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A NUMERICAL STUDY OF THE SIMULTANEOUS NATURAL CONVECTIVE HEAT TRANSFER FROM THE TOP AND BOTTOM SURFACES OF A THIN HORIZONTAL PLATE HAVING A TRAPEZOIDAL WAVY SURFACE

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Abstract. A numerical study of natural convective heat transfer from a thin, two-sided, two-dimensional horizontal plate having a uniform surface temperature has been undertaken. The surface shape is wavy, and attention has been given to the case where the surface waves have a trapezoidal shape with a constant height. The temperature of the top and bottom plate surfaces, which are the same, is higher than the temperature of the surrounding fluid. Here, the conditions considered are such that laminar flow, transitional flow, and turbulent flow can occur over the plate. The flow around the entire plate has been considered and the heat transfer from both the top and bottom surfaces of the plate have been studied. The flow has been assumed to be two-dimensional and steady and the Boussinesq approach has been adopted. The k -epsilon turbulence model has been used with full account being taken of buoyancy force effects. The commercial CFD solver ANSYS FLUENT[®] has been used to obtain the solution. The mean heat transfer rates from the top and bottom surfaces of the plate has been expressed in terms of Nusselt numbers based on the plate width. Results have been obtained only for a Prandtl number of 0.71, i.e., essentially the value for air. Results for a wide range of dimensionless surface wave heights and the ratio between the dimensionless wave larger base and the dimensionless wave minor base have been obtained.

Keywords: natural convective heat transfer, wavy surface, trapezoidal waves, heat transfer enhancement.

1. INTRODUCTION

Natural convective heat transfer occurs in many practical situations and remains an area of considerable basic and applied interest. In the present article attention will be restricted to external natural convective flows, that is, flow situations in which there are no constraining boundary surfaces near enough to the surface being considered to have any significant influence on the natural convective flow over this surface. Increasing the heat transfer rate in a given situation involving natural convective flows is often difficult to accomplish. Using surfaces that have a wavy surface is one method of attempting to enhance natural convective heat transfer rates.

The enhancement of the heat transfer rate produced by using a wavy surface arises from the increase in the surface area exposed to the fluid to which the heat is being transferred and, in some cases, to the changes in the near surface flow produced by the presence of the surface waves. The total enhancement of the heat transfer rate will depend on the shape and relative size of the surface waves. Many wavy shapes have been considered in past studies but the most common shapes considered remain rectangular, triangular and sinusoidal waves. The enhancement of the heat transfer rate produced by using a wavy surface will also depend on the flow situation being considered, for example, flow over a plane surface or flow over a cylinder, and on the thermal boundary conditions at the surface. The two surface boundary conditions most commonly considered being those in which there is a uniform temperature over the surface and those in which there is a uniform heat flux over the surface. Another factor that influences the natural convective heat transfer rate from a surface is its orientation, that is, is it horizontal or is it vertical, or is it inclined to the vertical and whether, when inclined, it is facing upward or downward (Oosthuizen, 2016).

While there have been some limited previous studies of natural convective heat transfer from thin, two-sided horizontal plates, these studies have mainly considered only the case where the plate is flat (non-wavy) and where the flow over the plate is laminar. Here, the conditions considered are such that laminar flow, transitional flow, and turbulent flow can occur over the plate. Numerical studies of heat transfer from a horizontal surface having rectangular waves and

triangular waves for conditions under which laminar, transitional, and turbulent flow exist are described in (Oosthuizen, 2016a) and (Oosthuizen, 2016b). Other studies of natural convective heat transfer from horizontal wavy surfaces are described in (Prétot *et al.*, 2000), (Prétot *et al.*, 2003), (Siddiqa and Hossain, 2013) and (Siddiqa *et al.*, 2015). In all of these studies, the natural convective heat transfer rate was obtained from a thin, one-sided, two-dimensional horizontal wavy plate having a uniform surface temperature. Studies of natural convective heat transfer from a thin, two-sided, two-dimensional wavy plates having a uniform surface temperature are described in (Oliveira and Oosthuizen, 2018) and (Oliveira and Oosthuizen, 2019).

The purpose of the present article is to undertake a numerical study of natural convective heat transfer from a thin, two-sided, two-dimensional horizontal plate having a uniform surface temperature. The surface shape is wavy, and attention has been given to the case where the surface waves have a trapezoidal shape with constant height. The importance of the present work is related to the increase of the natural convective heat transfer rate in situations where the implementation of a forced convection would be difficult to apply or even impossible. This may be the case, for example, when cooling a circuit board assembly or in cases where the thermal management of electronic components must be performed in more detail.

2. PHYSICAL SITUATION

The physical situation analyzed in this study can be visualized in Fig. 1:

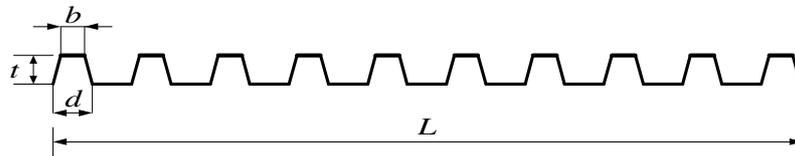


Figure 1. Horizontal surface with nineteen trapezoidal waves.

The situation under analysis consists of a thin, two-sided, two-dimensional horizontal plate having a uniform surface temperature T_w . The surface shape is wavy and attention has been given to the case where the surface waves have a trapezoidal shape with constant height t , larger base with width d and minor base with width b . The nineteen trapezoidal waves are equally spaced. The plate is in contact with a surrounding fluid at constant temperature T_f . For a heated surface, $T_w > T_f$ and both the top and bottom surfaces will exchange energy with the surrounding fluid by natural convection. The horizontal surface has unit width L and unit depth w . The purpose of this study is to calculate the heat transfer rate by natural convection between the heated surface and the surrounding fluid. Due to symmetry, only half of the surface has been analyzed. The heat transfer rates have been expressed per unit projected (i.e., base) area. This was done in order to clearly illustrate the effect waves on the heat transfer rate. The mean Nusselt number related to the natural convective heat transfer from the top and bottom surfaces can be calculated using the Newton law of cooling based on the surface projected area and the definition of the mean Nusselt number based on the width of the surface, that is:

$$\overline{\text{Nu}}_{L,\text{top/bottom}} = \frac{q_{\text{top/bottom}} L}{k(Lw)(T_w - T_f)} \quad (1)$$

where $\overline{\text{Nu}}_L$ is the mean Nusselt number based on L and on the mean heat transfer rate, q is the mean heat transfer rate, k is the thermal conductivity of the fluid and w is the surface depth considered which is here taken as unity.

3. SOLUTION PROCEDURE

In obtaining the numerical results discussed above the mean flow has been assumed to be steady. The Boussinesq approximation has been used, i.e., fluid properties have been assumed to be constant except for the density change with temperature in momentum equation. This gives rise to the buoyancy forces and the density change being assumed to be proportional to the temperature change. Radiant heat transfer effects have been neglected. Allowance has been made for the possibility that turbulent flow can occur in the system. In order to deal with this the basic *k-epsilon* turbulence model with standard wall functions and with full account being taken of buoyancy force effects has been used.

The governing equations subject to the boundary conditions have been solved numerically using the commercial CFD solver ANSYS FLUENT[®]. The numerical approach used here in order to determine when turbulence develops which involves solving the Reynolds averaged governing equations together with a turbulence model, in which the effects of buoyancy forces are taken into account, for all conditions considered and then monitoring the results obtained with increasing Rayleigh numbers to determine when significant turbulence effects develop. This approach has been used quite

extensively in the study of forced convective flows, e.g., see (Schmidt and Patankar, 1991) and (Zheng *et al.*, 1998). The solutions presented in this work all basically have the following parameters:

1. The Rayleigh number, Ra_L , based on the reference length scale L of the heated surface and the difference between the temperature of the heated surface, T_w , and the temperature of the undisturbed fluid well away from the system, T_f , i.e.:

$$Ra_L = \frac{g\beta(T_w - T_f)L^3}{\nu\alpha} \quad (2)$$

2. The dimensionless width of the larger base, $D = d/L$;
3. The dimensionless width of the minor base, $B = b/L$;
4. The dimensionless height of the surface waves, $H = t/L$ and
5. The Prandtl number, Pr .

In Eq. (2), Ra_L is the Rayleigh number based on L , g is the gravitational acceleration, β is the bulk coefficient of thermal expansion, L is the width of the heated surface, ν is the kinematic viscosity of the fluid and α is the thermal diffusivity of the fluid. Results have only been obtained for a Prandtl number of 0.71, i.e., effectively the value for air at 300 K. All the results obtained for the wave surfaces were compared with results obtained for the same surface without waves, in terms of the mean Nusselt numbers.

4. RESULTS AND DISCUSSION

Numerical results were obtained for Rayleigh numbers varying between 10^6 and 10^{14} . Results for the trapezoidal wave case were obtained for ratios between D and B equal of 1.25, 1.5, 1.75 and 2 and for H values equal of 0.02, 0.05 and 0.08. The surface temperature in both sides of the surface is equal of 310 K and the fluid temperature is equal of 290 K. All results obtained for the trapezoidal case were compared to that from a plane (non-wavy) horizontal surface. Variations of the mean top and mean bottom Nusselt number with the Rayleigh number for $H = 0.02$, $H = 0.05$, $H = 0.08$ and $D/B = 1.25, 1.5, 1.75, 2$ are shown in Figs. 2 to 4, respectively:

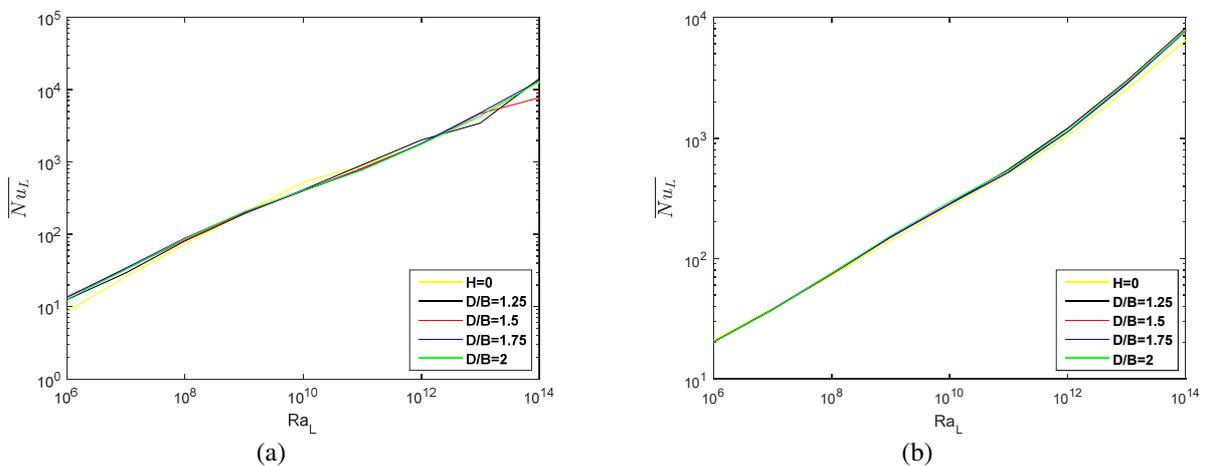


Figure 2. Variations of mean Nusselt number with Rayleigh number for $H = 0.02$. (a) top. (b) bottom.

According to Figs. 2 to 4, it can be seen that the mean top Nusselt number and the mean bottom Nusselt number increases with the increase of the Rayleigh number. Also, it can be seen that the mean top Nusselt number and the mean bottom Nusselt number generally increases with the increasing of D/B . A trapezoidal wavy height of 0 corresponds to the case of a flat non-wavy (plane) heated surface. Results for $H = 0$ were plotted for comparison and to show the influence of the waves on the mean heat transfer rate both for top and bottom surfaces. It can be noted that the presence of waves generally causes an increase in the mean Nusselt number, both for top and bottom surfaces. This is further illustrated by the results presented in Figs. 5 to 8, which show typical variations of the mean top Nusselt number and of the mean bottom Nusselt number with trapezoidal wave height for various values of the Rayleigh number and for $D/B = 1.25, 1.5, 1.75$ and 2, respectively.

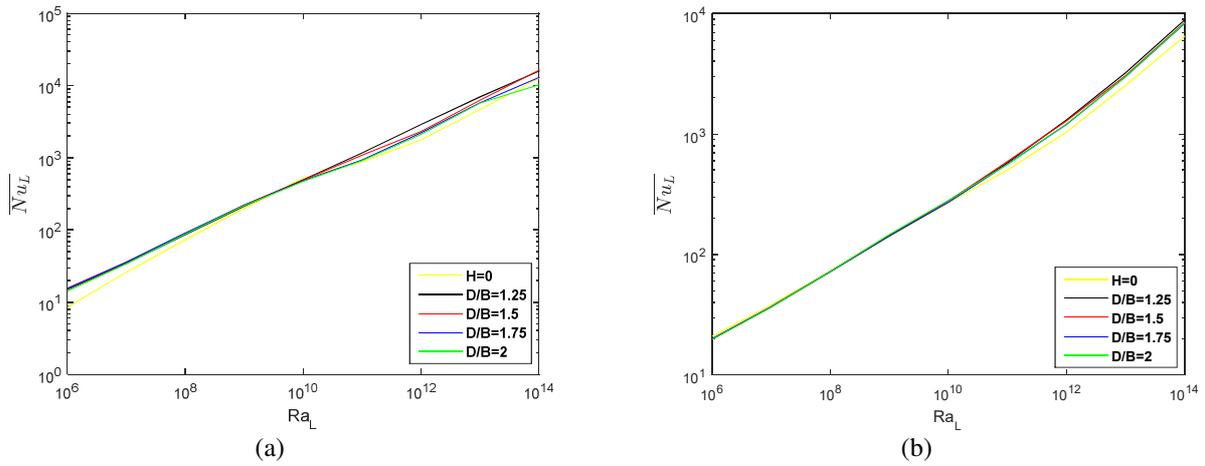


Figure 3. Variations of mean Nusselt number with Rayleigh number for $H = 0.05$. (a) top. (b) bottom.

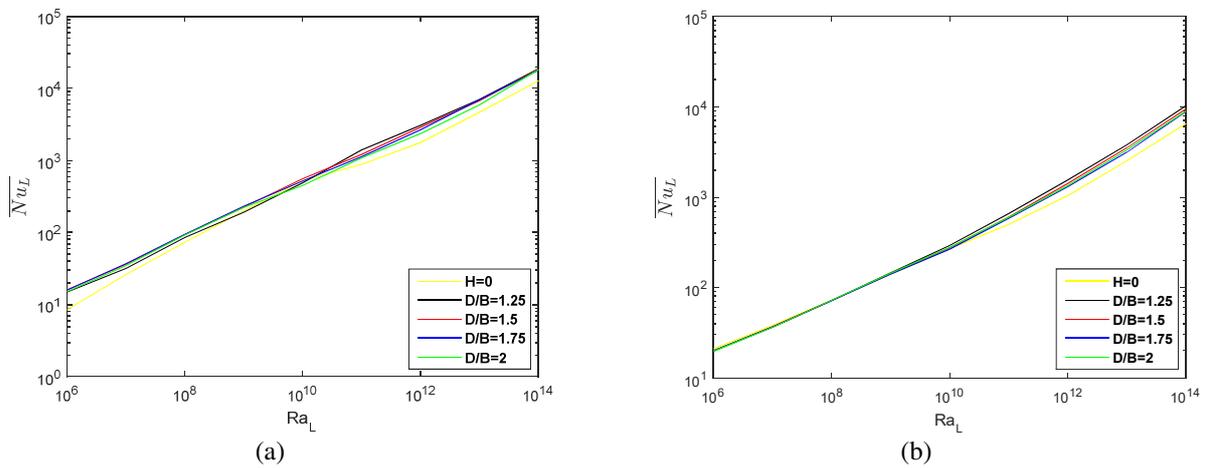


Figure 4. Variations of mean Nusselt number with Rayleigh number for $H = 0.08$. (a) top. (b) bottom.

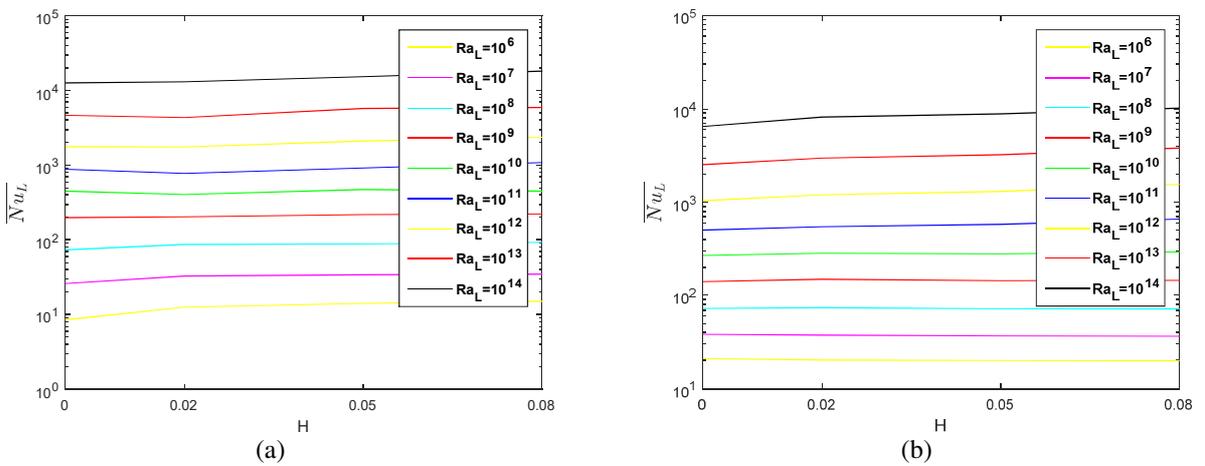


Figure 5. Variations of mean Nusselt number with dimensionless wavy height for $D/B = 1.25$. (a) top. (b) bottom.

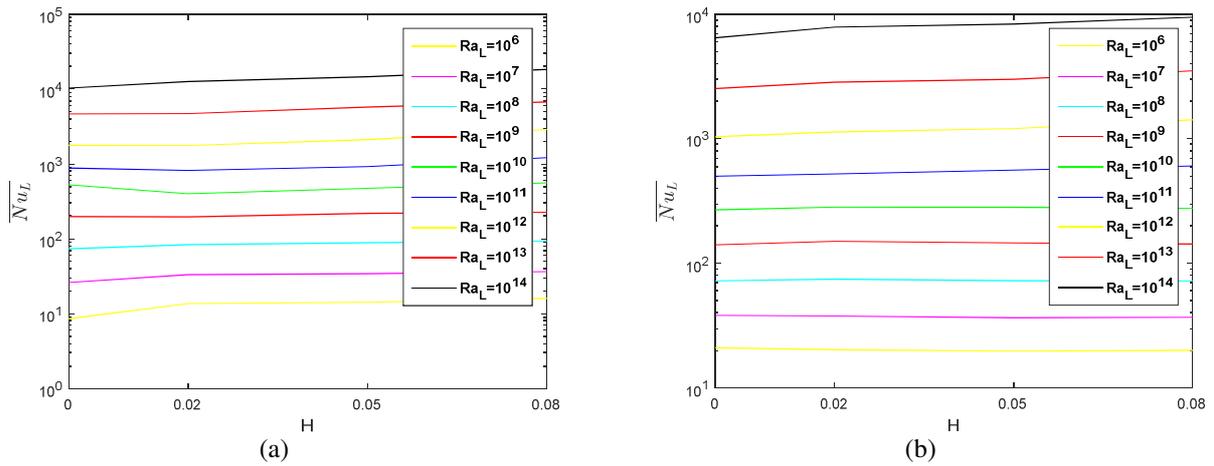


Figure 6. Variations of mean Nusselt number with dimensionless wavy height for $D/B = 1.5$. (a) top. (b) bottom.

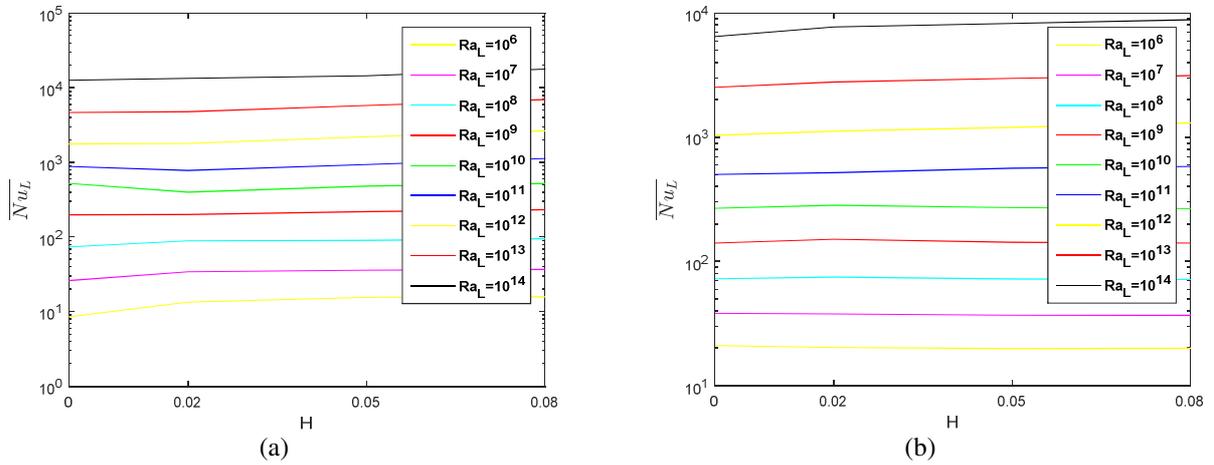


Figure 7. Variations of mean Nusselt number with dimensionless wavy height for $D/B = 1.75$. (a) top. (b) bottom.

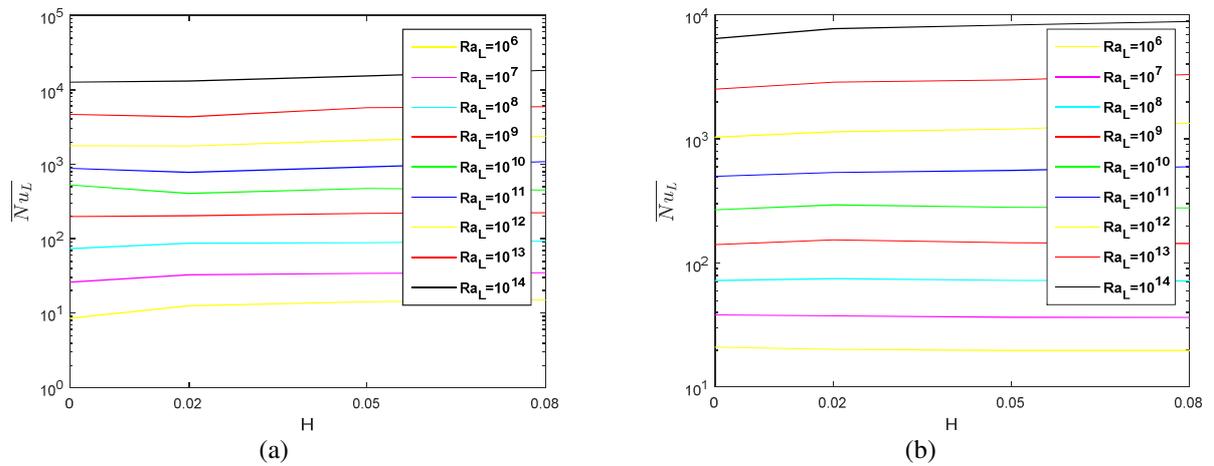


Figure 8. Variations of mean Nusselt number with dimensionless wavy height for $D/B = 2$. (a) top. (b) bottom.

Table 1 shows the percentual variations of the mean top Nusselt number and of the mean bottom Nusselt number with H for $D/B = 1.25, 1.5, 1.75$ and 2 . These results were calculated using the mean Nusselt number values for $H = 0$ and $H = 0.08$. In general, for low Rayleigh numbers there is a decrease in the mean bottom Nusselt numbers. However, for

higher Rayleigh numbers, there is an increase in the mean Nusselt numbers, both for top and bottom surfaces. These results show that, in general, there is an enhancement in the natural convective heat transfer in trapezoidal wavy surfaces when compared to a non-wavy surface. Variations of the mean top and mean bottom Nusselt number with D/B for various values of the Rayleigh number and for $H = 0.02$, $H = 0.05$ and $H = 0.08$ are shown in Figs. 9 to 11, respectively:

Table 1. Percentual variation of the $\overline{Nu}_{L,top/bottom}$ with H .

Ra_L	$D/B=1.25$		$D/B=1.5$		$D/B=1.75$		$D/B=2$	
	$\overline{Nu}_{L,top}$	$\overline{Nu}_{L,bottom}$	$\overline{Nu}_{L,top}$	$\overline{Nu}_{L,bottom}$	$\overline{Nu}_{L,top}$	$\overline{Nu}_{L,bottom}$	$\overline{Nu}_{L,top}$	$\overline{Nu}_{L,bottom}$
10^6	75.21 %	-4.94 %	87.55 %	-4.61 %	84.91 %	-4.89 %	77.11 %	-5.99 %
10^7	22.13 %	-4.43 %	39.62 %	-3.49 %	41.33 %	-3.72 %	33.96 %	-4.62 %
10^8	16.20 %	-1.13 %	26.62 %	-0.30 %	28.59 %	-0.98 %	25.91 %	-0.61 %
10^9	-4.04 %	3.06 %	13.83 %	1.56 %	16.90 %	-0.26 %	12.31 %	2.13 %
10^{10}	-6.87 %	8.86 %	6.15 %	2.63 %	-0.88 %	-0.79 %	-14.50 %	3.58 %
10^{11}	57.87 %	31.24 %	37.55 %	20.38 %	28.44 %	16.17 %	23.03 %	19.40 %
10^{12}	73.36 %	49.68 %	62.65 %	36.83 %	49.72 %	25.86 %	33.47 %	29.06 %
10^{13}	50.48 %	50.46 %	44.56 %	38.87 %	49.13 %	23.93 %	26.95 %	31.37 %
10^{14}	47.97 %	57.72 %	44.84 %	46.17 %	40.77 %	36.71 %	43.82 %	37.40 %

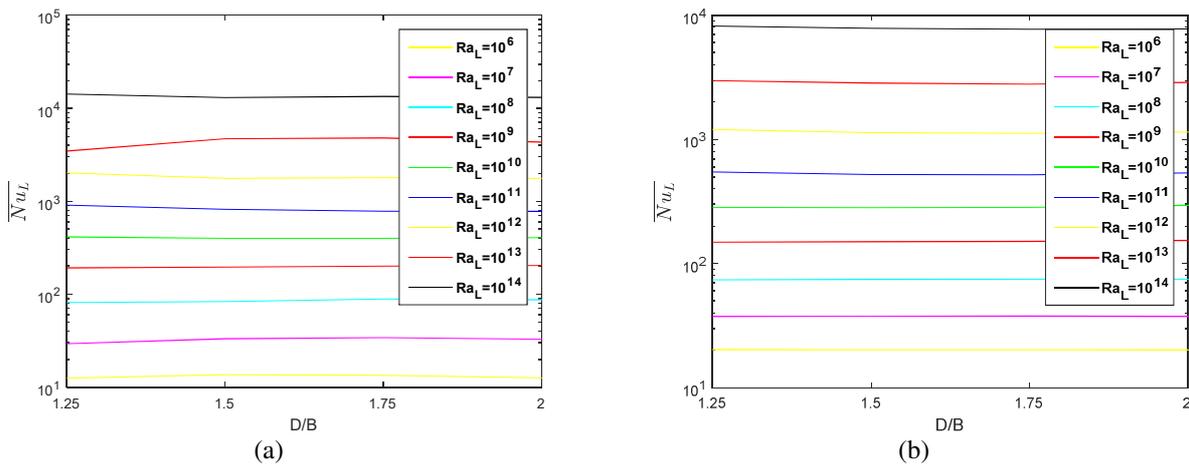


Figure 9. Variations of mean Nusselt number with D/B for $H = 0.02$. (a) top. (b) bottom.

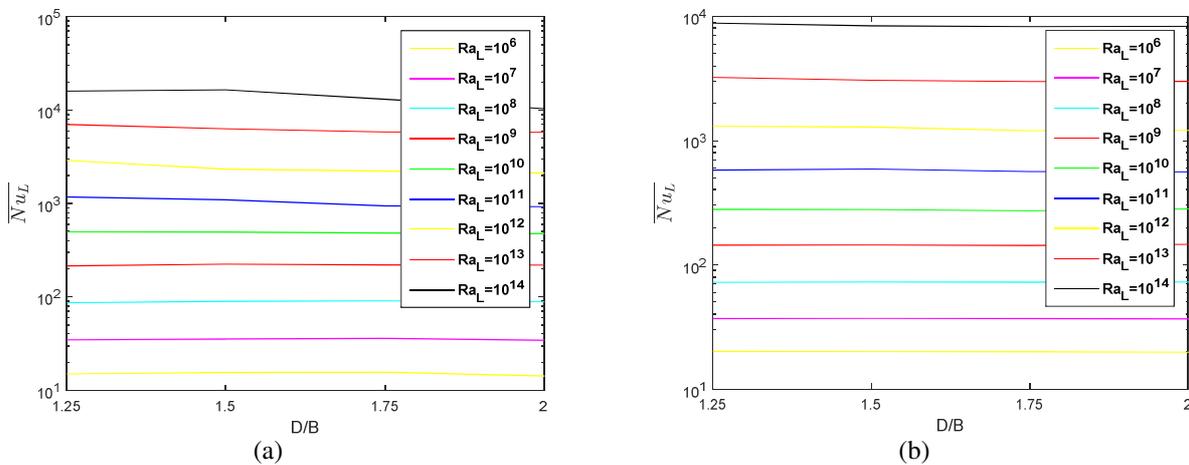


Figure 10. Variations of mean Nusselt number with D/B for $H = 0.05$. (a) top. (b) bottom.

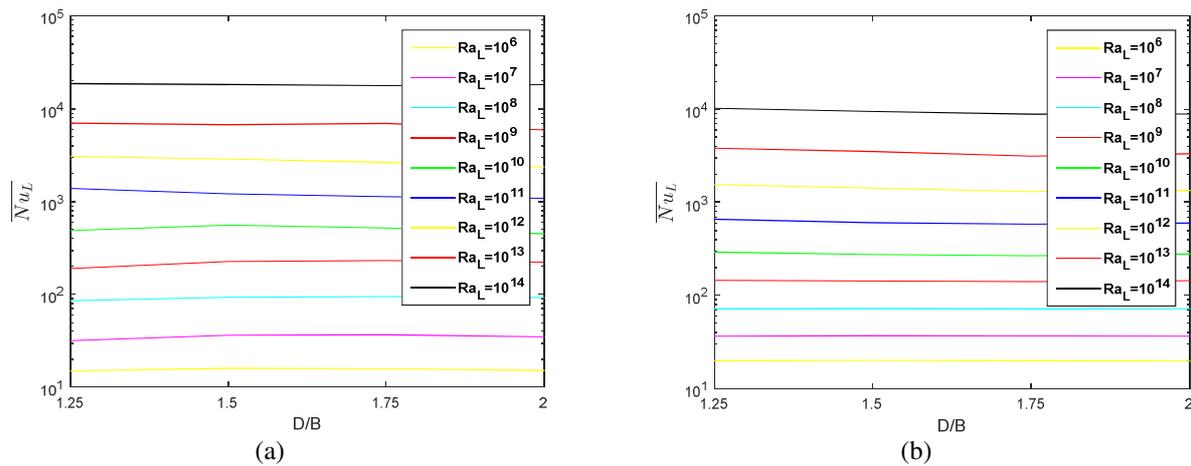


Figure 11. Variations of mean Nusselt number with D/B for $H = 0.08$. (a) top. (b) bottom.

Table 2 shows the percentual variations of the mean top Nusselt number and of the mean bottom Nusselt number with D/B for $H = 0.02$, 0.05 and 0.08 . These results were calculated using the mean Nusselt number values for $D/B = 1.25$ and $D/B = 2$. In general, the mean top Nusselt number increases with the increase of D/B and the mean bottom Nusselt number decreases with the increase of D/B . Besides, the increase in the mean top Nusselt number is higher for lower Rayleigh numbers than for higher Rayleigh numbers. In this manner, the effect of waves was more relevant in terms of heat transfer enhancement on the top surface than on the bottom surface.

Table 2. Percentual variation of the $\overline{Nu}_{L,top/bottom}$ with D/B .

Ra_L	$H = 0.02$		$H = 0.05$		$H = 0.08$	
	$\overline{Nu}_{L,top}$	$\overline{Nu}_{L,bottom}$	$\overline{Nu}_{L,top}$	$\overline{Nu}_{L,bottom}$	$\overline{Nu}_{L,top}$	$\overline{Nu}_{L,bottom}$
10^6	47.46 %	-0.52 %	66.03 %	-1.36 %	77.11 %	-1.11 %
10^7	25.84 %	0.13 %	31.15 %	-0.61 %	33.96 %	-0.20 %
10^8	18.23 %	1.23 %	20.15 %	0.65 %	25.91 %	0.52 %
10^9	2.51 %	3.19 %	10.02 %	1.52 %	12.31 %	-0.91 %
10^{10}	-22.81 %	3.83 %	-9.89 %	0.83 %	-14.50 %	-4.85 %
10^{11}	-11.85 %	-1.69 %	4.32 %	-3.28 %	23.03 %	-9.02 %
10^{12}	-0.50 %	-4.57 %	19.32 %	-7.54 %	33.47 %	-13.77 %
10^{13}	-7.06 %	-3.04 %	23.41 %	-7.04 %	26.95 %	-12.69 %
10^{14}	3.55 %	-5.04 %	-18.43 %	-5.85 %	43.82 %	-12.88 %

Correlations for the mean top Nusselt number and mean bottom Nusselt number as a function of the Rayleigh number, the dimensionless surface wave height and ratio between the dimensionless wave larger base and the dimensionless wave minor base are shown in Figs. 12 and 13, respectively. These correlations were obtained using a multivariable linear regression through the least square method. The Rayleigh number interval for each correlation it was determined seeking the smallest mean squared error between the simulated data and the multivariable linear regression. In both figures results for Rayleigh number equal of 10^{14} were excluded due to the high deviation in the mean squared error in the considered interval. Besides, it can be seen that the mean squared error increases with the increase of the Rayleigh number. A general equation for the mean Nusselt number can be written as:

$$\overline{Nu}_L = C Ra_L^m \left(\frac{D}{B} \right)^n H^p \quad (3)$$

where the constants C , m , n e p were obtained through the multivariable linear regression. Tables 3 and 4 shown these constants for the top surface and for the bottom surface, respectively:

Table 3. Constants C, m, n e p for the top surface.

Ra_L	C	m	n	p
$10^6 \leq Ra_L \leq 10^9$	0.079	0.391	0.109	0.078
$10^9 \leq Ra_L \leq 10^{12}$	0.353	0.343	-0.225	0.198
$10^{12} \leq Ra_L \leq 10^{13}$	0.138	0.391	-0.283	0.307

Table 4. Constants C, m, n e p for the bottom surface.

Ra_L	C	m	n	p
$10^6 \leq Ra_L \leq 10^{10}$	0.335	0.289	-0.002	-0.025
$10^{10} \leq Ra_L \leq 10^{13}$	0.122	0.348	-0.136	0.083

For the mean top Nusselt number the following correlations and the mean squared error (MSE) were obtained:

$$\overline{Nu}_{L,top} = 0.079 Ra_L^{0.391} \left(\frac{D}{B}\right)^{0.109} H^{0.078} \quad 10^6 \leq Ra_L \leq 10^9 \quad (MSE = 5.018) \quad (4)$$

$$\overline{Nu}_{L,top} = 0.353 Ra_L^{0.343} \left(\frac{D}{B}\right)^{-0.225} H^{0.198} \quad 10^9 \leq Ra_L \leq 10^{12} \quad (MSE = 116.795) \quad (5)$$

$$\overline{Nu}_{L,top} = 0.138 Ra_L^{0.391} \left(\frac{D}{B}\right)^{-0.283} H^{0.307} \quad 10^{12} \leq Ra_L \leq 10^{13} \quad (MSE = 401.798) \quad (6)$$

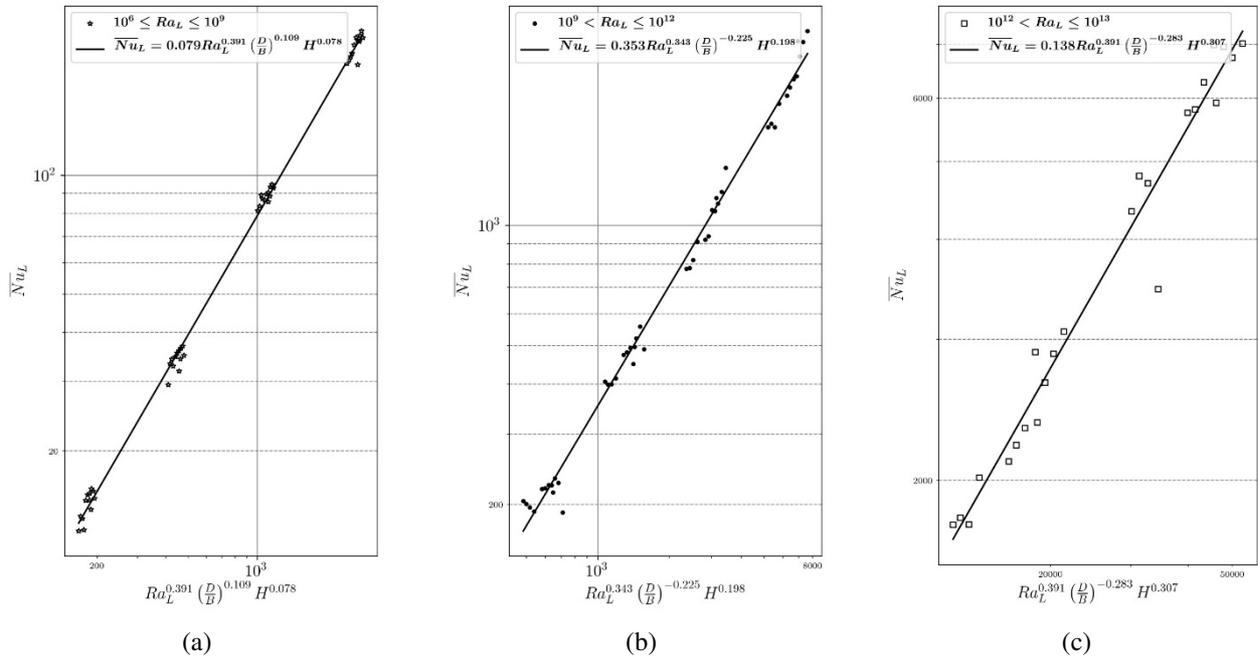


Figure 12. Correlation for the mean top Nusselt number as function of the Rayleigh number, the dimensionless surface wave height and ratio between the dimensionless wave larger base and the dimensionless wave minor base.

For the mean bottom Nusselt number the following correlations and the mean squared error (MSE) were obtained:

$$\overline{Nu}_{L,bottom} = 0.335 Ra_L^{0.289} \left(\frac{D}{B}\right)^{-0.002} H^{-0.025} \quad 10^6 \leq Ra_L \leq 10^{10} \quad (MSE = 3.514) \quad (7)$$

$$\overline{Nu}_{L,\text{bottom}} = 0.122 Ra_L^{0.348} \left(\frac{D}{B} \right)^{-0.136} H^{0.083} \quad 10^{10} \leq Ra_L \leq 10^{13} \quad (MSE = 129.371) \quad (8)$$

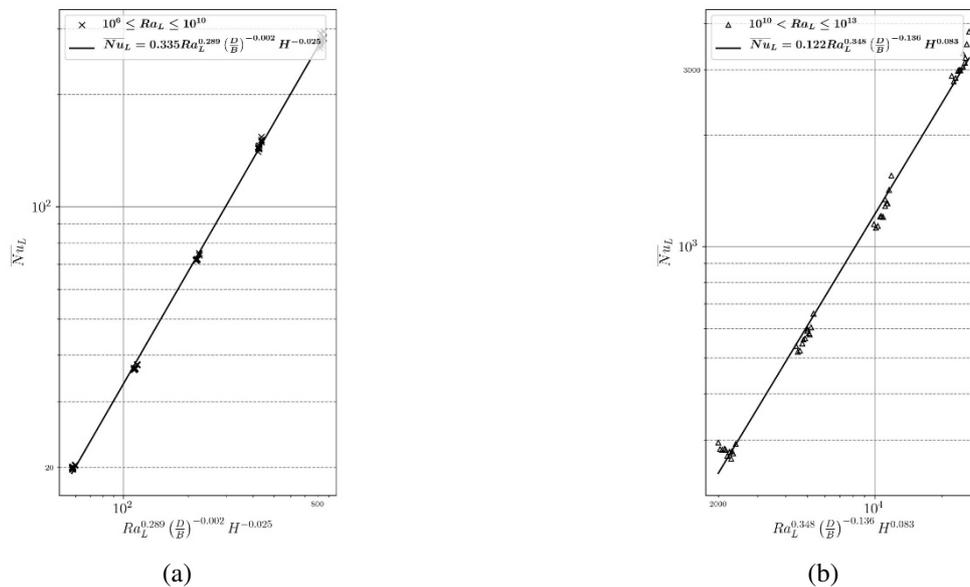


Figure 13. Correlation for the mean bottom Nusselt number as function of the Rayleigh number, the dimensionless surface wave height and ratio between the dimensionless wave larger base and the dimensionless wave minor base.

5. CONCLUSIONS

A numerical study of natural convective heat transfer from a thin, two-sided, two-dimensional horizontal plate having a uniform surface temperature has been undertaken. The surface shape is wavy, and attention has been given to the case where the surface waves have a trapezoidal shape with a constant height. Some conclusions were found after the numerical calculations:

1. It can be noted that the presence of wavy shapes generally causes an increase in the mean Nusselt number, both for top and bottom surfaces.
2. In general, for low Rayleigh numbers there is a decrease in the mean bottom Nusselt numbers. However, for higher Rayleigh numbers, there is an increase in the mean Nusselt numbers, both for top and bottom surfaces. These results show that, in general, there is an enhancement in the natural convective heat transfer in trapezoidal wavy surfaces when compared to a non-wavy surface.
3. In general, the mean top Nusselt number increases with the increase of D/B and the mean bottom Nusselt number decreases with the increase of D/B . Besides, the increase in the mean top Nusselt number is higher for lower Rayleigh numbers than for higher Rayleigh numbers.

Future studies include the influence of the trapezoidal wave number and effects of the variable height of the trapezoidal waves along the horizontal surface width on the natural convective heat transfer from the surface. Besides, all these features and parameters analyzed in this work can be studied using a boundary condition of uniform heat flux over the surface.

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7. RESPONSIBILITY NOTICE

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