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ANALYSIS OF THE FEASIBILITY OF A PARABOLIC HEAT CONDUCTION MODEL FOR SELECTIVE SURFACES THERMAL BEHAVIOR

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Abstract. *In order to increase the efficiency of solar energy capitation in solar collectors, selective coatings formed by thin films are used. These coatings are subject to temperature variations from solar radiation which can cause changes in microfractures. The objective of this work is to evaluate if the microscale parabolic heat conduction model can be used as a tool for knowledge by thermal behavior of Selective Surfaces for thermosolar energy conversion. It evaluated the impact of the terms: coefficient related to the dimensionless power of the internal heat source (ψ_0), dimensionless absorption coefficient (β) and dimensionless time (τ). Therefore, it is possible use the parabolic heat conduction model in thin films as a tool for design of selective surfaces in relation to their thermal behavior.*

Keywords: *solar collectors, selective coatings, parabolic microscale heat conduction model.*

1. INTRODUCTION

The simplest way to convert solar energy is thermal conversion. This type of conversion occurs in equipment called solar thermal collectors, the most common is flat plat and parabolic troughs collectors. Solar thermal conversion is commonly used for fluid heating in power cycles in the generation of electricity (Rankine Cycle) (Duffie; Beckman, 2006) .

In order to increase the efficiency of solar radiation uptake, such collectors may have coatings on their absorptive surface that increase the efficiency of solar radiation absorption (Neto, 2017). These coatings are called Selective Surfaces or Selective Coatings and are subject to temperature variations from solar radiation, which can cause changes in their optical characteristics and even microfractures (Tabor, 1961). Thus, assessing the thermal stability is important to ensure optimum operation of the selective surface.

The temperature of the selective coatings is directly related to its own optical and morphological characteristics (Zheng et. al., 2013). In this case, the study of the temperature gradient present in the coating due to solar heating may show evidence of how the thermal stability of a surface will behave. To estimate the temperature gradient in a coating it is important to study the heat conduction in microscale.

Several mathematic models have been developed to explain the conduction of heat at the microscale for a variety of circumstances, such as Two-Equations (Yilbas, Manson and Ali, 2018), Hyperbolic (Ai, Li, 2014; Lewandowska, 2001; Torii, Yang 2005) and the Parabolic (Lewandowska, 2001; Yilbas, Mansoor, Ali, 2018).

Hyperbolic and Two-Equations models were better to represent the thin film heat conduction, due they account the thermal transporters interaction (electrons and phonons) (Yilbas; Mansoor; Ali, 2018). Although the Parabolic model does not consider these interactions, this one presented close results to hyperbolic model.

The mathematical effort to solve hyperbolic equations is very large. On the other hand, parabolic equations have relatively simple answers to be found. So, using the parabolic model does not generate many errors in the final answer.

A good strategy to solve the Parabolic Heat Conduction Equation (PHCE) is using numeric method. Finite difference approximation can be used for this kind of application. The PHCE solution should be generalized for all kinds of thin films, so dimensionless will be used to generalize the solution.

By Lewandowska (2001) to assess theoretically changes in the temperature field inside the body, the heat conduction equation should be solved. The Following PHCE in x direction it is based on the classical Fourier law.

$$kT''(x) + \varphi = \rho c_p T'(t)$$

Where T is the Temperature in K, t is Time in s, x is range of the coating in m, k is thermal conductivity in W/mK, ρ and cp are specific mass in kg/m³ and specific heat in J/kgK respectively, and φ is the internal heat generation.

By Zubair (1996) the internal heat generation (φ) can be replaced by the term $g(x,t)$ which represents radiation absorption in molecules up to 1 µm from the surface. The $g(x,t)$ depends on the intensity of incident radiation ($I(t)$) given in W/m², reflectance (R) and absorption coefficient (μ). $g(x,t)$ can be written as:

$$g(x,t) = I(t)\,(1 - R)\,\mu\,exp(-\mu x) \qquad (2)$$

The objective of this work is to evaluate if the parabolic heat conduction model in thin films can be used as a tool to prediction of the thermal behavior of solar collector coatings. The studied model was solved numerically by using an implicit finite-difference method.

2. MATHEMATICAL MODEL

According to the microscopic behavior of the selective surfaces, the heat conduction mechanism must be compatible to the interactions that occur between the energy carriers (Yilbas; Mansoor; Ali, 2018). The microscale heat conduction parabolic model proposed by Lewandownska (2001) attend these requirements. The model is represented by Equation (3) below.

$$T'(t) = \alpha T''(x) + [I(t)\,(1 - R)\,exp(-\mu x)]\,/\rho c_p \qquad (3)$$

Where α is thermal diffusivity in m²/s, that comes from ratio between k and ρc_p. The Equation (3) was formed by union of Equations (1) and (2).

In order to obtain a solution for Equation (3), independent of characteristics of the selective surfaces, it will be dimensionless technique.

2.1 Parabolic equation in dimensionless coordinates

According to Lewandowska (2001) the dimensionless variables related to space (X), time (τ) and temperature (θ) are defined:

$$X = wx/2\alpha \qquad (4)$$

$$\tau = t/2t_k \qquad (5)$$

$$\theta = (T - T_0)/(T_m - T_0) \qquad (6)$$

Where w is the speed of heat propagation in m/s, t_k is the relaxation time of heat flux in s, T_0 and T_m are arbitrary reference temperature both given in K.

The dimensionless parabolic heat conduction equation and the new dimensionless terms are defined as:

$$\psi_0 = [I_r\,(1 - R)\mu t_k]/[\rho c_p(T_m - T_0)] \qquad (7)$$

$$\eta(\tau) = I(2t_k\tau)/I_r \qquad (8)$$

$$\beta = 2w\mu t_k \qquad (9)$$

$$4\psi_0\,\eta(\tau)\,exp(-\beta X) = 2\theta'(\tau) + \theta''(X) \qquad (10)$$

Where ψ_0 is dimensionless coefficient related to the power of the internal heat source, $\eta(\tau)$ is the dimensionless rate of energy absorbed in the medium, β is the dimensionless absorption coefficient, $I(2t_k\tau)$ is radiation incident intensity in W/m², I_r is related to an arbitrary reference radiation intensity and is also given in W/m².

Solar absorbed energy on a selective surface is a source of constant strength, therefore $\eta(\tau)$ should be equal to 1 (Torii, 2005). Equation (10) now can be replaced by:

$$4\psi_0 \exp(-\beta X) = 2\theta'(\tau) + \theta''(X) \quad (11)$$

2.2 Numeric solution

Computational methods and computational techniques to solve heat conduction problems are becoming more and more common. The finite difference is a mathematical method to solve differential equation based on the approximation of derivatives by algebraic differences, is shown to be effective and be used to solve Equation (10). The approximation of the differential equations by an algebraic system whose solution is known to approximate to the continuous problem, which is done by fragmenting domains in space and time on discrete points is called discretization (Ozisik, 1994).

Transient problems can be discretized in two different ways: explicit or implicit schemes (Ozisik, 1994). The explicit methods calculate the state of the system later than the current state of the system, the implicit methods already solving an equation involving both states (Birchall et al. 2009).

When applying the implicit finite difference method in Equation (11), we can obtain the differentiated equation:

$$\theta_i^{\,n} + 2(\Delta\tau)\psi_0 \exp(-\beta i(\Delta X)) = -((\Delta\tau)/2(\Delta X)^2)\,\theta_{i+1}^{\,n+1} + [1+2((\Delta\tau)/2(\Delta X)^2)]\,\theta_i^{\,n+1} - ((\Delta\tau)/2(\Delta X)^2)\,\theta_{i-1}^{\,n+1} \quad (12)$$

Where the subscript i denotes the grid points in the space domain and superscript n denotes the time level, $\Delta\tau$ and ΔX are time and space steps, respectively. By Torii (2005), throughout numerical calculations, the number of grids is properly selected between 1000 and 5000 to obtain a grid-independent solution, resulting in no appreciable difference between the numerical results with different grid spacing. The range of the non-dimensional thickness is between 0 and 8.

The dimensionless initial and boundary conditions to be applied to Equation 12 are as follows:

$$\theta(X, 0) = 0 \quad (13a)$$

$$\theta'(0, \tau) = 0 \quad (13b)$$

$$\theta(\infty, 0) = 0 \quad (13c)$$

3. METHODOLOGY

Initially the verification of the numerical solution was performed, these evaluations were made by comparing the analytical solutions obtained by Lewandowska (2001) and the numerical solutions by implicit finite difference obtained in this work.

After this a parametric study was performed varying the variables (ψ_0, τ and β) in order to evaluate the influence of these variables on the temperature gradient in the film. Table 1 summarizes the values of (ψ_0, τ and β) that was evaluated in this work.

Table 1. Parameters for evaluating the terms ψ_0, τ and β.

Terms	Evaluated Conditions		
	1th Evaluation	2th Evaluation	3th Evaluation
$\boldsymbol{\beta}$	0,5 1 5	1	1
$\boldsymbol{\psi_0}$	1	0,3 1 3	1
$\boldsymbol{\tau}$	1	1	1 2 3

4. RESULTS AND DISCUSSION

In the next topics, will be showed how it was done the numerical solution validation and the evaluated conditions of parameters analyzes.

4.1 numerical solution validation

Initially, it is necessary to make the numerical solution validation. This validation will be done by comparing the analytical and the numerical solution. If the solution shows a similarity, the numerical way becomes validated. Figure 1 shows the comparison between the analytical solutions obtained by Lewandowska (2001) and the finite implicit differences obtained in this work for $\tau = 1$ and $\tau = 3$ with $\beta = 1$ and $\psi_0 = 1$.

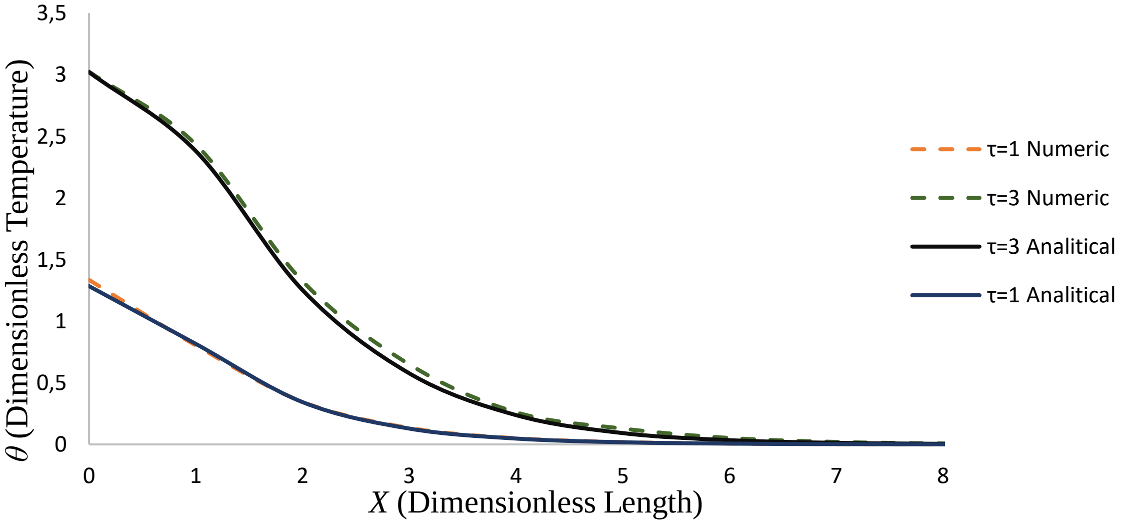

Figure 1. Numerical and analytical solution of Equation (3) for $\tau = 1$ and $\tau = 3$ with $\beta = 1$ and $\psi_0 = 1$.

By the Figure 1 it is possible to notice a great similarity between the two solutions (numerical and analytical). For $\tau = 1$ there is a small difference in the first values of X, for $\tau = 3$ there is a small difference for X ranging from 1 to 4.

The residual difference between the two solutions was applied. It was observed that for the first X values the residual difference is high, which happens due to the accumulation of truncation error disregarding the calculation. The residual difference decreases with increasing X. Therefore the numerical solution proposed by this work is validated for further heat conduction analysis on selective surfaces.

4.2 Model parameters evaluation

The resolution of Equation 11 with the conditions presented in Table 1 shows the film temperature distribution curves.

Figure 2 indicates the influence of the parameter τ on the temperature distribution in the coating as described in Table 1. Was fixed $\psi_0 = 1$ and $\beta = 1$ with $\tau = 1$, 2 and 3.

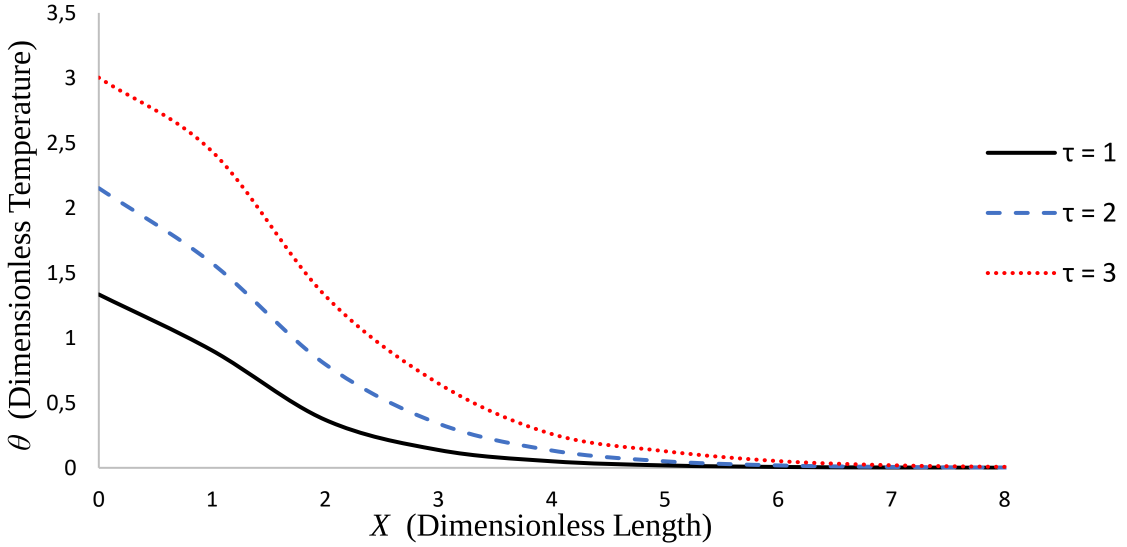

Figure 2. Temperature distribution for $\beta = 1$, $\psi_0 = 1$ with $\tau = 1$, 2 and 3.

In Figure 14 it can be observed that increasing the value of τ causes an increase in the temperature of the edge receiving solar radiation. The following points of the mesh also have their temperature increased but in smaller quantity. Over time (τ increase) the curve tends to be steeper due to the resistance of the coating to temperature diffusion. This is a natural effect of a selective surface exposed to solar radiation.

Figure 3 shows the influence of the parameter ψ_0 on the temperature distribution in the coating, as described in Table 1. Was fixed τ = 1 and β = 1 with ψ_0 = 0.3, 1 and 3.

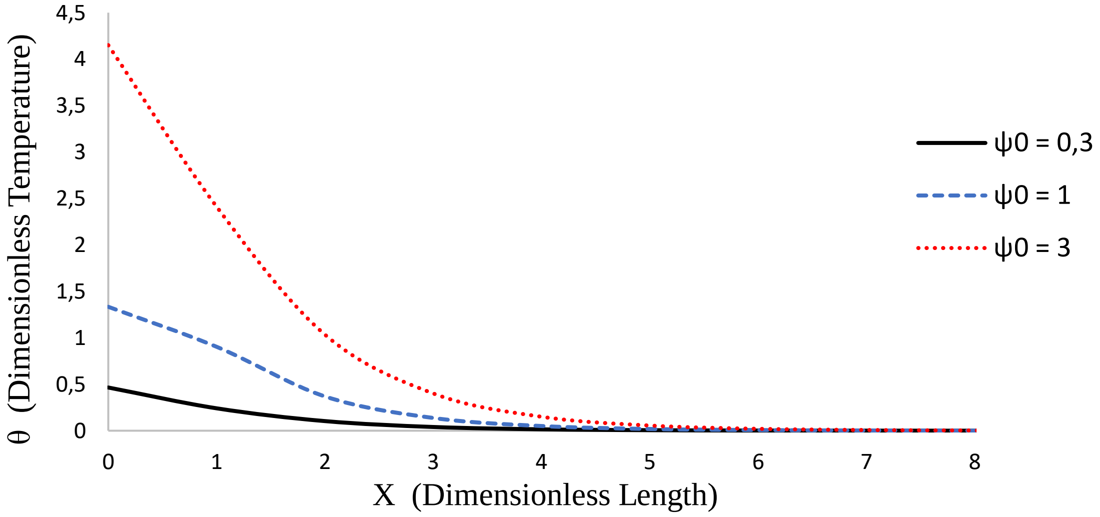

Figure 3. Temperature distribution for β = 1, τ = 1 with ψ_0 = 0.3, 1 and 3.

Increasing ψ_0 produces an increase in the rate of absorption of solar radiation by the coating, as shown in Figure 3. This is represented by the temperature gain shown by the three curves. Another important point is that the increase in de ψ_0 will also cause an increase in the temperature reduction rate per coating length. Similarly, the reduction in ψ_0 indicates that the coating is absorbing less radiation.

Figure 4 shows the influence of the parameter β on the temperature distribution in the coatings as described on Table 1. Was fixed τ = 1 and ψ_0 = 1 with β = 0.5, 1 and 5.

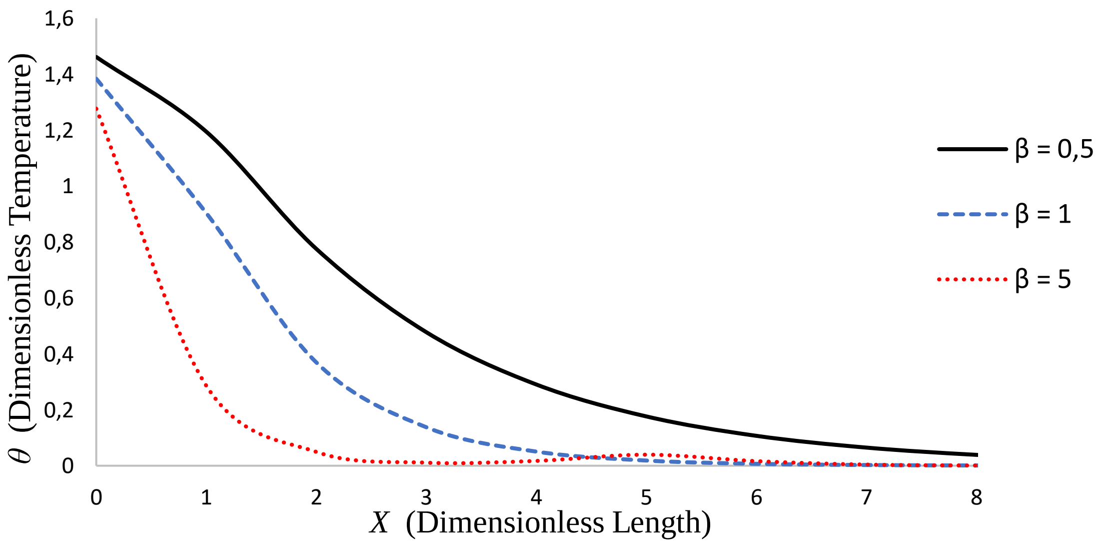

Figure 4. Temperature distribution for ψ_0= 1, τ = 1 with β = 0.5, 1 and 5.

Looking at Figure 4 it is possible to notice that the temperature distribution for β equal to 0.5 has lower slope when compared to the β curve equal to 5. This is because with the increase of β, the energy tends to be absorbed faster by the coating. This implies that coatings which have high β values cause large differences in temperature inside.

5. CONCLUSION

With the obtained results it was possible to understand how the parameters (τ, β and ψ_0) can influence the thermal behavior of the film, whose understanding is fundamental to evaluate the thermal stability of a selective surface. This is because the fast temperature change will produce thermal stresses inside the coating and may produce microcracks and increase optical properties changes.

For the increase in τ (over time) there is a gain in the rate of change in temperature as shown in Figure 15. Therefore, according to the parabolic heat conduction model, over time will naturally increase the thermal stresses of the coating, it is thus an effect that cannot be controlled.

The increment of ψ_0 indicated a temperature increase at the same time as the growth in the temperature reduction rate increased. However, the reduction in ψ_0 will indicate that the coating is absorbing less radiation. Thus, the ideal becomes to use materials in the manufacture of selective coatings that can optimize the ψ_0 value making it a coating that can heat at high temperatures but not produce large temperature differences.

The increase in β resulted in a higher rate of temperature reduction per length. however, coatings which have high β cause large temperature differences inside increasing thermal stresses. Thus, the use of larger β will lead to a potential chance of microcracking reducing the coating efficiency. Therefore, the choice of beta is also critical to ensure the thermal stability of the film.

Consequently, the parameters evaluated showed a consistent response to what they represent physically in the model. This allows adapting the control parameters to the type of coating that is evaluated to use them as a tool for the design of selective surfaces for their thermal behavior.

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7. RESPONSIBILITY NOTICE

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