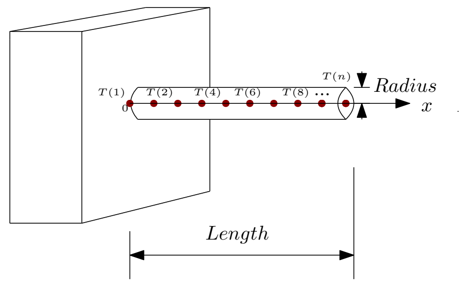
3. RESULTS AND DISCUSSIONS

3.1 Numerical analysis of the temperature gradient developed in the fins varying its geometric profile to rectangular profile pin and straight rectangular profile

As stated by the proposed research, for the convection, adiabatic and specified temperature contour conditions at the tip of the fin, one of the conditions to demonstrate the heat transfer efficiency of a fin stems from the temperature gradient developed from its base to the its end. As claimed by (Çengel and Ghajar, 2015) the maximum efficiency occurs when the temperature of the fin is uniform and equal to its base value $(\mathrm{T_{base}})$, since the heat transfer from the fin is maximum, since in this condition its thermal conductivity (k) tends to infinity. In reality, the temperature decreases along the fin, so the heat transfer is less because of the decrease in the temperature difference towards the tip of the fin. It follows in Fig. 1.a and Fig. 1.b the geometries of the fins used.



(a) Straight Fins.



(b) Pin Fins.

Figure 1. Geometries of the fins

The data used in the code for the modeling of the rectangular profile pin and the straight rectangular profile fin were: the thermal conductivity of copper $(\mathrm{k_{cop}})$ = 398.7 W/mK, the external convection coefficient $(\mathrm{h_{ext}})$ = 211.7 W/m^{2}K , the ambient air temperature $(\mathrm{T_{C(in)}})$ = 20 °C and the fin length divided by the number of discretizing elements $(\mathrm{h_e})$ = 0.0111 m, for a total fin length (L) = 0.045 m, the perimeter of the fin (P) and cross-sectional area $(\mathrm{A_c})$, which are supplied as a function of the half of the outside diameter of the pipe $(\mathrm{D_{ext}})$ = 0.105 m for the rectangular profile pin, on the other hand, for the straight rectangular profile the same parameters are supplied as a function of the width (w) = 0.003 m and thickness (th) = 0.00033 m of the fin.

At first, the modeling took into account 4 linear intervals for the analytic and finite difference theories; however, for finite element theory it took into account 2 quadratic elements and 4 linear elements. This was done to demonstrate that finite element theory requires a discretization in fewer elements when using more precise curves, as is the case of the quadratic in counterpart of the linear one. An initial observation to be made solely based on the temperature at the end of the fin shown in Fig. 2 and in Fig. 3, is that the adiabatic condition denotes higher efficiency, followed by the convection condition and the temperature condition specified at the tip of the fin.

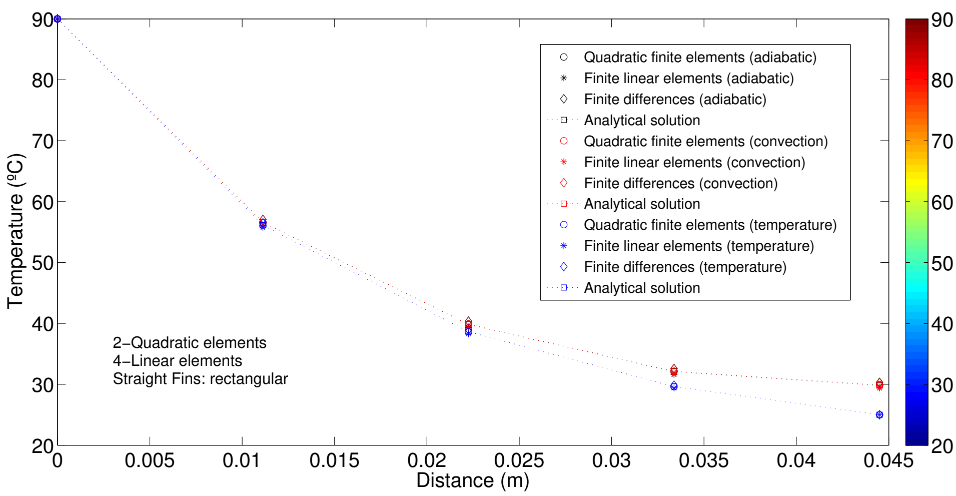


Figure 2. Temperature gradient for each boundary condition at the tip of the fin using 2 quadratic and 4 linear elements to straight fins.

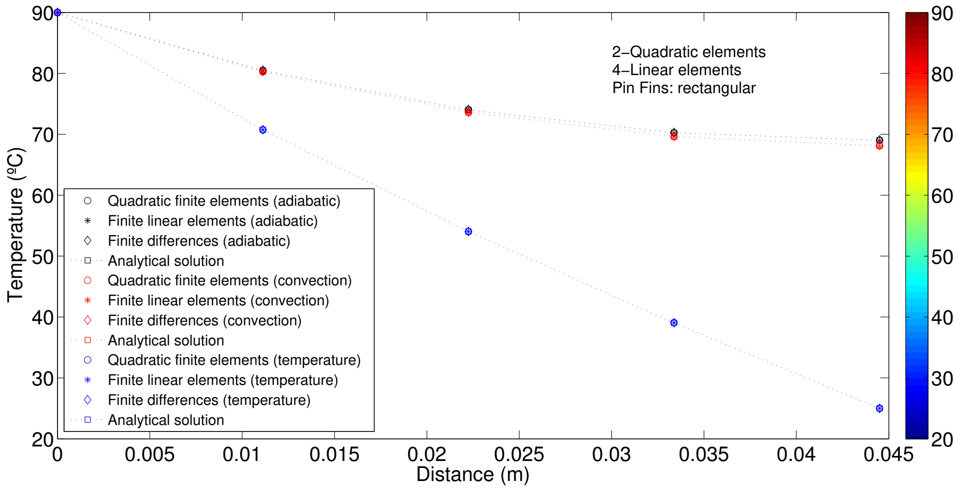


Figure 3. Temperature gradient for each boundary condition at the tip of the fin using 2 quadratic and 4 linear elements to pin fins.

Analyzing the data in Table 1 and Table 2, it can be seen that the efficiency of the fin with straight rectangular geometry is inferior to the efficiency of the pin fin in all numerical methods employed, in addition, for the adiabatic condition at the tip of the fin, present in Fig. 2 and in Fig. 3 and represented in Tab. 1 and in Tab. 2, it can be seen that the numerical solution of quadratic finite elements is much closer to the analytic than the too much. This precision follows in the data for the temperature contour condition specified in Tab. 3 and in Tab. 4, satisfying analytical theory and validating methods.

Table 1. Comparison of the numerical and analytical results for 2 quadratic elements and 4 linear elements: temperature gradient in the rectangular fin straight for the adiabatic contour condition at the tip.

x [m]	Analytical solution T [°C]	Linear finite elements T [°C]	Quadratic finite elements T [°C]	Finite differences T [°C]	$\eta_{average}$ fins [-]
0	90.0000	90.0000	90.0000	90.0000	1.0000
0.0111	56.6079	56.1139	56.5784	57.0634	0.5230
0.0222	39.8483	39.2960	39.9165	40.3646	0.2835
0.0334	32.1066	31.5978	32.1400	32.5878	0.1730
0.0445	29.8655	29.3810	29.9259	30.3259	0.1409

Table 2. Comparison of the numerical results with the analytical results for 2 quadratic elements and 4 linear elements: temperature gradient in the rectangular pin fin to the adiabatic contour condition at the tip.

x [m]	Analytical solution T [°C]	Linear finite elements T [°C]	Quadratic finite elements T [°C]	Finite differences T [°C]	$\eta_{average}$ fins [-]
0	90.0000	90.0000	90.0000	90.0000	1.0000
0.0111	80.4844	80.4533	80.4835	80.5153	0.8641
0.0222	74.0100	73.9589	74.0107	74.0605	0.7716
0.0334	70.2511	70.1889	70.2508	70.3126	0.7179
0.0445	69.0188	68.9531	69.0197	69.0838	0.7003

The temperature specified at the tip of the fin is 25 °C, which disadvantageously increases the temperature gradient along the fin compared to the other boundary conditions, but it should be noted in Fig. 5 and in Fig. 7 that even for such a condition, the lines continue to show that the quadratic finite element method is closer to the analytical solution, in addition, this boundary condition clearly demonstrates the heterogeneous behavior among the selected geometries, as seen in Fig. 4 and in Fig. 6.

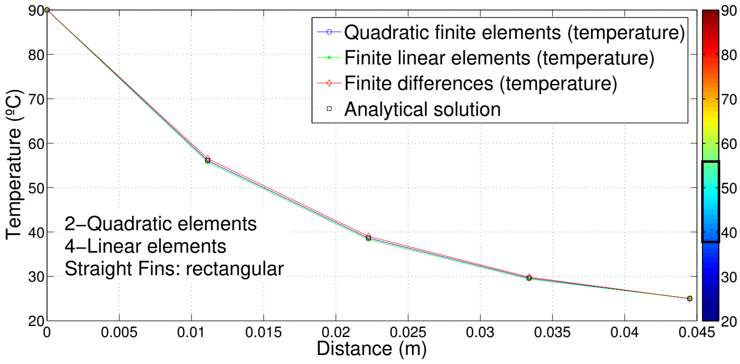


Figure 4. Temperature gradient in the straight rectangular fin to the contour condition temperature specified at the tip.

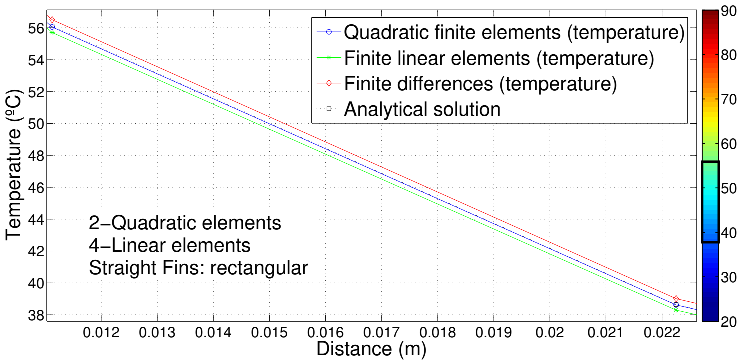


Figure 5. Zoom: temperature gradient x straight fin length to the boundary condition temperature specified at the tip

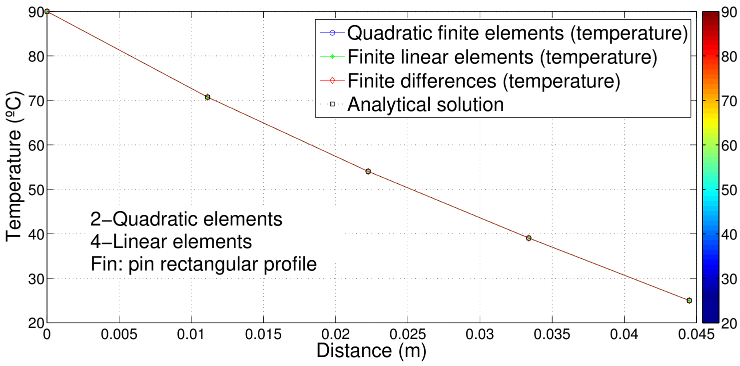


Figure 6. Temperature gradient in the pin fin to the contour condition temperature specified at the tip.

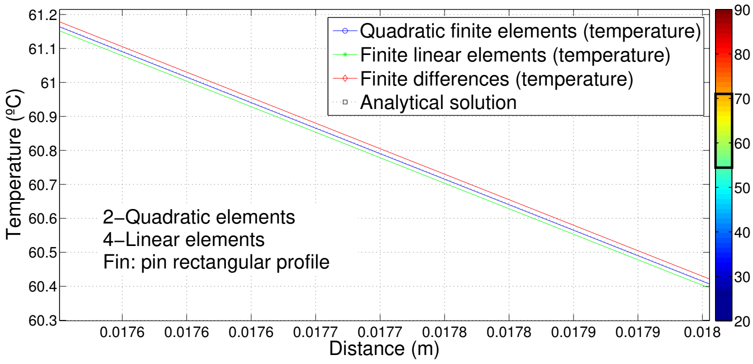


Figure 7. Zoom: temperature gradient x pin fin length to the boundary condition temperature specified at the tip

Observing the data from Table 3 and Table 4, it is noted that although the contour condition specifies the temperature at the tip of the fin, the temperature gradient develops completely heterogeneously in each of the chosen geometries, which consequently leads to a greater or lesser temperature gradient between the base of the fin and the nodal point prior to its tip, it is found that there is almost twice the efficiency of the geometry of the pin fin to the straight rectangular.

Table 3. Comparison of the numerical and analytical results for 2 quadratic elements and 4 linear elements: temperature gradient on the pin fin with rectangular profile for the boundary condition temperature specified at the tip.

x [m]	Analytical solution T [°C]	Linear finite elements T [°C]	Quadratic finite elements T [°C]	Finite differences T [°C]	$\eta_{\text{average fins}}$ [-]
0	90.0000	90.0000	90.0000	90.0000	1.0000
0.0111	70.7416	70.7291	70.7394	70.7533	0.7249
0.0222	54.0345	54.0195	54.0321	54.0478	0.4862
0.0334	39.0386	39.0276	39.0308	39.0471	0.2720
0.0445	25.0000	25.0000	25.0000	25.0000	0.0714

Table 4. Comparison of the numerical and analytical results for 2 quadratic elements and 4 linear elements: temperature gradient in the rectangular fin straight to the boundary condition temperature specified at the tip.

x [m]	Analytical solution T [°C]	Linear finite elements T [°C]	Quadratic finite elements T [°C]	Finite differences T [°C]	$\eta_{\text{average fins}}$ [-]
0	90.0000	90.0000	90.0000	90.0000	1.0000
0.0111	56.1153	55.7084	56.0508	56.5096	0.5159
0.0222	38.6392	38.2932	38.6195	39.0144	0.2663
0.0334	29.6316	29.3964	29.6085	29.8496	0.1376
0.0445	25.0000	25.0000	25.0000	25.0000	0.0714

The temperature gradient analyzes for the convection contour condition at the fin tip developed for the straight rectangular geometric profile fin follow the same methodology used for the rectangular profile pin fin. The obtained results also present the same observations as is shown in Fig. 9 and in Fig. 11: that the numerical solution of quadratic finite elements is more precise and close to the analytical solution, this when it is compared to the other numerical solutions and also that the condition of contour is inferior in efficiency in comparison to the condition of adiabatic control, because it presents a higher temperature gradient from its base to its tip.

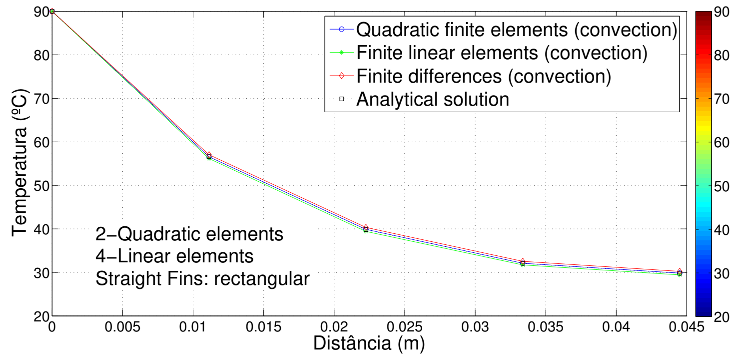


Figure 8. Temperature gradient in the straight rectangular fin to the convection contour condition at the fin tip.

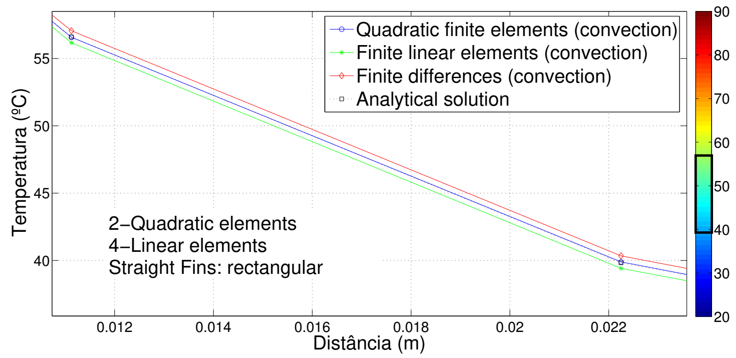


Figure 9. Zoom: temperature gradient x straight rectangular fin length to the convection contour condition at the fin tip.

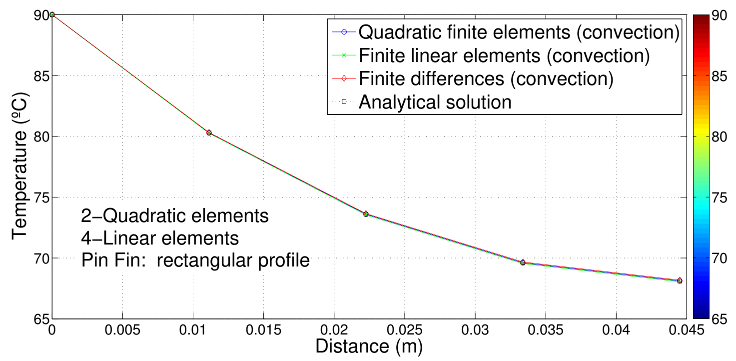


Figure 10. Temperature gradient in the pin fin to the convection contour condition at the fin tip.

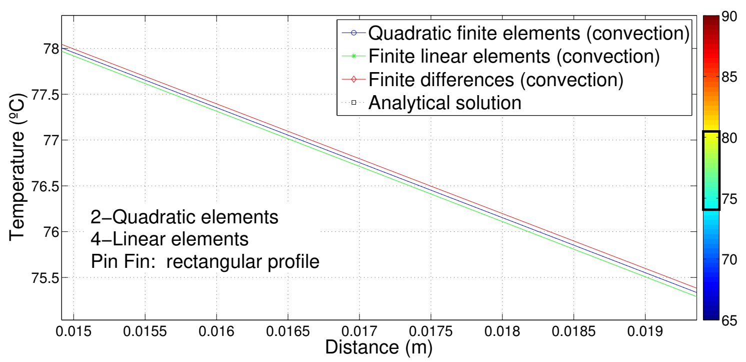


Figure 11. Zoom: temperature gradient x pin fin length to the convection contour condition at the fin tip.

3.2 Mesh refinement and comparison of fin efficiencies

In a second moment, the number of finite elements for 4 quadratic elements and 8 linear elements for the demonstration of the refinement of the discretized mesh was increased. As expected, the quadratic finite element model presented the highest accuracy in comparison to the other numerical solutions as in the Tab. 5 and in Tab. 6, for the convection boundary condition, is verified. The value of η_{average} is an average of the efficiencies of all models proposed at each point.

Table 5. Comparison of the numerical and analytical results for 4 quadratic elements and 8 linear elements: temperature gradient in the rectangular pin fin to the contour condition convection at the tip.

x [m]	Analytical solution T [°C]	Linear finite elements T [°C]	Quadratic finite elements T [°C]	Finite differences T [°C]	η_{average} fins [-]
0	90.0000	90.0000	90.0000	90.0000	1.0000
0.0056	84.7363	84.7321	84.7362	84.7407	0.9248
0.0111	80.2838	80.2761	80.2838	80.2918	0.8612
0.0167	76.5866	76.5763	76.5866	76.5976	0.8084
0.0222	73.5985	73.5861	73.5986	73.6118	0.7657
0.0278	71.2821	71.2680	71.2821	71.2971	0.7326
0.0334	69.6082	69.5931	69.6082	69.6246	0.7087
0.0389	68.5559	68.5402	68.5559	68.5732	0.6937
0.0445	68.1121	68.0962	68.1122	68.1298	0.6873

Table 6. Comparison of the numerical and analytical results for 4 quadratic elements and 8 linear elements: temperature gradient in the rectangular fin straight to the contour condition convection at the tip.

x [m]	Analytical solution T [°C]	Linear finite elements T [°C]	Quadratic finite elements T [°C]	Finite differences T [°C]	η_{average} fins [-]
0	90.0000	90.0000	90.0000	90.0000	1.0000
0.0056	70.5082	70.4269	70.5049	70.5878	0.7215
0.0111	56.5992	56.4797	56.6027	56.7164	0.5228
0.0167	46.7355	46.6023	46.7364	46.8664	0.3819
0.0222	39.8269	39.6928	39.8308	39.9590	0.2832
0.0278	35.1097	34.9803	35.1119	35.2378	0.2159
0.0334	32.0627	31.9390	32.0664	32.1854	0.1723
0.0389	30.3490	30.2297	30.3513	30.4677	0.1478
0.0445	29.7791	29.6617	29.7826	29.8965	0.1397

However, the temperature gradient values obtained for the straight rectangular fin profile in each contour condition are well above those of the rectangular pine profile. In other words, there is an increase in the temperature difference that goes from the base of the fin to its tip, which sums up in efficiency numbers, for the same conditions of contour, much smaller. Therefore, when analyzing the data in Tab. 6, for the convection contour condition, a fairly sharp drop in the efficiency of the straight rectangular geometric profile fin is observed in comparison with Tab. 5, for the same contour conditions of the profile pin fin rectangular.

The analysis of the temperature gradient developed for the straight rectangular geometric profile fin results that the numerical solution of quadratic finite elements is more accurate and close to the analytical solution. There is also an increase in the approximation of numerical results with the analytical results for all theories as the number of elements increases, that is, as the domain discretization mesh is refined, as can be seen in Fig. 12 and in Fig. 14.

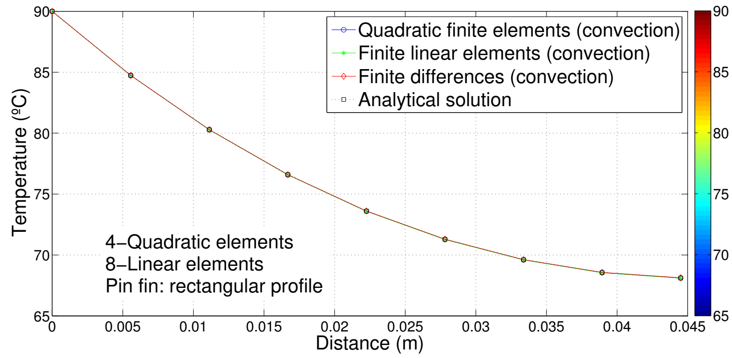


Figure 12. Temperature gradient in the pin fin to the convection contour condition at the fin tip.

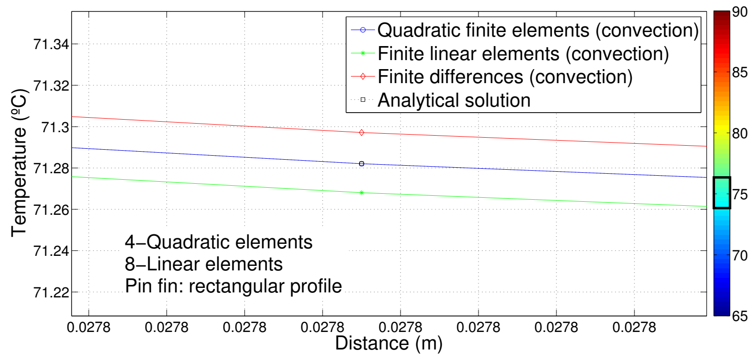


Figure 13. Zoom: temperature gradient x pin fin length to the convection contour condition at the fin tip.

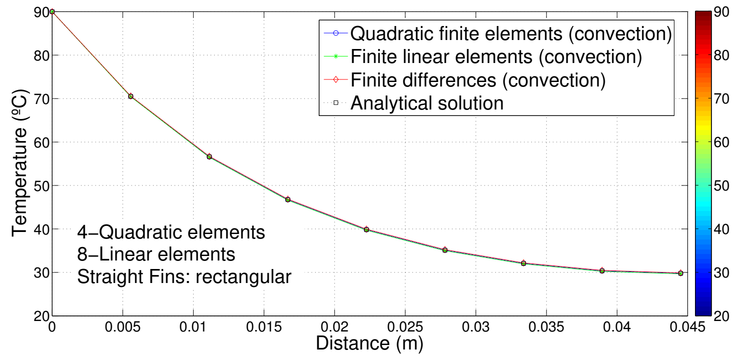


Figure 14. Temperature gradient in the straight rectangular fin to the convection contour condition at the fin tip.

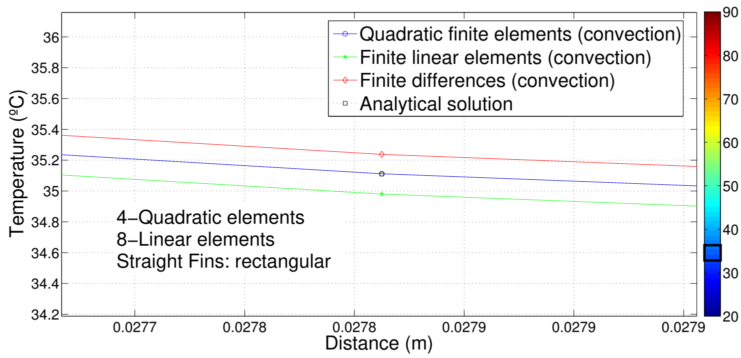


Figure 15. Zoom: temperature gradient x straight rectangular fin length to the convection contour condition at the fin tip.

4. CONCLUSIONS

This research analyzed the three contour conditions for fins with rectangular profile and rectangular pin geometry modeled on finite elements and finite differences and showed that, for the adiabatic condition, the temperature difference between the base and the end of the fin is lower, culminating in greater fin efficiency, followed by the convection condition and specified temperature.

The refinement of the finite element meshes and finite differences in the numerical code developed in MATLAB® showed an increase in the accuracy of the results when compared with the analytical solution. Whereas the numerical results of the finite element theory for quadratic elements have always been higher in accuracy of linear finite element theory and finite difference theory.

Therefore, the modeling developed presented satisfactory results, because, through the manipulations of the proposed parameters, the results of the three contour conditions and numerical theories were verified in concordance with the analytical solution available in the literature.

5. ACKNOWLEDGEMENTS

The authors are thankful to Coordenação de Aperfeiçoamento de Pessoal de Nível Superior (CAPES), for the financial support.

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